

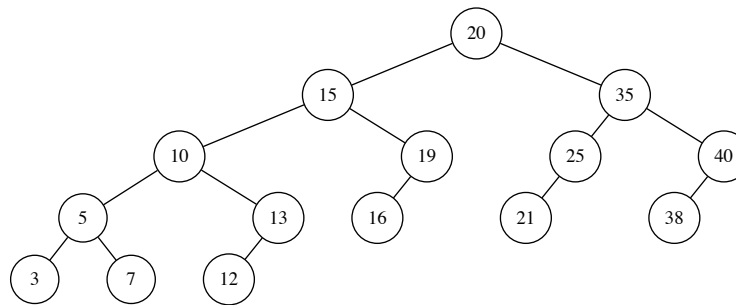
Assignment #2
Matt Langlois - 7731813
October 25

Question 1

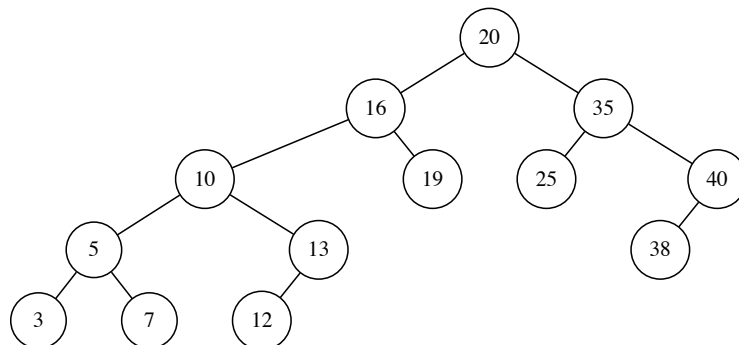
- a) 20, 15, 10, 5, 3, 7, 13, 12, 19, 16, 35, 25, 40, 38
- b) 3, 7, 5, 12, 13, 10, 16, 19, 15, 25, 38, 40, 35, 20
- c) Searches for the largest value in a binary tree.

```
findMax(Node n) {  
    if (n.hasRightChild()) {  
        return findMax(n.rightChild());  
    }  
    return n;  
}
```

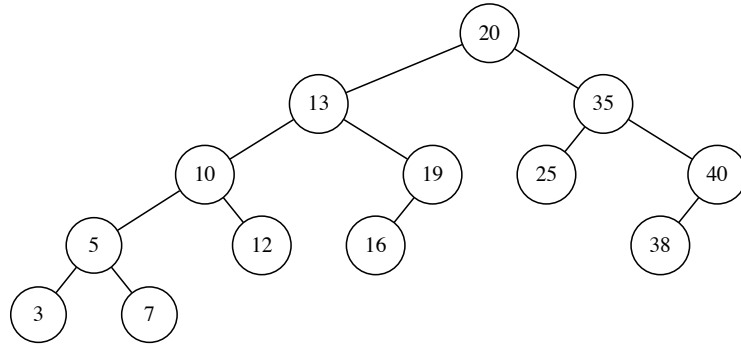
- d) *Insert element:* 21



- e) Case 1: Replace 15 with the left most node of the right subtree. Then remove the leftmost node of the right subtree.

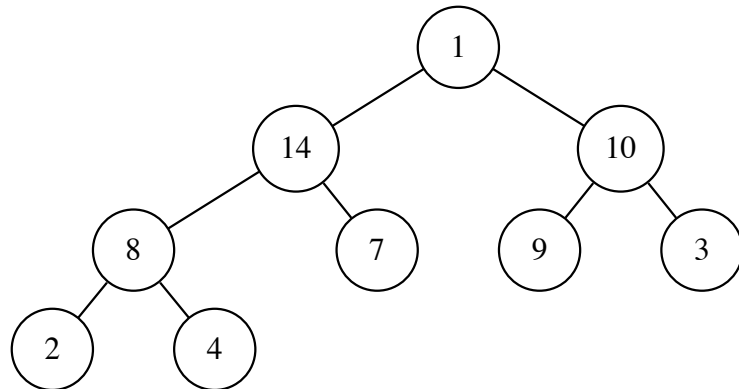
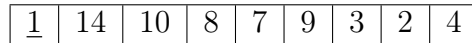


Case 2: Replace 15 with the right most node of the left subtree. Any children of the right most node become children of the right most node's parent.

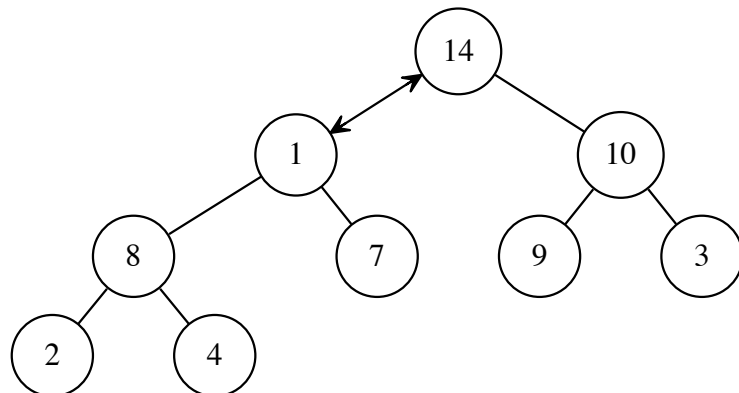
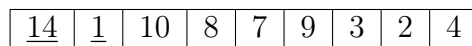


Question 2

a) Step 1: Replace node with the last node in the heap

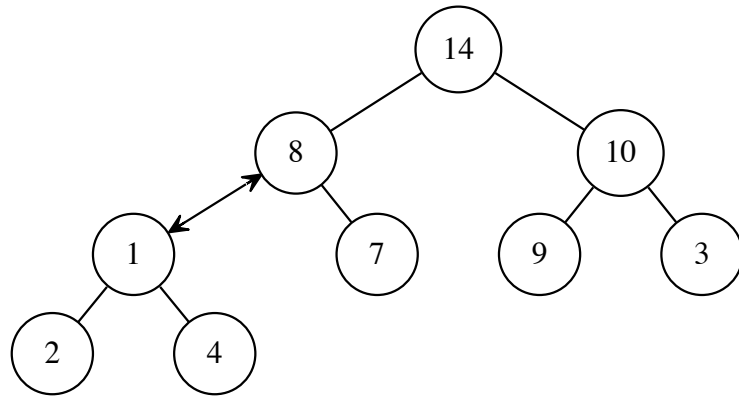


Step 2: Downheap while the children are larger



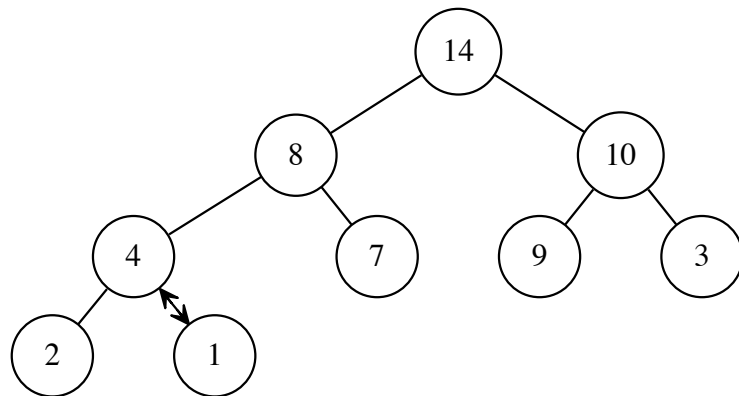
Step 3: Downheap while the children are larger

14	<u>8</u>	10	<u>1</u>	7	9	3	2	4
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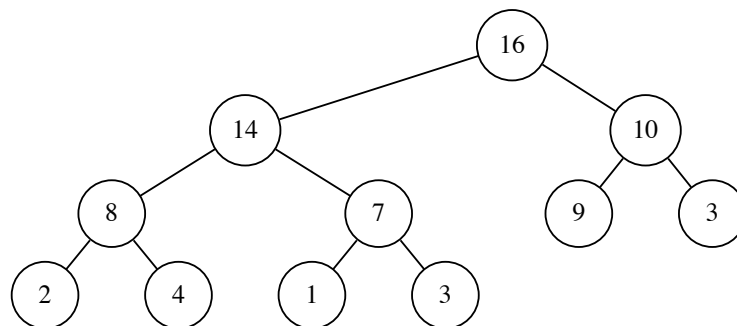
Step 4: Downheap to become a leaf node

14	8	10	<u>4</u>	7	9	3	2	<u>1</u>
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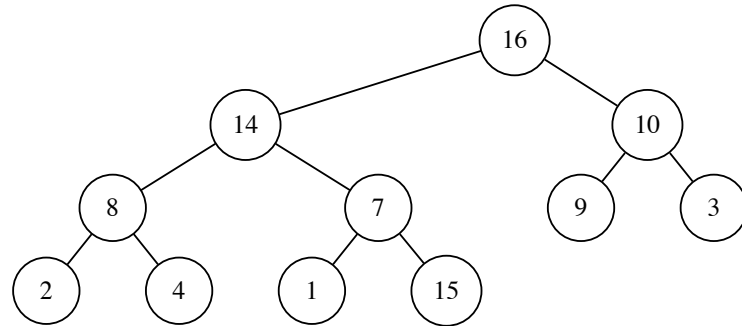
- b) Insert element 3 at the left most position on the empty row. No further changes are required as the $7 \geq 3$ property of the max heap is satisfied

16	14	10	8	7	9	3	2	4	1	<u>3</u>
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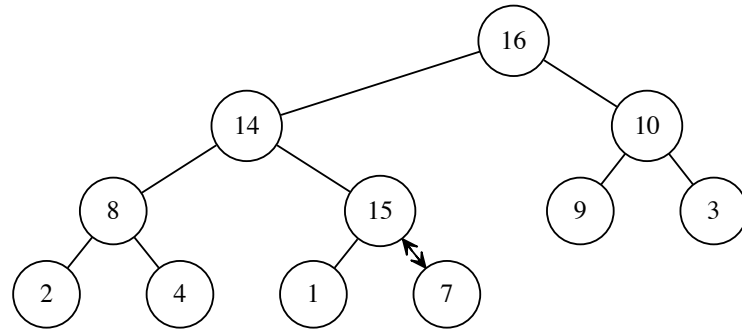
c) Step 1: Insert at left most node in the empty row

16	14	10	8	7	9	3	2	4	1	<u>15</u>
----	----	----	---	---	---	---	---	---	---	-----------



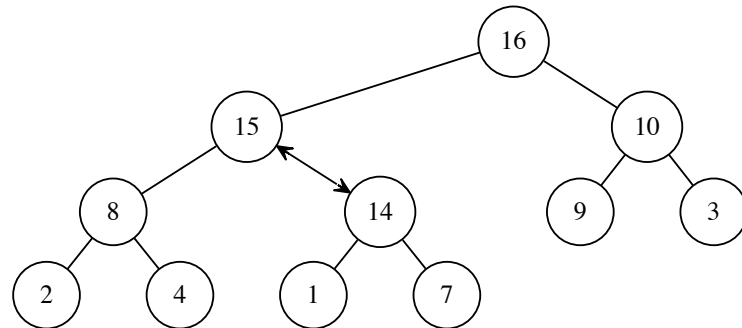
Step 2: Upheap 15 until max heap property $parent \geq child$ is met

16	14	10	8	<u>15</u>	9	3	2	4	1	<u>7</u>
----	----	----	---	-----------	---	---	---	---	---	----------



Step 3: Upheap once more to satisfy the max-heap property.

16	<u>15</u>	10	8	<u>14</u>	9	3	2	4	1	7
----	-----------	----	---	-----------	---	---	---	---	---	---



d) Calculate the height of a 2000 element heap:

$$h(n) = \lfloor \log_2(n) \rfloor$$

$$h(2000) = \lfloor \log_2(2000) \rfloor$$

$$h(2000) = 10$$

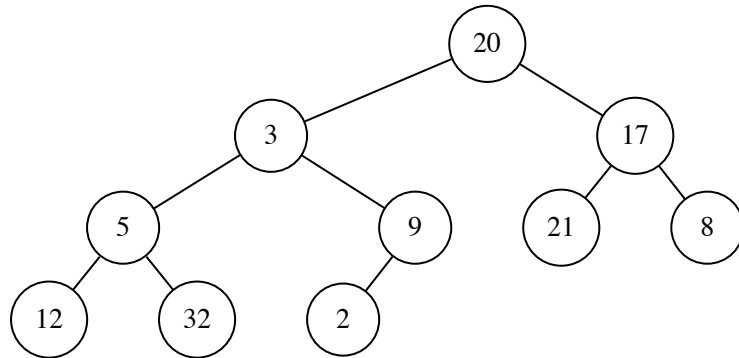
\therefore the height of a 2000 node heap is 10.

Question 3

Step 1: Heapify the contents of the array into a max-heap

Heap:

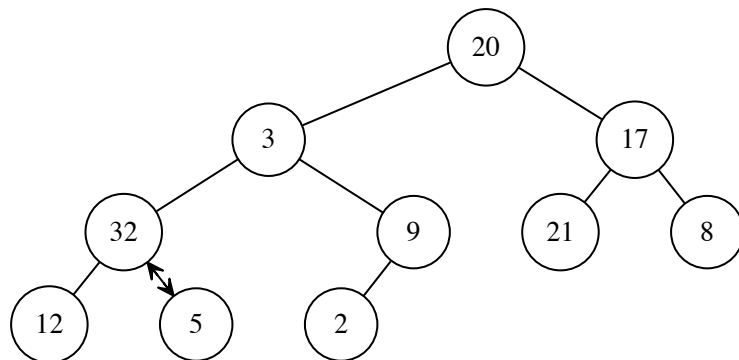
20	3	17	5	9	21	8	12	32	2
----	---	----	---	---	----	---	----	----	---



Step 2: Upheap while the parents are smaller

Heap:

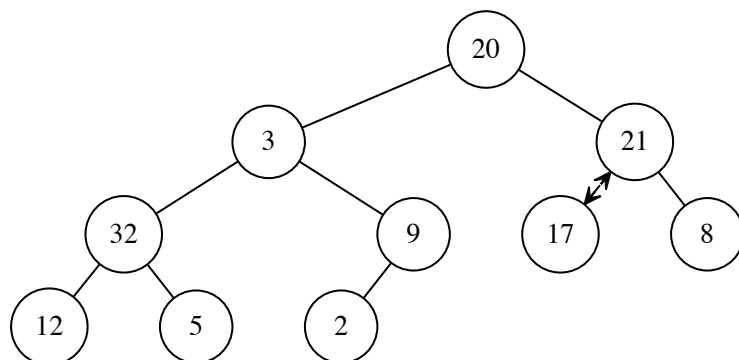
20	3	17	<u>32</u>	9	21	8	12	<u>5</u>	2
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Step 3: Upheap while the parents are smaller

Heap:

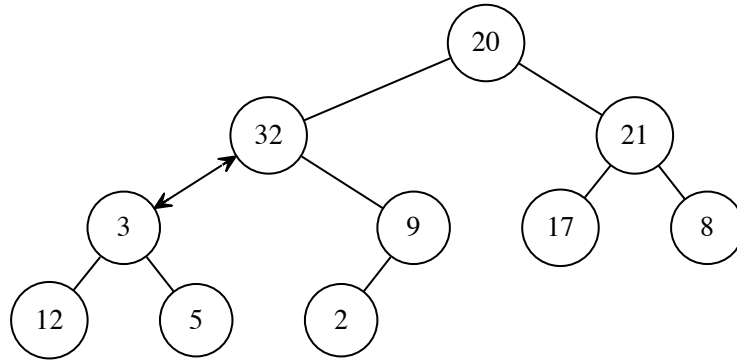
20	3	<u>21</u>	32	9	<u>17</u>	8	12	5	2
----	---	-----------	----	---	-----------	---	----	---	---



Step 4: Upheap while the parents are smaller

Heap:

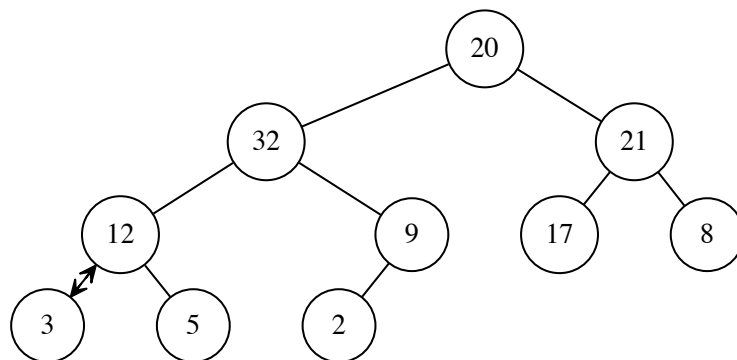
20	<u>32</u>	21	<u>3</u>	9	17	8	12	5	2
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Step 5: Upheap while the parents are smaller

Heap:

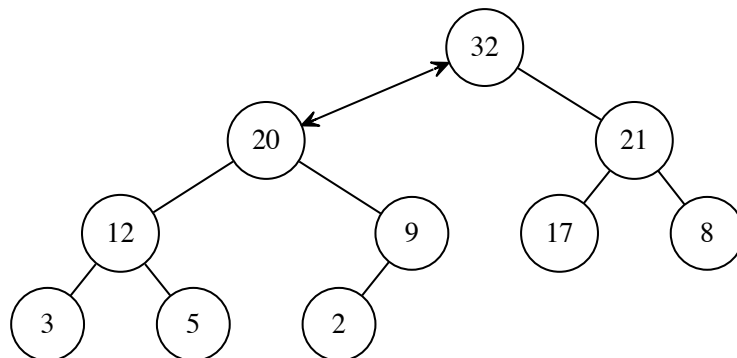
20	32	21	<u>12</u>	9	17	8	<u>3</u>	5	2
----	----	----	-----------	---	----	---	----------	---	---



Step 6: Form a max heap

Max-heap:

<u>32</u>	<u>20</u>	21	12	9	17	8	3	5	2
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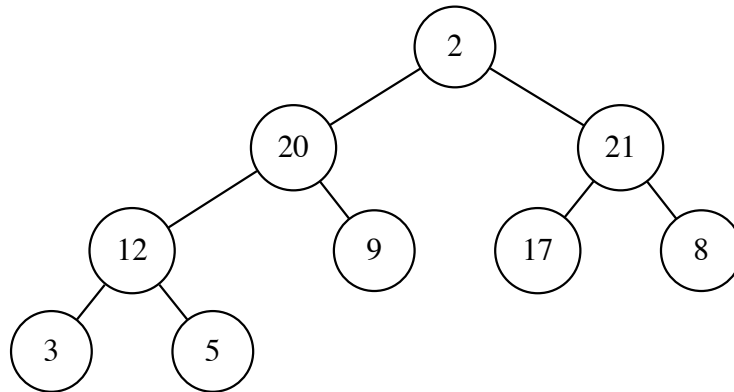
Step 7: Remove the max from the heap and add it to the front of the sorted sequence

Heap:

<u>2</u>	20	21	12	9	17	8	3	5
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Sorted Sequence:

<u>32</u>



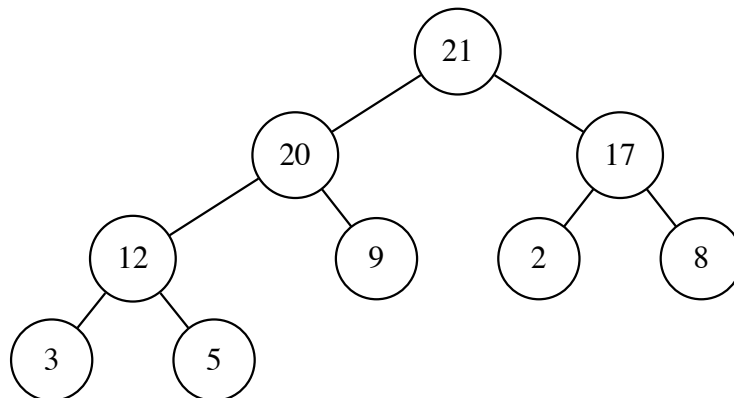
Step 8: Form a max-heap with the remaining elements

Max-heap:

21	20	17	12	9	2	8	3	5
----	----	----	----	---	---	---	---	---

Sorted Sequence:

32



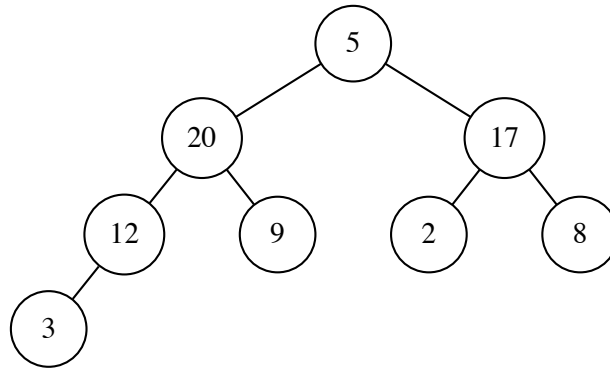
Step 9: Remove the max from the heap and add it to the front of the sorted sequence

Heap:

<u>5</u>	20	17	12	9	2	8	3
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Sorted Sequence:

<u>21</u>	32
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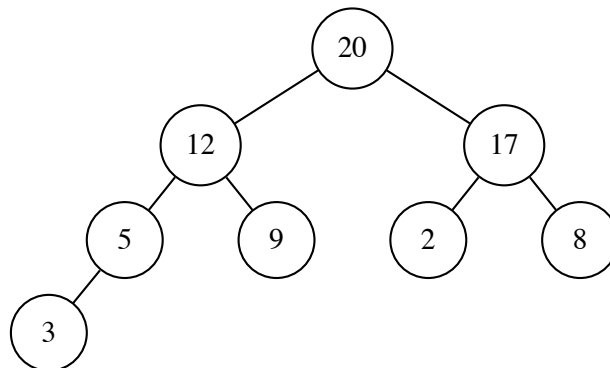
Step 10: Form a max-heap with the remaining elements

Max-heap:

20	12	17	5	9	2	8	3
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Sorted Sequence:

21	32
----	----



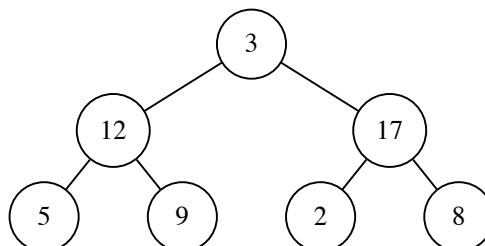
Step 11: Remove the max from the heap and add it to the front of the sorted sequence

Heap:

3	12	17	5	9	2	8
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Sorted Sequence:

<u>20</u>	21	32
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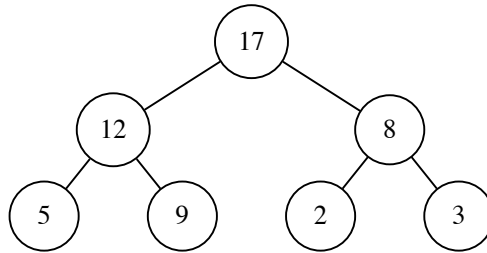
Step 12: Form a max-heap with the remaining elements

Max-heap:

17	12	8	5	9	2	3
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Sorted Sequence:

20	21	32
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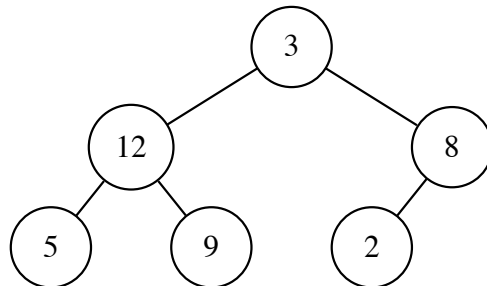
Step 13: Remove the max from the heap and add it to the front of the sorted sequence

Heap:

3	12	8	5	9	2
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Sorted Sequence:

<u>17</u>	20	21	32
-----------	----	----	----



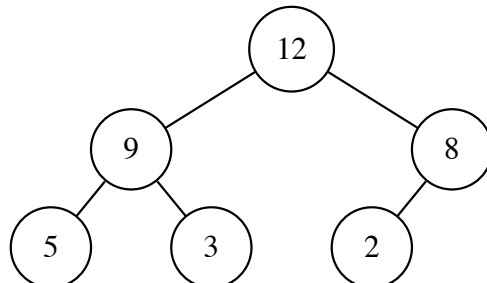
Step 14: Form a max-heap with the remaining elements

Max-heap:

12	9	8	5	3	2
----	---	---	---	---	---

Sorted Sequence:

17	20	21	32
----	----	----	----



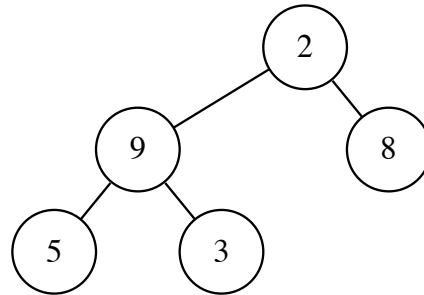
Step 15: Remove the max from the heap and add it to the front of the sorted sequence

Heap:

2	9	8	5	3
---	---	---	---	---

Sorted Sequence:

<u>12</u>	17	20	21	32
-----------	----	----	----	----



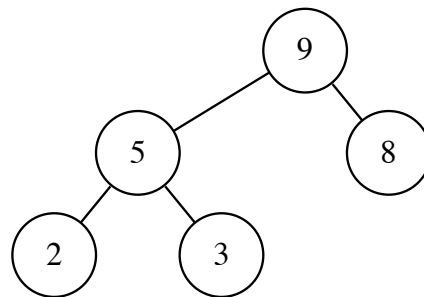
Step 16: Form a max-heap with the remaining elements

Max-heap:

9	5	8	2	3
---	---	---	---	---

Sorted Sequence:

12	17	20	21	32
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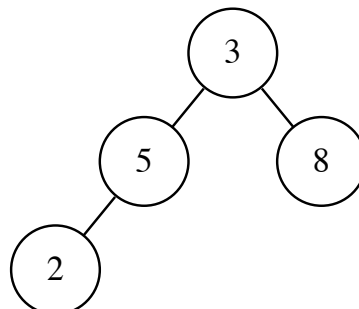
Step 17: Remove the max from the heap and add it to the front of the sorted sequence

Heap:

3	5	8	2
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Sorted Sequence:

<u>9</u>	12	17	20	21	32
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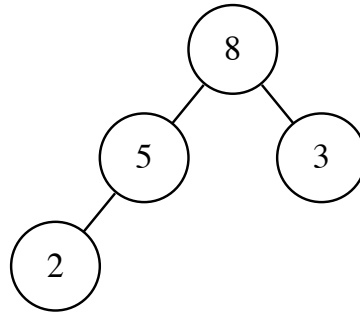
Step 18: Form a max-heap with the remaining elements

Max-heap:

8	5	3	2
---	---	---	---

Sorted Sequence:

9	12	17	20	21	32
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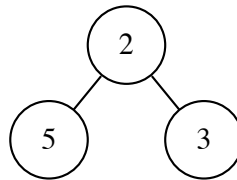
Step 19: Remove the max from the heap and add it to the front of the sorted sequence

Heap:

2	5	3
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Sorted Sequence:

<u>8</u>	9	12	17	20	21	32
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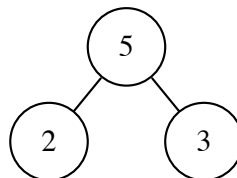
Step 20: Form a max-heap with the remaining elements

Max-heap:

5	2	3
---	---	---

Sorted Sequence:

8	9	12	17	20	21	32
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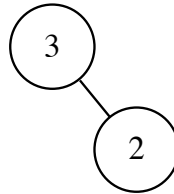
Step 21: Remove the max from the heap and add it to the front of the sorted sequence

Heap:

3	2
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Sorted Sequence:

<u>5</u>	8	9	12	17	20	21	32
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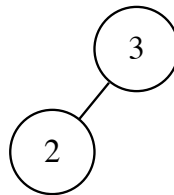
Step 22: Form a max-heap with the remaining elements

Max-heap:

3	2
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Sorted Sequence:

5	8	9	12	17	20	21	32
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Step 23: Remove the max from the heap and add it to the front of the sorted sequence

Max-heap:

2

Sorted Sequence:

<u>3</u>	5	8	9	12	17	20	21	32
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Step 24: Take the final element and add it to the sorted sequence to complete the heapsort

Sorted Sequence:

<u>2</u>	3	5	8	9	12	17	20	21	32
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Sorted sequence after heapsort is complete:

