LCS CA Tank

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Find the equilibrium point of the system considering the given operating point. Linearize the system around the obtained equilibrium point, find the transfer function of the system, and analyze its stability.

$$A\frac{dh_1}{dt} = Q_1 - Q_{out} \quad , \quad Q_{out} = \frac{1}{2} S\sqrt{2gh_1}$$

$$x = h_1 \to \dot{x} = \dot{h}_1 \qquad \to \frac{dh_1}{dt} = \frac{u}{A} - \frac{1}{2A} S\sqrt{2gh_1} \to \dot{x} = \frac{u}{A} - \frac{1}{2A} S\sqrt{2gx}$$

$$u = Q_1$$

$$\text{نقطه تعادل} \to \frac{u}{A} - \frac{1}{2A} S\sqrt{2gx} = 0 \to u = \frac{1}{2} S\sqrt{2gx}$$

$$\to \sqrt{2gx} = \frac{2u}{S} \to 2gx = \left(\frac{2u}{S}\right)^2 \to x = \left(\frac{2u}{S}\right)^2 \times \frac{1}{2g}$$

$$\alpha \quad x^* = h_1^* = 147\,cm = 1.47\,m \quad \to 1.47^{m} = \left(\frac{2u}{5\times10^{-9}\,m^2}\right)^2 \times \frac{1}{9.81\,\frac{m}{s^2}} \to 4u^2 = 360.5175\times10^{-10}\,\frac{m^6}{s^2} \to u^* = 9.49349194\times10^{-5}\,\frac{m^3}{s}$$

$$\tilde{x} = x - x^* \qquad \dot{\tilde{x}} = \frac{\partial f}{\partial x}\Big|_{x^*}\tilde{x} + \frac{\partial f}{\partial u}\Big|_{u^*}\tilde{u} \to \dot{\tilde{x}} = -\frac{gS}{2^{\frac{3}{2}}a\sqrt{gx}}\Big|_{x^*}\tilde{x} + \frac{1}{A}\tilde{u}$$

$$\tilde{u} = u - u^*$$

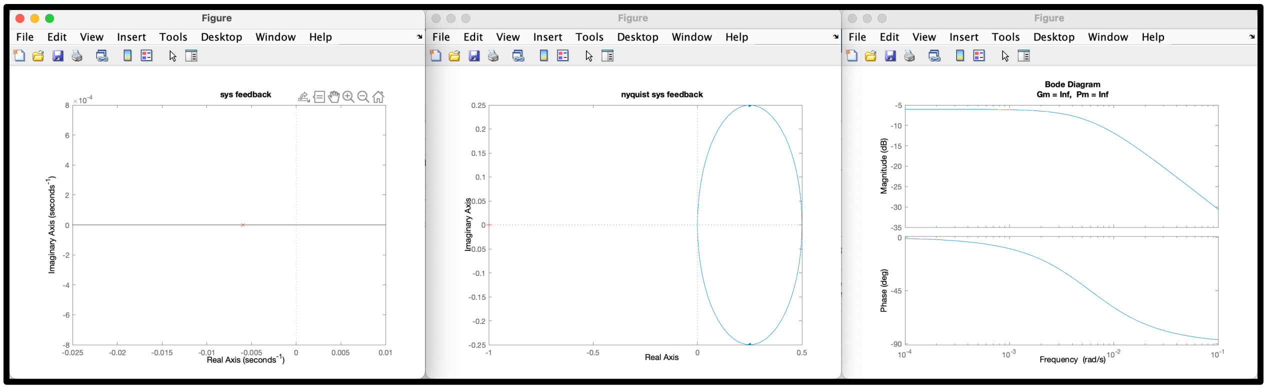
$$\to \dot{\tilde{x}} = \frac{-gS_1}{2^{\frac{3}{2}}a\sqrt{g\times1.47}}\tilde{x} + \frac{1}{A}\tilde{u}$$

$$y = \frac{1}{2} S\sqrt{2gx} \to \tilde{y} = \frac{gS}{2^{3/2}\sqrt{g\times1.47}}\tilde{x}$$

From the equation above we can extract the A,B,C and D matrix.
The we will use MATLAB to find the transfer function :

$$sys = \frac{0.002965}{s + 0.002965}$$

The system is stable.



We can calculate the k value which makes the system on stable :

$$K: \;\to\; 1+KGH=0 \;\to\; K=\frac{-1}{GH} \;\to\; K=\left.\frac{-1}{\frac{0.004950}{s+0.004959}}\right|_{s=0} \;\to\; K<-1 \;\to\; K\geqslant -1$$

Stable

Calculating zeta and wn for the given conditions:

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}}$$

$M_p \leqslant 15\%$ $\quad t_s \leqslant 200\,s$ $\quad \rightsquigarrow \quad M_p = 12\%$ $\quad t_s = 180\,s$

۱ مرتبه $t_s = 4\tau \to G = \frac{K}{\tau s+1}$

۲ مرتبه $t_s = \frac{4}{\zeta\omega_n} \to G = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$

$$M_p = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} \to \ln M_p = \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} \to (\ln M_p)^2 = \frac{\zeta^2\pi^2}{1-\zeta^2}$$

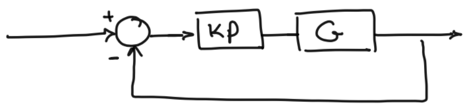
$$\to \zeta^2(\pi^2+(\ln M_p)^2) = (\ln M_p)^2 \to \zeta = \frac{\sqrt{(\ln M_p)^2}}{\sqrt{\pi^2+(\ln M_p)^2}}$$

$$t_s = \frac{4}{\zeta\omega_n} \to \omega_n = \frac{4}{\zeta t_s} \xrightarrow{t_s=180\,s}$$

$$\omega_n = \frac{4}{180\times\zeta}$$

P Controller:

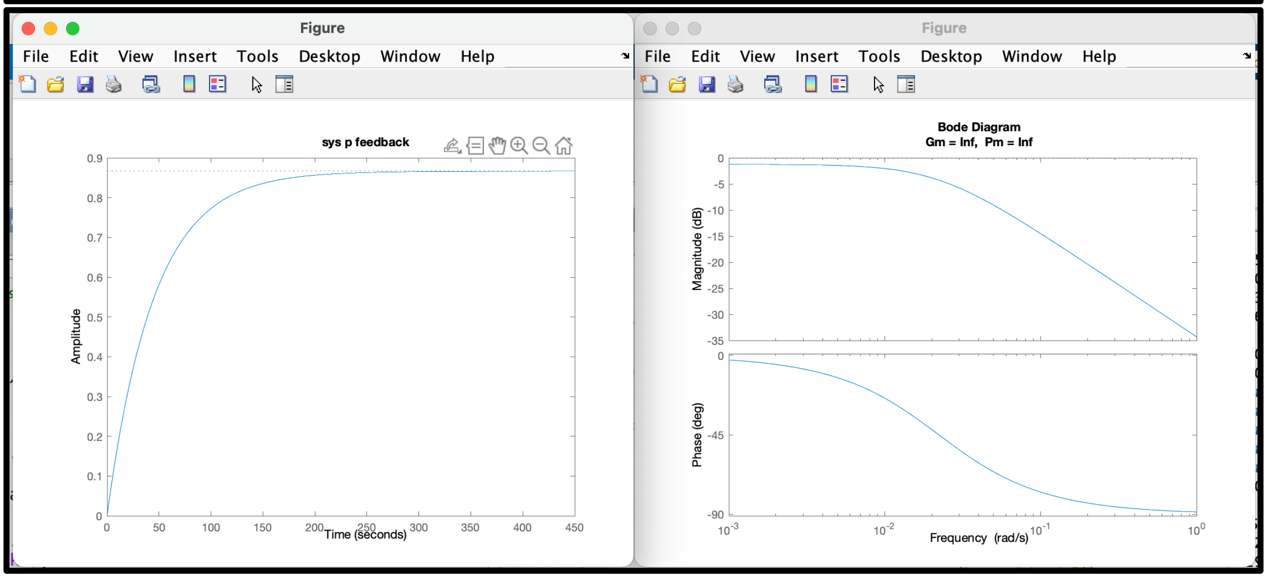
P کنترلر :



$G = \frac{a}{s+a}$ $\rightsquigarrow$ حلقه بسته $= \frac{K_P a}{s + a + K_P a}$ $\rightsquigarrow t_s = ۴\tau_{new}$

$\tau_{new} = \frac{۱}{(K_P+۱)a}$

$\rightarrow \tau_{new} = ۱۸۰ \rightarrow t_s = ۴ \times \frac{۱}{(K_P+۱)۰,۰۰۲۹۴۵} \rightarrow K_P = \frac{۴}{t_s \times ۰,۰۰۲۹۴۵} - ۱ =$



stepinfo_P

1x1 struct with 9 fields

Field	Value
RiseTime	98.8527
TransientTime	176.0209
SettlingTime	176.0209
SettlingMin	0.7838
SettlingMax	0.8666
Overshoot	0
Undershoot	0
Peak	0.8666
PeakTime	474.5022

For this type of controller, we can’t adjust the overshoot% because the system only has one pole.

PI Controller:

PI برای → $G_c = \frac{K_p s + K_i}{s}$

حلقه بسته $= \frac{G_c \times G}{1 + G_c \times G}$

$$G_{open} = \left(K_p + \frac{K_i}{s}\right) \times \frac{۰٫۰۰۲۴۹۵}{s + ۰٫۰۰۲۴۹۵}$$

$$G_{closed} = \frac{(K_p s + K_i) \times ۰٫۰۰۲۴۹۵}{s(s + ۰٫۰۰۲۴۹۵) + (K_p s + K_i) \times ۰٫۰۰۲۴۹۵}$$

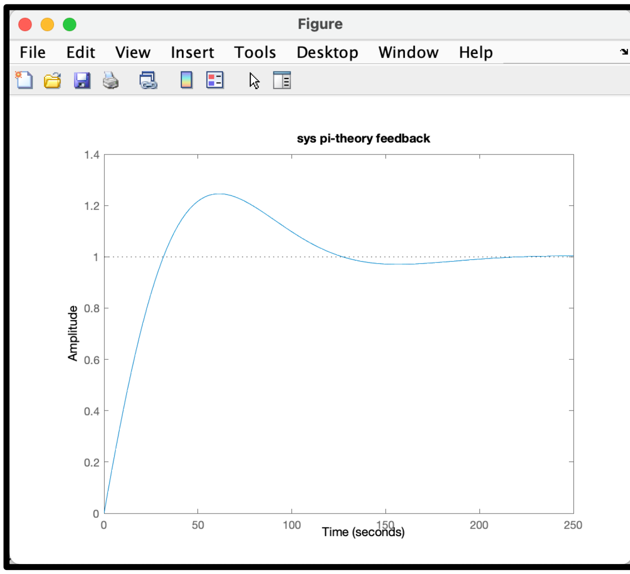
$$s^۲ + (K_p \times ۰٫۰۰۲۴۹۵ + ۰٫۰۰۲۴۹۵)s + K_i \times ۰٫۰۰۲۴۹۵ = s^۲ + ۲\zeta\omega_n s + \omega_n^۲$$

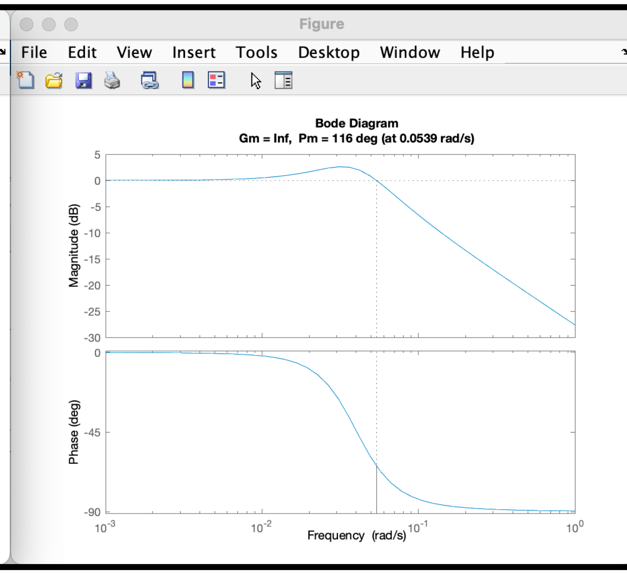
$\rightarrow K_p \times ۰٫۰۰۲۴۹۵ + ۰٫۰۰۲۴۹۵ = ۲\zeta\omega_n$

$\rightarrow K_p = \frac{۲\zeta\omega_n - ۰٫۰۰۲۴۹۵}{۰٫۰۰۲۴۹۵}$

$\rightarrow K_i = \frac{\omega_n^۲}{۰٫۰۰۲۴۹۵}$

First we used the calculations:



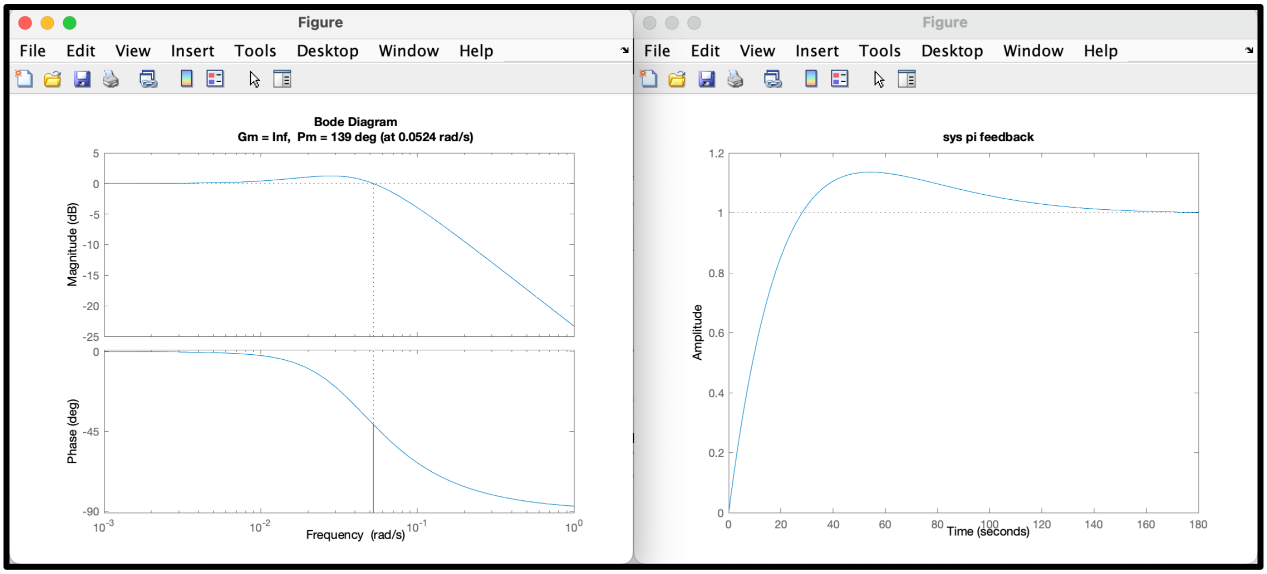


stepinfo_PItheory

1x1 struct with 9 fields

Field	Value
RiseTime	24.4180
TransientTime	181.5071
SettlingTime	181.5071
SettlingMin	0.9020
SettlingMax	1.2457
Overshoot	24.5700
Undershoot	0
Peak	1.2457
PeakTime	62.1619

As we can see we couldn't set the overshoot to be less than 15% so we will change the kp value manually.



stepinfo_PI

1x1 struct with 9 fields

Field	Value
RiseTime	20.6354
TransientTime	130.0479
SettlingTime	130.0479
SettlingMin	0.9223
SettlingMax	1.1358
Overshoot	13.5786
Undershoot	0
Peak	1.1358
PeakTime	54.3543

This time we achieved both the settling time and the overshoot.

PD Controller:

برای PD $\rightarrow G_c = K_P + K_D S$ حلقه بسته $= \dfrac{G_c \times G}{1 + G_c \times G}$

$$G_{closed} = \frac{(K_P + K_D S)\,\text{۰٫۰۰۲۹۴۵}}{S + \text{۰٫۰۰۲۹۴۵} + (K_P + K_D S)\,\text{۰٫۰۰۲۹۴۵}}$$

$$K_D = \frac{\text{۲}\zeta\omega_n - \text{۰٫۰۰۲۹۴۵}}{\text{۰٫۰۰۲۹۴۵}} \qquad K_P = \frac{\omega_n^{\text{۲}}}{\text{۰٫۰۰۲۹۴۵}}$$

$$S + \text{۰٫۰۰۲۹۴۵} + K_P \times \text{۰٫۰۰۲۹۴۵} + K_D S \times \text{۰٫۰۰۲۹۴۵}$$

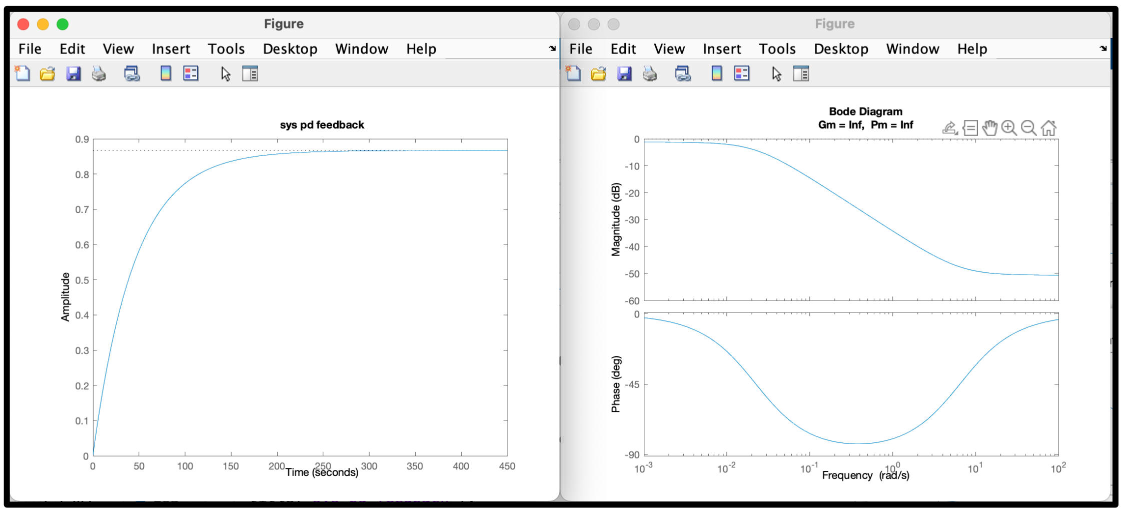
$$S(K_D \times \text{۰٫۰۰۲۹۴۵} + \text{۱}) + \text{۰٫۰۰۲۹۴۵} + K_P \times \text{۰٫۰۰۲۹۴۵}$$

سیستم مرتبه ۱ $\rightarrow S(K_D \times \text{۰٫۰۰۲۹۴۵} + 1) + \text{۰٫۰۰۲۹۴۵} + K_P \times \text{۰٫۰۰۲۹۴۵} \rightarrow \tau_{new} = \dfrac{(K_D \times \text{۰٫۰۰۲۹۴۵} + 1)}{\text{۰٫۰۰۲۹۴۵} + K_P \times \text{۰٫۰۰۲۹۴۵}}$

$$t_s = \text{۴}\tau_{new} \rightarrow t_s = \frac{\text{۴} \times (K_D \times \text{۰٫۰۰۲۹۴۵} + 1)}{\text{۰٫۰۰۲۹۴۵} + K_P \times \text{۰٫۰۰۲۹۴۵}} \rightarrow t_s \times \text{۰٫۰۰۲۹۴۵} + t_s \times K_P \times \text{۰٫۰۰۲۹۴۵} = \text{۴} \times (K_D \times \text{۰٫۰۰۲۹۴۵} + 1)$$

$$K_P = \frac{\text{۴} \times (K_D \times \text{۰٫۰۰۲۹۴۵} + 1)}{t_s \times \text{۰٫۰۰۲۹۴۵}} - \text{۱}$$

This time we have on equation and two variables, so we will give one of them a number and calculate the other one.
In this case the KD=1.



stepinfo_PD

1x1 struct with 9 fields

Field ▲	Value
RiseTime	98.8577
TransientTime	176.0209
SettlingTime	175.8701
SettlingMin	0.7806
SettlingMax	0.8669
Overshoot	0
Undershoot	0
Peak	0.8669
PeakTime	474.5024

As we can see we couldn't adjust the overshoot because the system only has one pole and one zero.

PID Controller:

$$PID \rightarrow G_c = K_P + K_D s + \frac{K_I}{s} \rightarrow G_c = \frac{K_P s + K_D s^2 + K_I}{s}$$

حلقه بسته $= \dfrac{G_c \times G}{1 + G_c \times G} = \dfrac{\dfrac{K_P s + K_D s^2 + K_I}{s} \times \dfrac{0.002945}{s + 0.002945}}{1 + \dfrac{K_P s + K_D s^2 + K_I}{s} \times \dfrac{0.002945}{s + 0.002945}}$

$$\rightarrow \frac{(K_P s + K_I + K_D s^2)\,0.002945}{s(s + 0.002945) + (K_P s + K_I + K_D s^2)\,0.002945}$$

$$\rightarrow s^2 + s \times 0.002945 + 0.002945 K_P s + 0.002945 K_I + 0.002945 K_D s^2$$

$$\rightarrow s^2(1 + 0.002945 K_D) + s(0.00294 + 0.002945 K_P) + 0.002945 K_I$$

$$= s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$s^2 + \frac{s(0.00294 + 0.002945 K_P)}{(1 + 0.002945 K_D)} + \frac{0.002945 K_I}{(1 + 0.002945 K_D)} = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$\frac{0.002945 K_I}{(1 + 0.002945 K_D)} = \omega_n^2 \rightarrow 0.002945 K_I - \omega_n^2 = 0.002945 K_D \omega_n^2 \rightarrow K_D = \frac{0.002945 K_I - \omega_n^2}{0.002945\,\omega_n^2}$$

K_I Constant ← روش اول

$$\frac{(0.002945 + 0.002945 K_P)}{(1 + 0.002945 K_D)} = 2\zeta\omega_n \rightarrow \quad K_P = \frac{2\zeta\omega_n(1 + 0.002945 K_D) - 0.002945}{0.002945}$$

$$K_D = \frac{1}{0.002945}\left(\frac{0.002945(K_P + 1)}{2\zeta\omega_n} - 1\right)$$

K_P constant ← روش دوم

$$K_i = \frac{\omega_n^2(1 + 0.002945 K_D)}{0.002945}$$

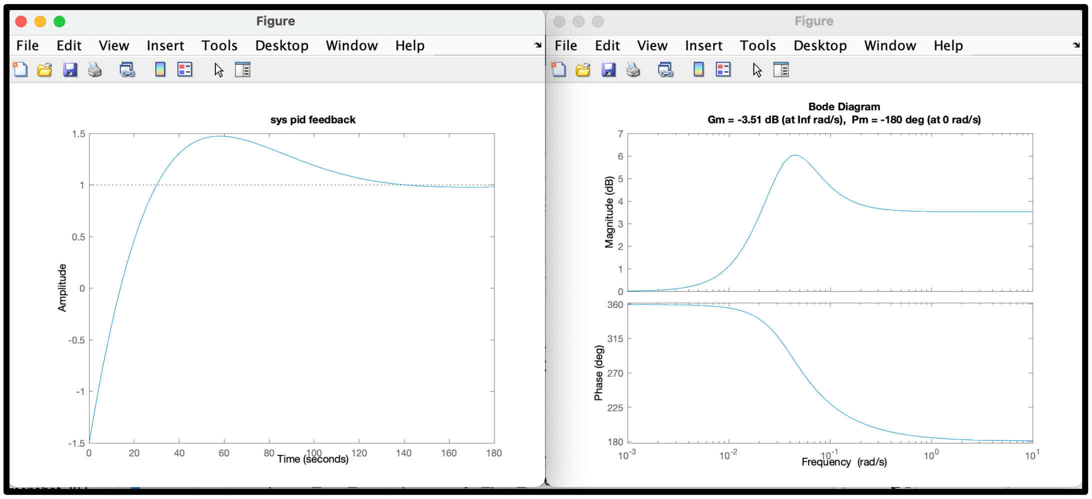
$$K_P = \frac{2\zeta\omega_n\, 0.002945\,(1 + 0.002945 K_D) - 1}{0.002945}$$

K_D Constant ← روش سوم

$$K_i = \frac{\omega_n^2(1 + 0.002945 K_D)}{0.002945}$$

This time we used three different methods:
First method: we had the KI constant and we calculated KP and KD
second method: we had the KP constant and we calculated KI and KD
third method: we had the KD constant and we calculated KP and KI
we did those methods above because we had two equations and three variables.

First method:

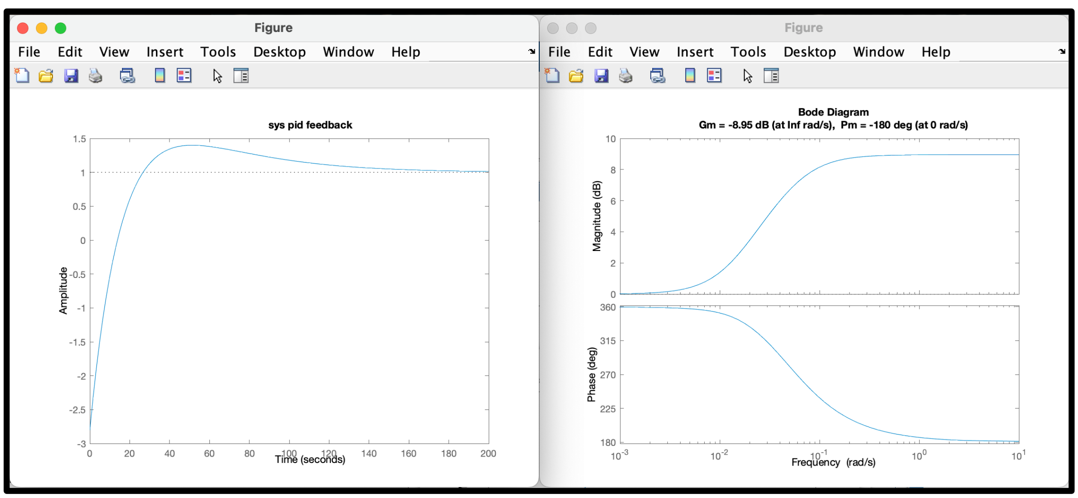


stepinfo_PID_1

1x1 struct with 9 fields

Field	Value
RiseTime	12.6737
TransientTime	123.5567
SettlingTime	180.5236
SettlingMin	0.9211
SettlingMax	1.4730
Overshoot	47.2989
Undershoot	149.8760
Peak	1.4988
PeakTime	0

Second method:

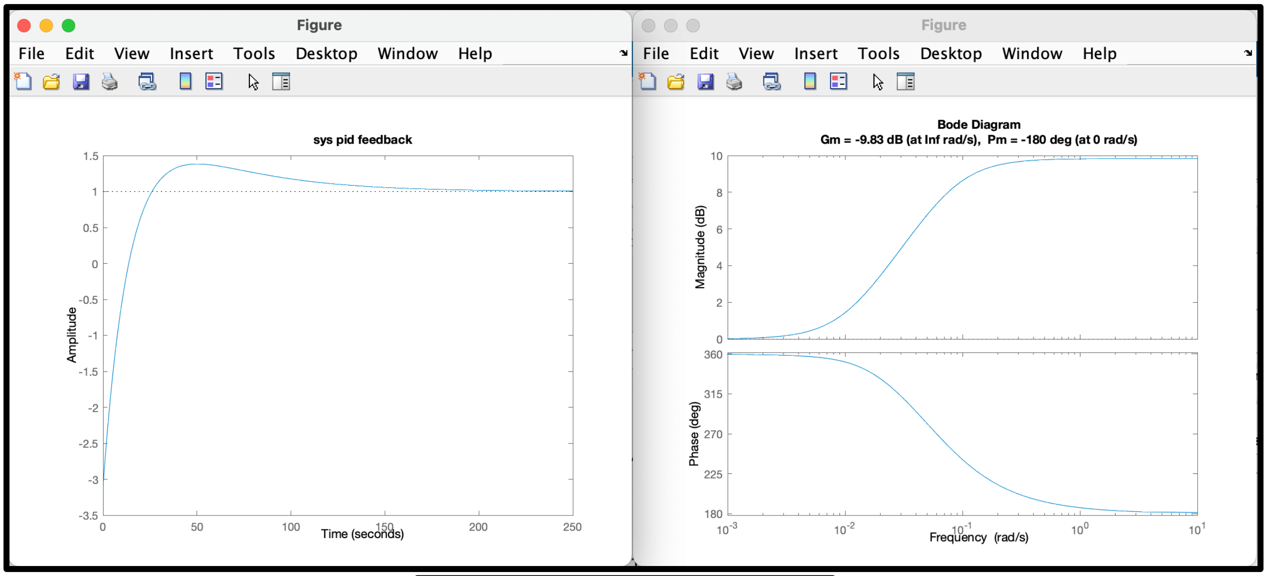


stepinfo_PID_2

1x1 struct with 9 fields

Field	Value
RiseTime	9.9607
TransientTime	133.4841
SettlingTime	182.7136
SettlingMin	0.9687
SettlingMax	1.3996
Overshoot	39.9648
Undershoot	280.2832
Peak	2.8028
PeakTime	0

Third method:

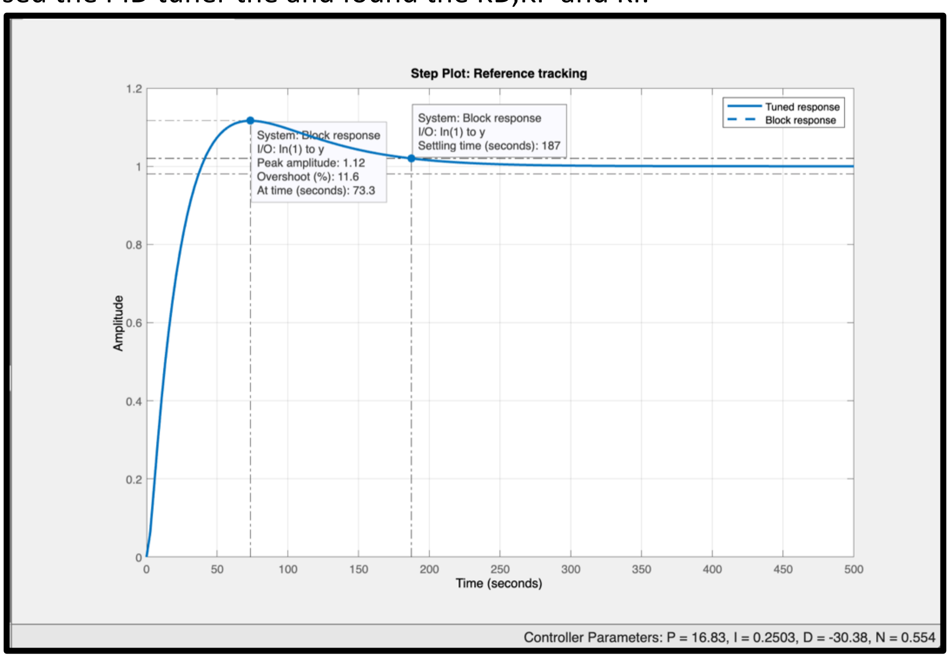


stepinfo_PID_3

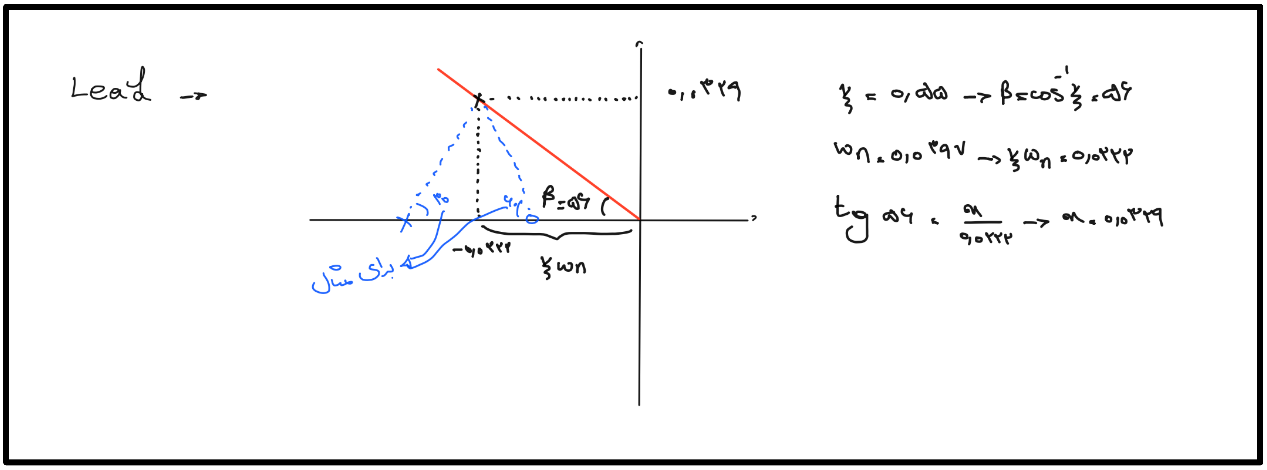
1x1 struct with 9 fields

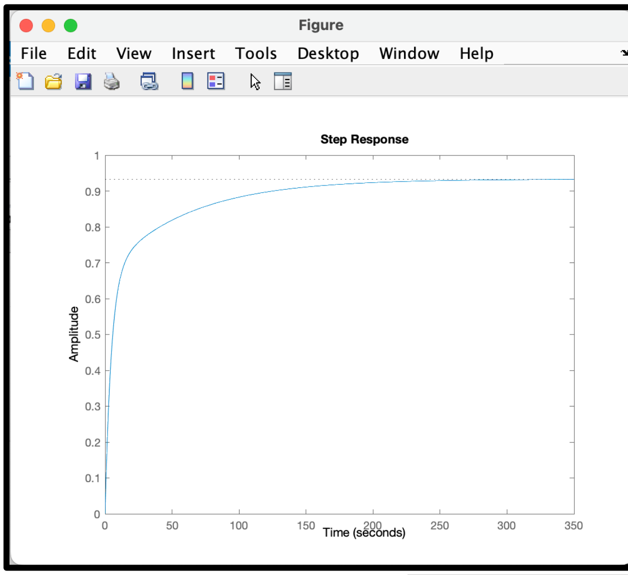
Field	Value
RiseTime	9.5621
TransientTime	133.9730
SettlingTime	194.9925
SettlingMin	0.9197
SettlingMax	1.3826
Overshoot	38.2605
Undershoot	310.1243
Peak	3.1012
PeakTime	0

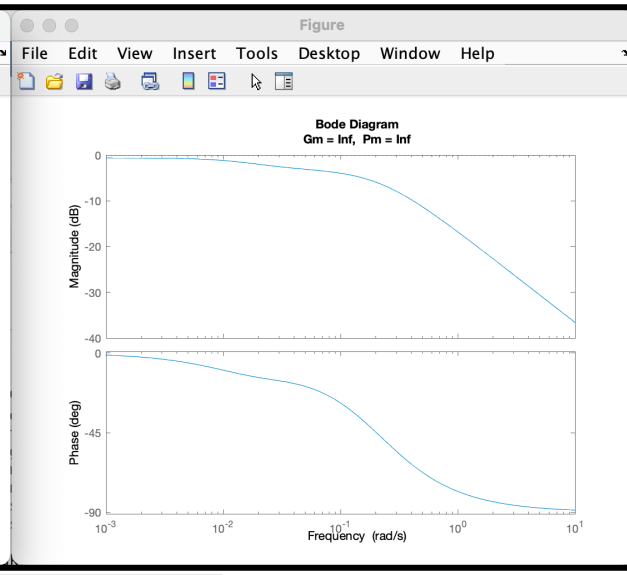
We couldn’t achieve the overshoot goal because of the systems order.
At last we used the PID tuner the and found the KD,KP and KI:



Lead Controller:







stepinfo_lead

1x1 struct with 9 fields

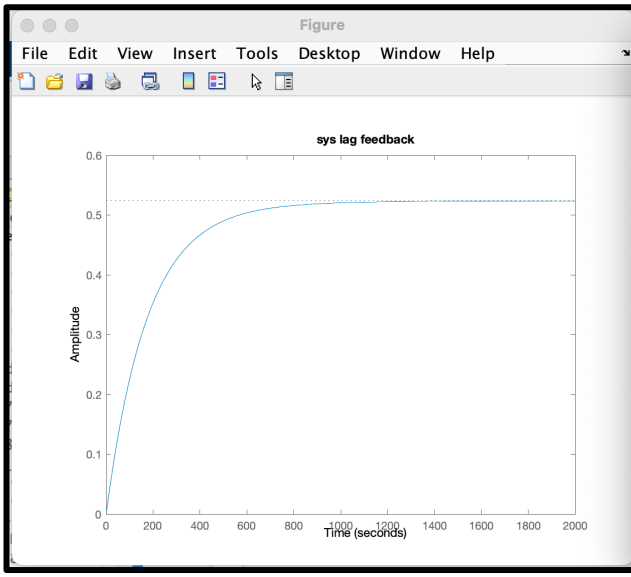
Field	Value
RiseTime	61.7760
TransientTime	160.3738
SettlingTime	160.3738
SettlingMin	0.8405
SettlingMax	0.9325
Overshoot	0
Undershoot	0
Peak	0.9325
PeakTime	361.0815

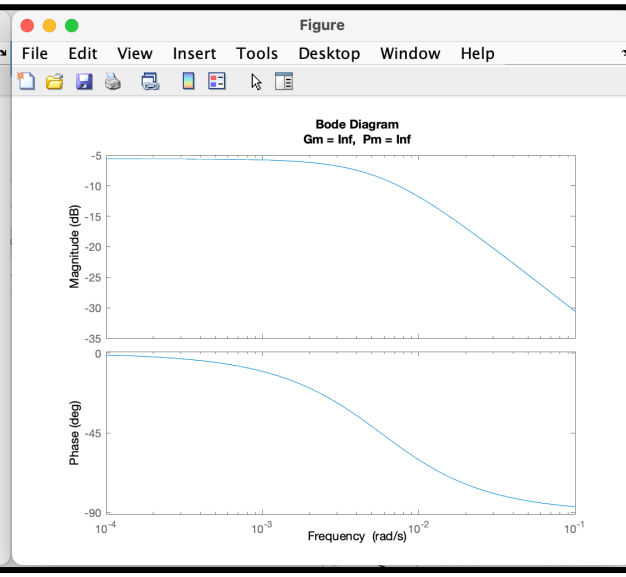
Lag Controller:

Lag $\rightarrow$ $K_a\,old = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{0.002945}{s+0.002945} = 1$

e_{ss} ۱۰٪ بهبود $\longrightarrow$ ۱۰٪ خودمان لغسم $\rightarrow \frac{K_a\,new}{K_a\,old} = 1.1 \rightarrow K_a\,new = 1.1 = \frac{z}{p}$

$z = \frac{\alpha}{10}$, $\alpha = 0.0222$ $\Big\} \Rightarrow z = 0.00222 \rightarrow \frac{z}{p} = 1.1 \rightarrow p = 2.018 \times 10^{-3} \rightarrow G_{lag} = \frac{s + 0.00222}{s + 0.002018}$

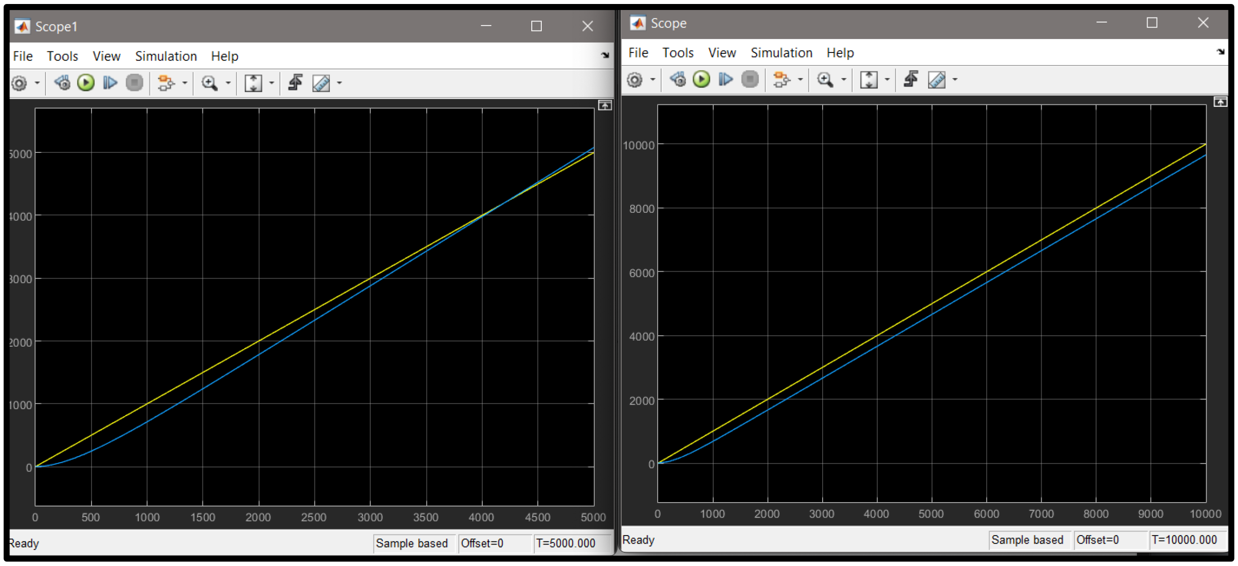




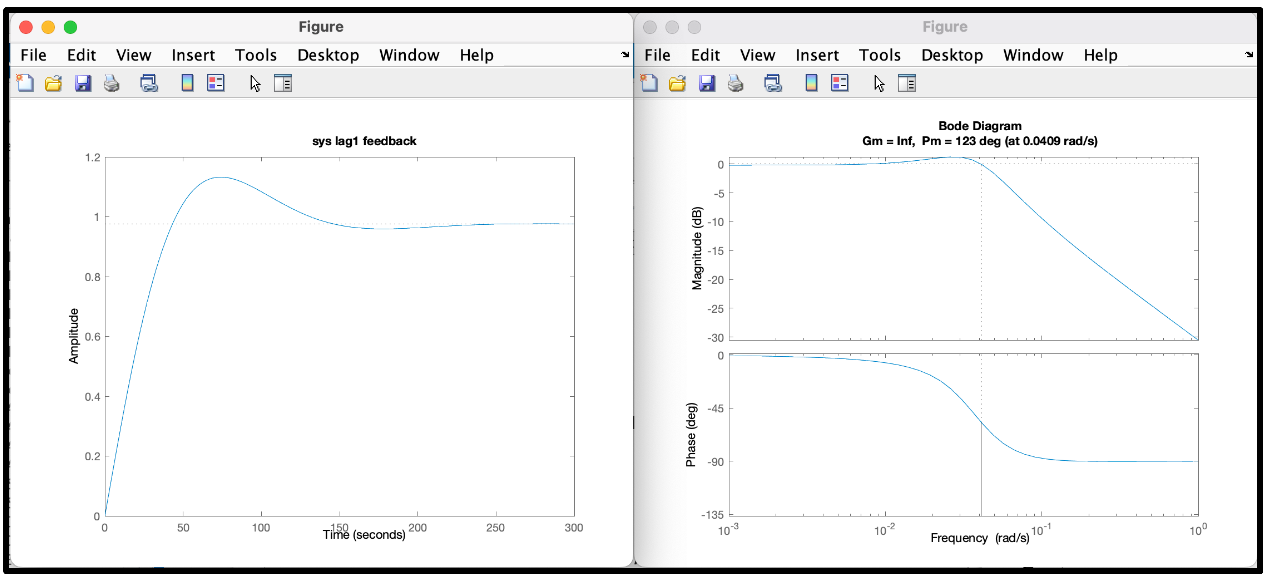
stepinfo_lag

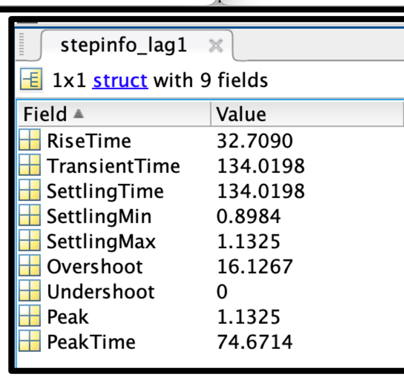
1x1 struct with 9 fields

Field	Value
RiseTime	397.8406
TransientTime	736.2314
SettlingTime	736.2314
SettlingMin	0.4732
SettlingMax	0.5237
Overshoot	0
Undershoot	0
Peak	0.5237
PeakTime	2.2267e+03



After designing the lag controller theoretically, we couldn't achieve the settling time goal
So we used experimental values for the controller.



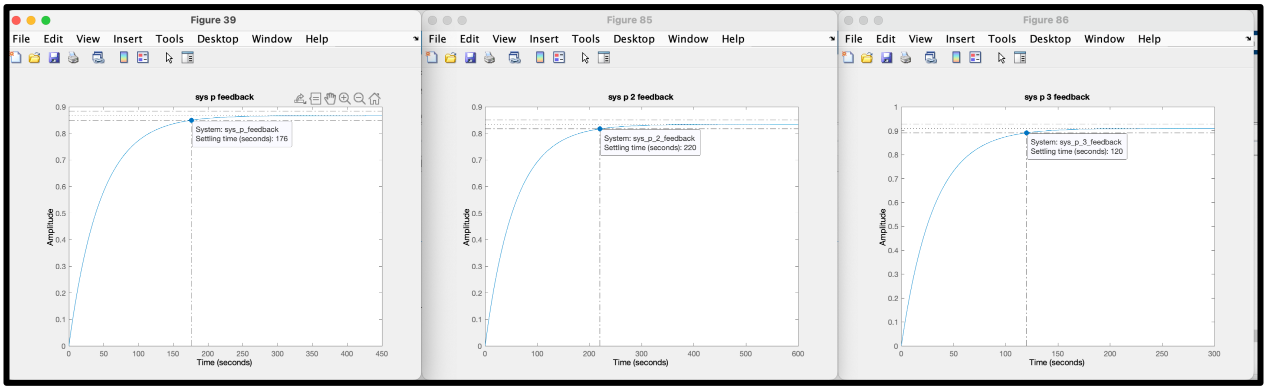


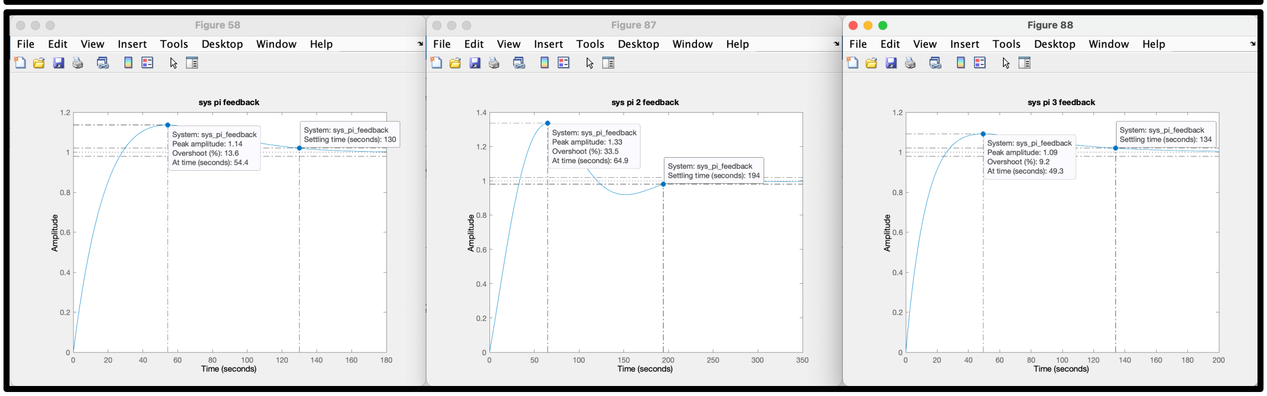
stepinfo_lag1

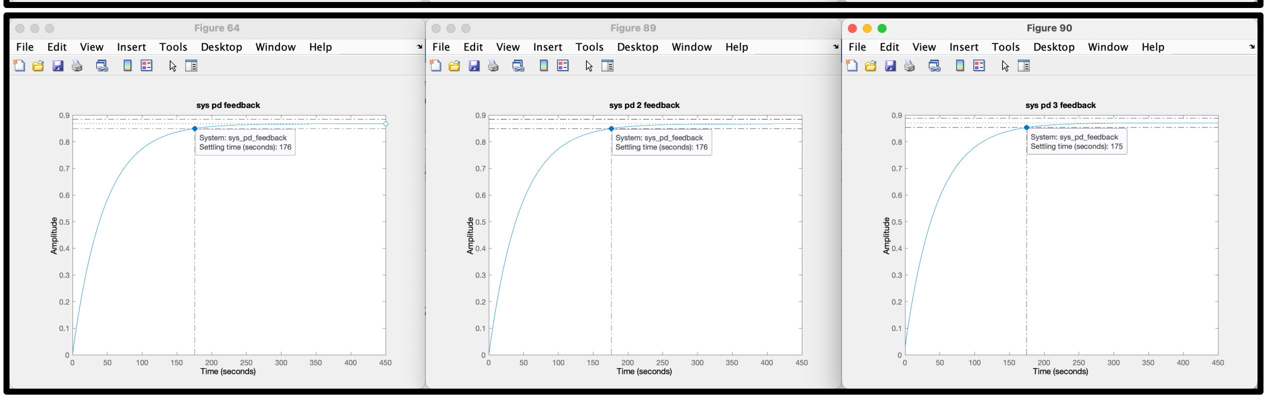
1x1 struct with 9 fields

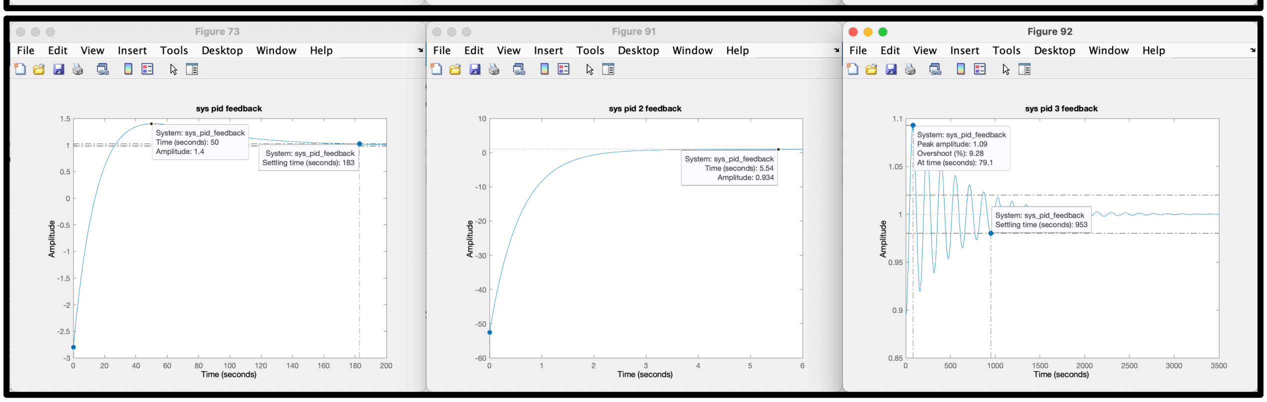
Field	Value
RiseTime	32.7090
TransientTime	134.0198
SettlingTime	134.0198
SettlingMin	0.8984
SettlingMax	1.1325
Overshoot	16.1267
Undershoot	0
Peak	1.1325
PeakTime	74.6714

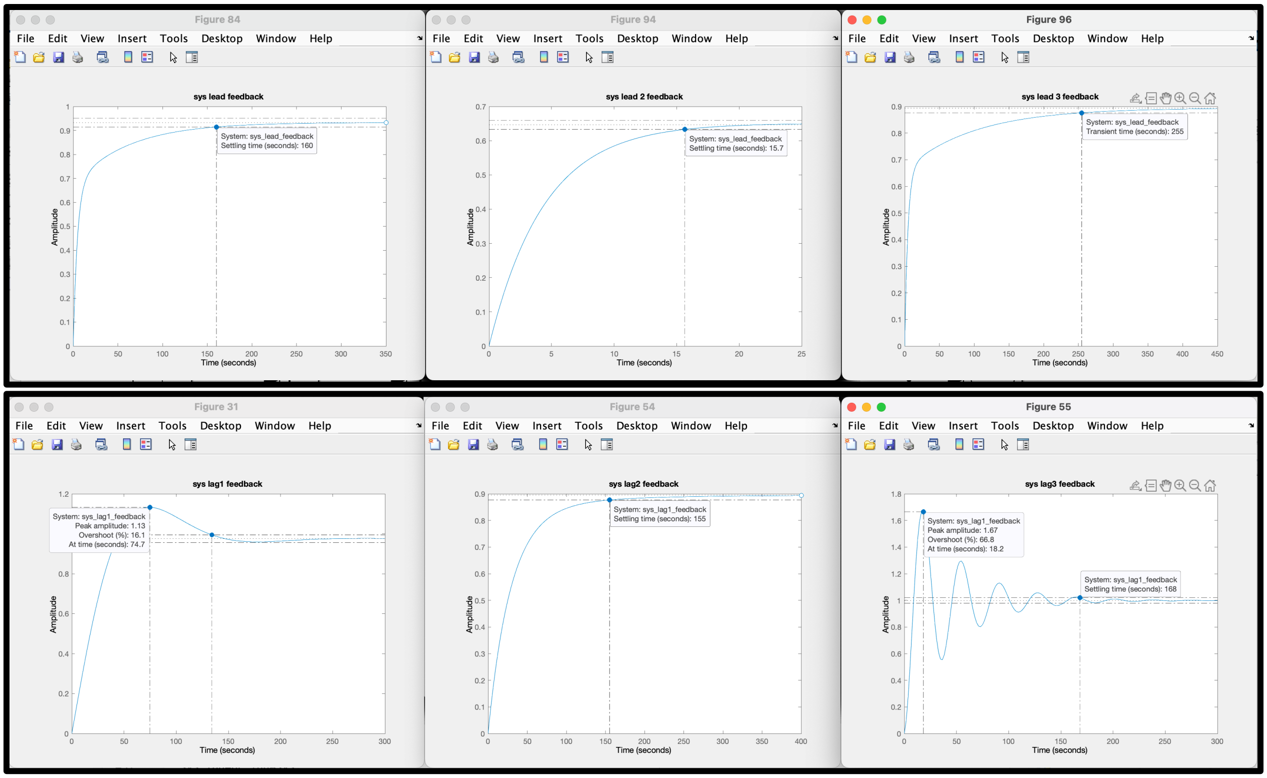
Changing variables:











First figure of each picture is for the original system
second figure of each picture is for the system that a value was changed and it was less than the original
third figure of each picture is for the system that a value was changed and it was greater than the original

P: in this controller when we change the KP to be less than the original the settling time will be greater
The opposite is for when its greater than the original.

PI: in this controller when we change the value to be less than the original the settling time will be greater and so will be the overshoot %.
when we change the value to be greater than the original the settling time will be greater(but close) but the overshoot % will be less than the original.

PD: in this controller when we change the value to be less than the original the settling time will be slightly greater
The opposite is for when its greater than the original.

PID: in this controller when we change the value to be less than the original, the settling time will be much smaller and the overshoot becomes less too.
When we change the value to be greater than the original, the settling time will be much higher but the overshoot will be smaller.

Lead: in this controller when we change the value to be less than the original the settling time will be less the overshoot in all cases is zero
The opposite is for when its greater than the original.

Lag: in this controller when we change the value to be less than the original the settling time will be grater but the overshoot will be less.
when we change the value to be greater than the original the settling time will be grater and the overshoot will be grater too.

Best controller:

PI

غیر خطی $A\frac{dh_1}{dt} = Q_1 - Q_{out}$, $Q_{out} = \frac{1}{2} S\sqrt{2gh_1}$

$x = h_1 \to \dot{x} = \dot{h}_1 \quad \to \frac{dh_1}{dt} = \frac{u}{A} - \frac{1}{2A} S\sqrt{2gh_1} \to \dot{x} = \frac{u}{A} - \frac{1}{2A} S\sqrt{2gx}$

$u = Q_1$

نقطه تعادل $\to \frac{u}{A} - \frac{1}{2A} S\sqrt{2gx} = 0 \to u = \frac{1}{2} S\sqrt{2gx}$

$\to \sqrt{2gx} = \frac{2u}{S} \to 2gx = \left(\frac{2u}{S}\right)^2 \to x = \left(\frac{2u}{S}\right)^2 \times \frac{1}{2g}$

$\dot{x} = \frac{-1}{2A} S\sqrt{2gx} + \frac{u}{A}$

$y = \frac{1}{2} S\sqrt{2gx}$

We chose this controller because it had the best settling time and it has overshoot as well
There were other controllers that had better settling time but zero overshoot.

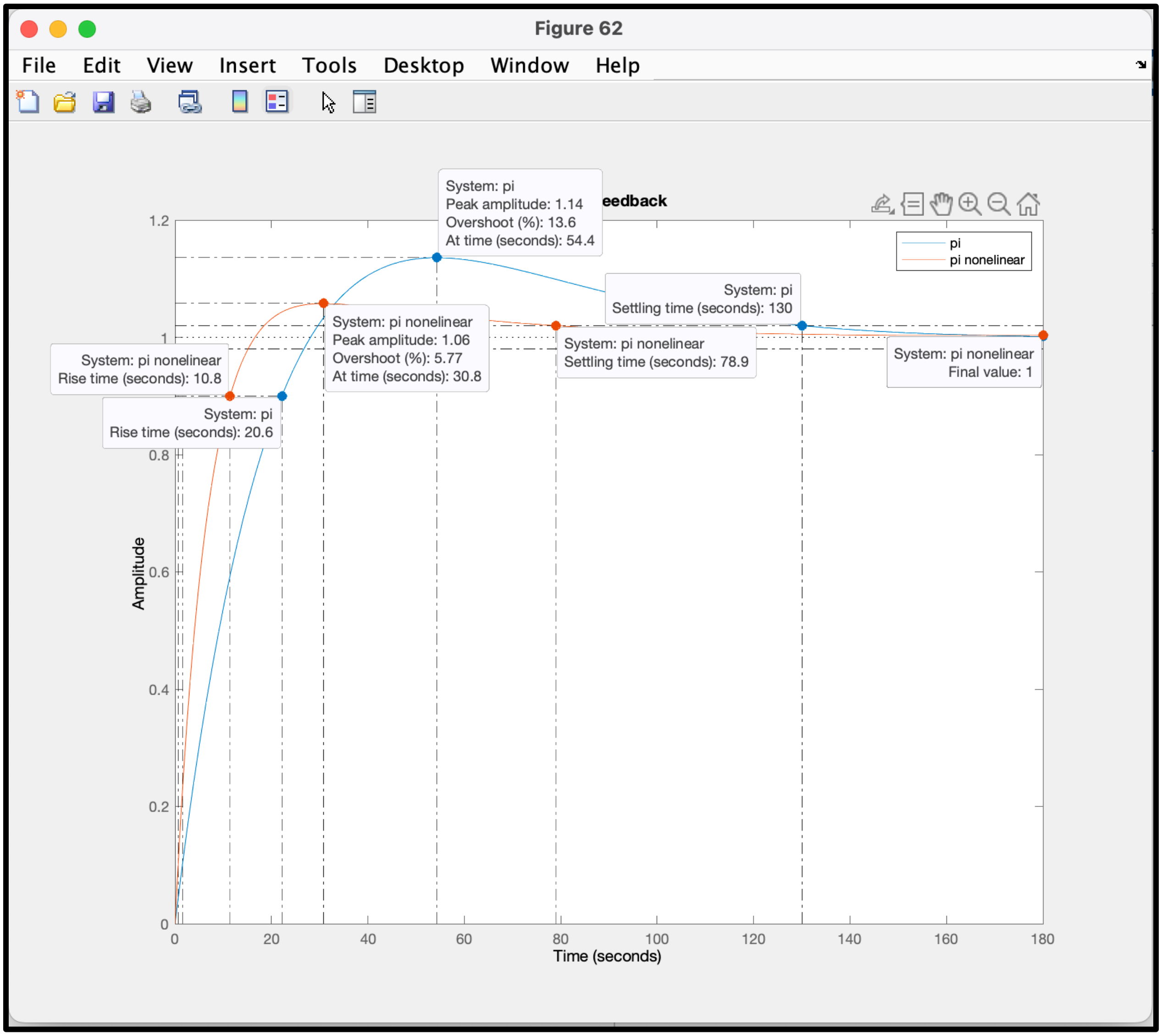


Figure 62
File
Edit
View
Insert
Tools
Desktop
Window
Help
eedback
System: pi
Peak amplitude: 1.14
Overshoot (%): 13.6
At time (seconds): 54.4
pi
pi nonelinear
System: pi
Settling time (seconds): 130
System: pi nonelinear
Peak amplitude: 1.06
Overshoot (%): 5.77
At time (seconds): 30.8
System: pi nonelinear
Settling time (seconds): 78.9
System: pi nonelinear
Final value: 1
System: pi nonelinear
Rise time (seconds): 10.8
System: pi
Rise time (seconds): 20.6
Amplitude
Time (seconds)
1.2
1
0.8
0.6
0.4
0.2
0
20
40
60
80
100
120
140
160
180