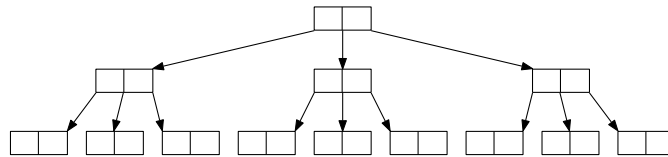


**Assignment #4**  
Matt Langlois - 7731813  
December 7

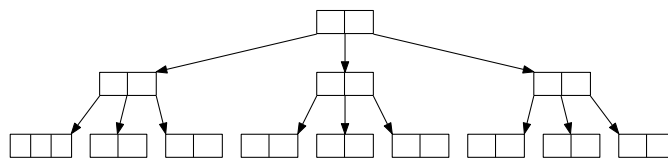
---

## Question 1

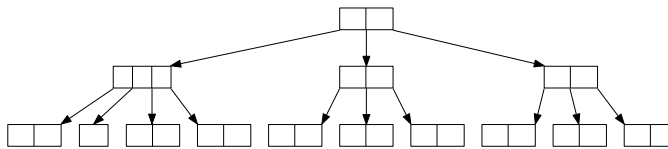
a) 2-4 Tree with 13 3-nodes



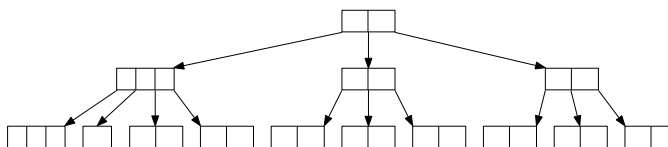
b) After 1 minimum insertion:



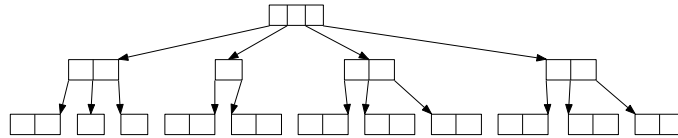
After 2 minimum insertions:



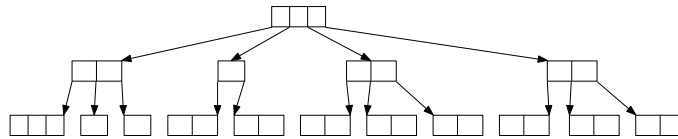
After 3 minimum insertions:



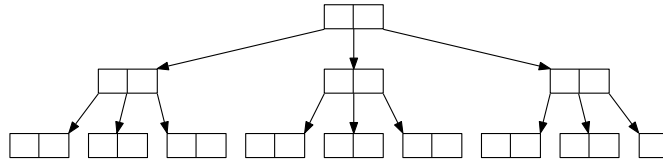
After 4 minimum insertions:



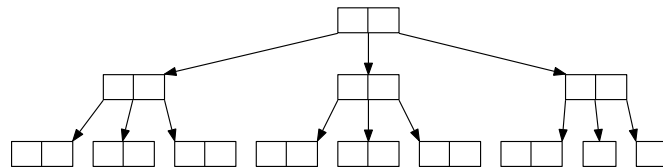
After 5 minimum insertions:



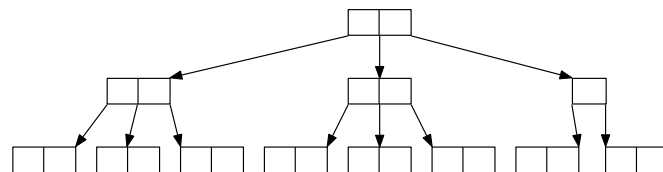
c) After 1 max deletion:



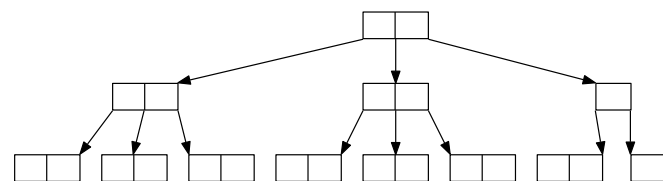
After 2 max deletions:



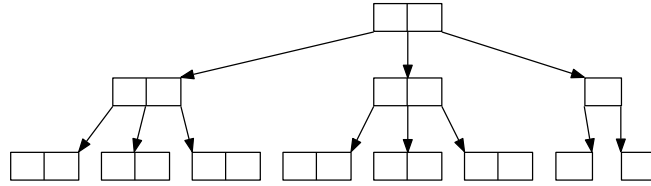
After 3 max deletions:



After 4 max deletions:



After 5 max deletions:



## Question 2

a) DFS Traversal order:  $v5 \rightarrow v4 \rightarrow v2 \rightarrow v4 \rightarrow v3 \rightarrow v1 \rightarrow v10 \rightarrow v9 \rightarrow v8 \rightarrow v6 \rightarrow v7$

Discovery Edges	Back Edges
$(v5, v4)$	$(v10, v3)$
$(v4, v2)$	$(v6, v3)$
$(v4, v3)$	$(v6, v5)$
$(v3, v1)$	$(v7, v8)$
$(v1, v10)$	
$(v10, v9)$	
$(v9, v8)$	
$(v8, v6)$	
$(v6, v7)$	

b) BFS Traversal Levels:

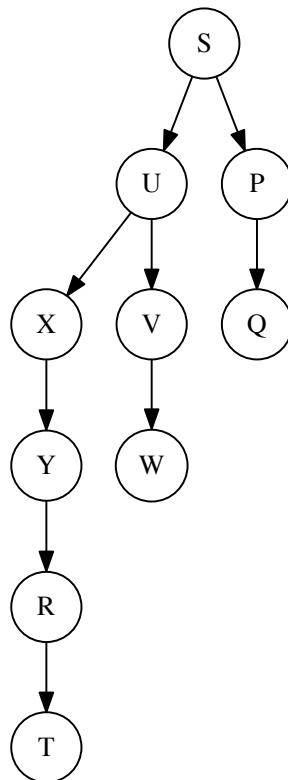
Level	Nodes	Discovery Edges	Cross Edges
L1	v4	$(v5, v4)$	
	v6	$(v5, v6)$	
L2	v2	$(v4, v2)$	$(v6, v3)$
	v3	$(v4, v3)$	
	v7	$(v6, v7)$	
	v8	$(v6, v8)$	
L3	v1	$(v3, v1)$	$(v7, v8)$
	v10	$(v3, v10)$	
	v9	$(v8, v9)$	
L4			$(v1, v10)$
			$(v10, v9)$

### Question 3

a) Dijkstra's algorithm trace:

New Vertex	S	P	U	X	Q	Y	V	R	W	T	New Edge
S	0✓	6	3*	∞	∞	∞	∞	∞	∞	∞	
U	0✓	6	3✓	4*	∞	∞	6	∞	∞	∞	(S, U)
X	0✓	6*	3✓	4✓	12	10	6	∞	∞	∞	(U, X)
P	0✓	6✓	3✓	4✓	11	10	6*	∞	∞	∞	(S, P)
V	0✓	6✓	3✓	4✓	11	10*	6✓	∞	10	∞	(U, V)
Y	0✓	6✓	3✓	4✓	11	10✓	6✓	11	10*	∞	(X, Y)
W	0✓	6✓	3✓	4✓	11*	10✓	6✓	11	10✓	15	(V, W)
Q	0✓	6✓	3✓	4✓	11✓	10✓	6✓	11*	10✓	15	(P, Q)
R	0✓	6✓	3✓	4✓	11✓	10✓	6✓	11✓	10✓	13*	(Y, R)
T	0✓	6✓	3✓	4✓	11✓	10✓	6✓	11✓	10✓	13✓	(R, T)

Minimum Spanning Tree:



b) SUXYRT is the shortest path from S to T.

## Question 4

- a) Insertion into a hash table using linear probing with the function  $h(X) = X \bmod 11$  on the following elements: 3, 32, 46, 58, 57, 26, 17, 74.

$h(X)$	0	1	2	3	4	5	6	7	8	9	10
x			46	3	58	57	26	17	74		32

- b) Four elements would have to be probed before we find 57. This is because  $57 \bmod 11 = 2$  but 57 is found at the 5th position.
- c) Eight elements would have to be probed before we realize 68 is not in the table. This is because  $68 \bmod 11 = 2$  but the first empty element is found at the 9th position.
- d) i) Insertion into a hash table using linear probing with the double hash function  $h_2(X) = 7 - (X \bmod 7)$  on the following elements: 3, 32, 46, 58, 57, 26, 17, 74.

$h(X)$	0	1	2	3	4	5	6	7	8	9	10
x	74		46	3	26		17		58	57	32

- ii) Four elements would have to be probed before we find 57. This is because  $57 \bmod 11 = 2$  but element 46 is found at position 2. So then the double hashing algorithm is applied  $7 - (57 \bmod 7) = 6$ . Jumping 6 buckets we find 58, jumping 6 more we find 3, jumping 6 more we find 57.  $\therefore$  57 is found on the 4<sup>th</sup> probe.
- iii) Six elements would have to be probed before we realize 68 is not in the table. This is because  $68 \bmod 11 = 2$  but element 46 is found at position 2. So then the double hashing algorithm is applied  $7 - (68 \bmod 7) = 2$ . Jumping 2 buckets we find 26, jumping 2 more we find 17, jumping 2 more we find 58, jumping 2 more we find 32 and finally jumping 2 more we find null.  $\therefore$  68 is determine to not be in the hash table on the 6<sup>th</sup> probe.