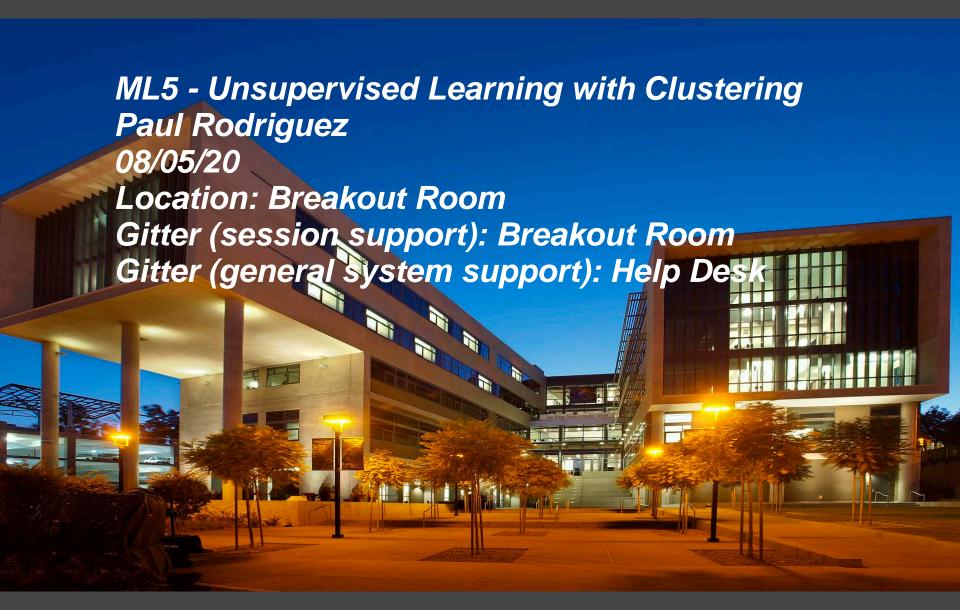
SDSC Summer Institute 2020





K-means cluster idea

Group points into clusters by some measure of distance

Assign points to 1 cluster only

Use clusters as a summary of data



Distance Measures

For numeric data

Euclidean distance (or sum squared differences)



Distance Measures

For numeric data

Euclidean distance (or sum squared differences)

For categorical data 'Gower' metric uses 'same' or 'different' counts

Lots of others...



Objective: minimize within-cluster distances



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Start with K initial cluster centers

spread out initial guesses



Objective: minimize within-cluster distances

Start with K initial cluster centers Loop:

spread out initial quesses

Assign each data point to nearest cluster center

Objective: minimize within-cluster distances

Start with K initial cluster centers Loop:

Assign each data point to nearest cluster center Calculate mean of cluster for new center



Objective: minimize within-cluster distances

Start with K initial cluster centers Loop:

Assign each data point to nearest cluster center Calculate mean of cluster for new center Stop when assignments don't change

converges (but not to global min)



Objective: minimize within-cluster distances

Start with K initial cluster centers Loop:

Assign each data point to nearest cluster center Calculate mean of cluster for new center Stop when assignments don't change

Kmeans works in a variety of situations, but choosing K is sometimes difficult



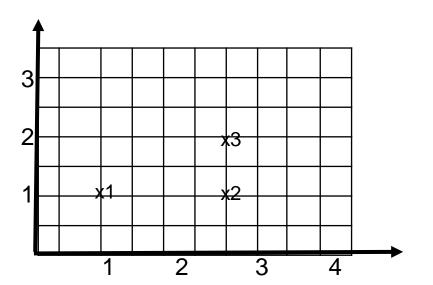
A quick simple example:

For these points

$$x1=(1,1)$$

$$x2=(3,1)$$

$$x3=(3,1.99)$$



For these points

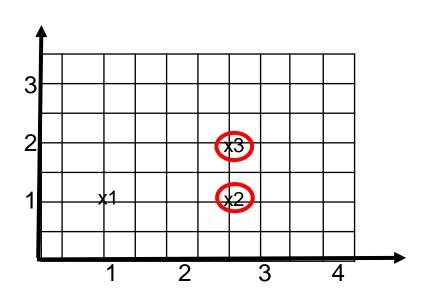
$$x1=(1,1)$$

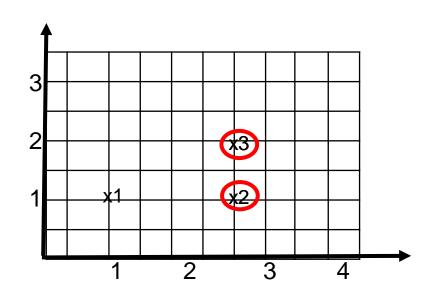
$$x2=(3,1)$$

$$x3=(3,1.99)$$

Use K=2, set initial centers as:

$$\mu_1 = x_2 \text{ and } \mu_2 = x_3$$



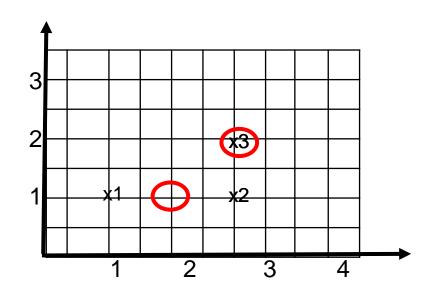


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Loop 1:

get distance of all points to all clusters means, a NxK distance matrix



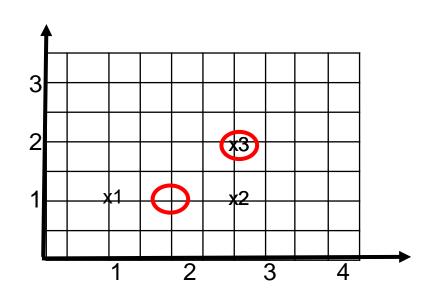
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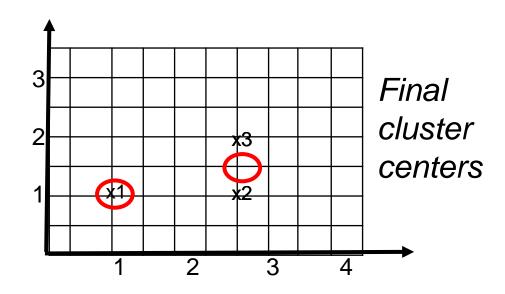
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After calculating new center, what happens to x2 assignment in next loop?





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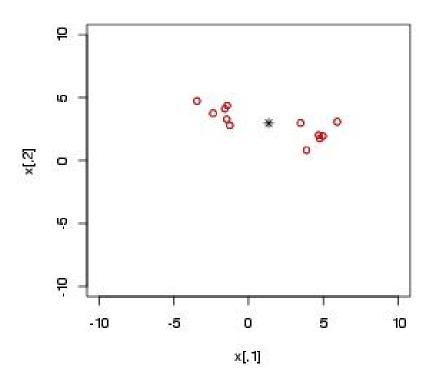
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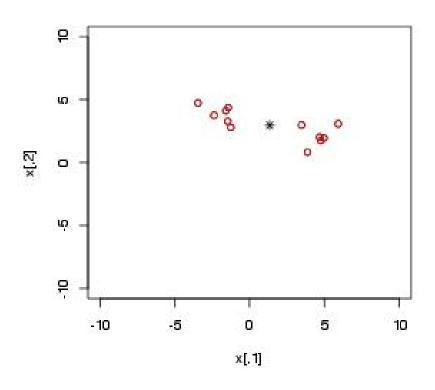
For K=1 where is the cluster center?



Essential R commands: Kresult = kmeans(X,1,10,1)

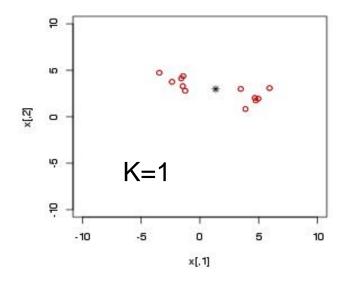
#K=1 #10 is max loop iterations #1 is number of initial sets to try

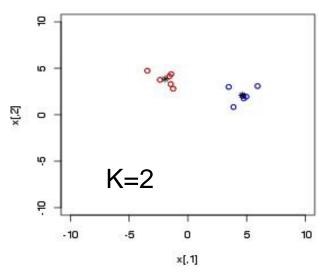
For K=1 where is the cluster center? At the overall mean

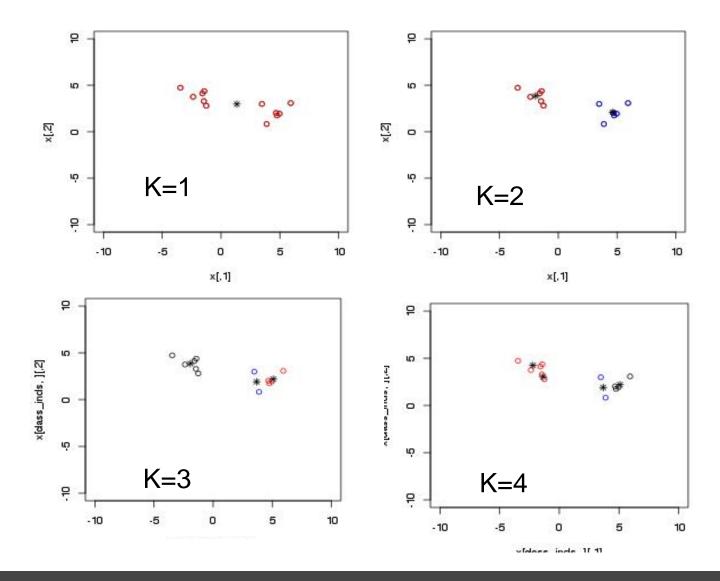


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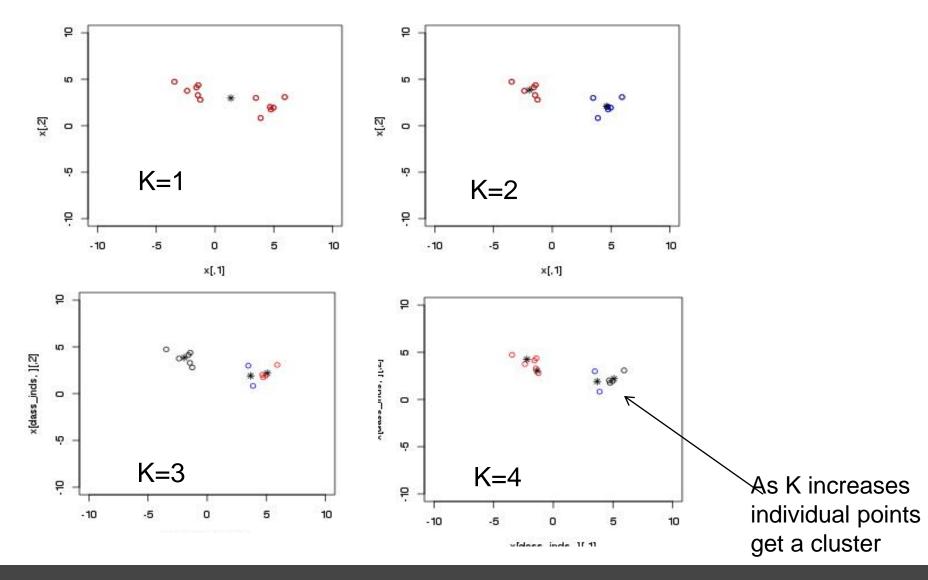
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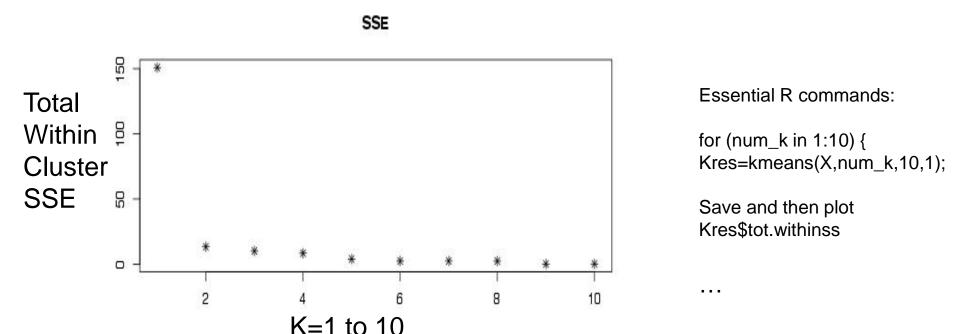








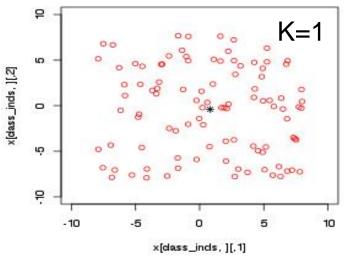
Choosing K for Kmeans

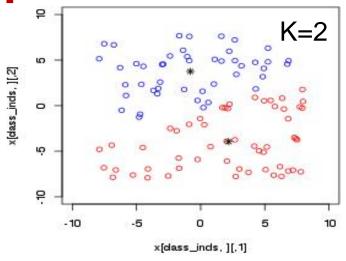


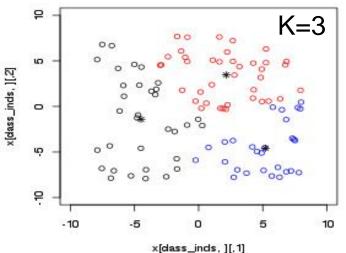
Not much improvement after K=2 ("elbow")

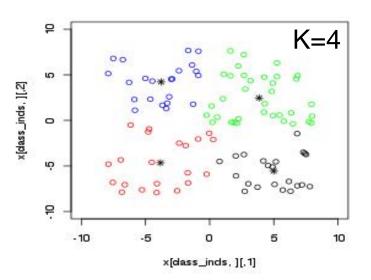


Kmeans Example: uniform dist.

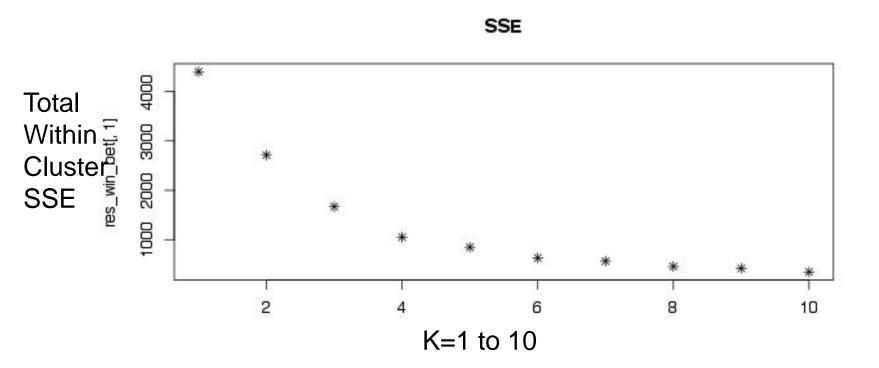








Choosing K - uniform



- Smooth decrease across K => less structure

"Elbow" of Sum Squared Error Within Clusters



"Elbow" of Sum Squared Error Within Clusters

 "Silhouette": mean SSE within a cluster vs to next best cluster

take maximum value over K=1...Kmax



"Elbow" of Sum Squared Error Within Clusters

 "Silhouette": mean SSE within a cluster vs to next best cluster

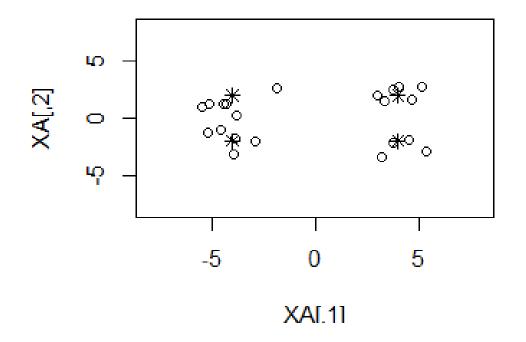
take maximum value over K=1...Kmax

"Gap" value of SSE-within-cluster of data vs uniform distribution

take maximum value over K=1 ...Kmax



Example, 4 clusters normal distribution, small sample

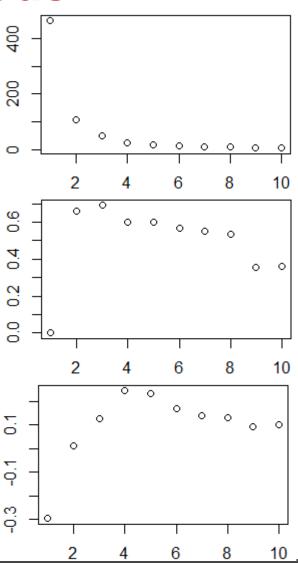




SSE within cluster (elbow)

Silhouette within vs next best

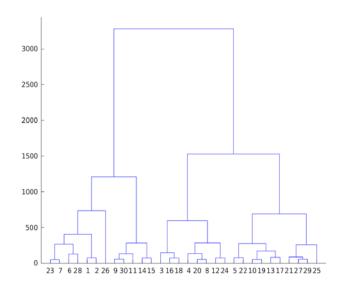
Gap SSE within cluster vs uniform data baseline

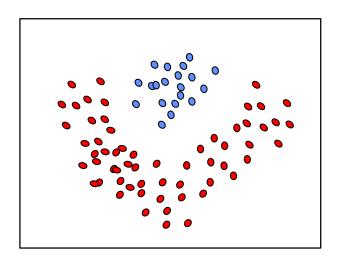


Many other clustering variations -

- Hierarchical clustering

 start with N clusters
 and merge points into
 large clusters (good to
 get whole tree)
- Density based clustering - build and link neighborhoods (good for spatial data)

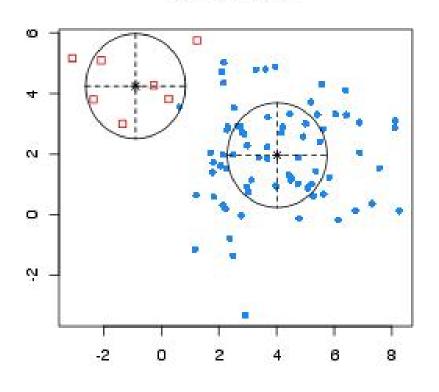




EM clustering

(expectation maximization)

Classification

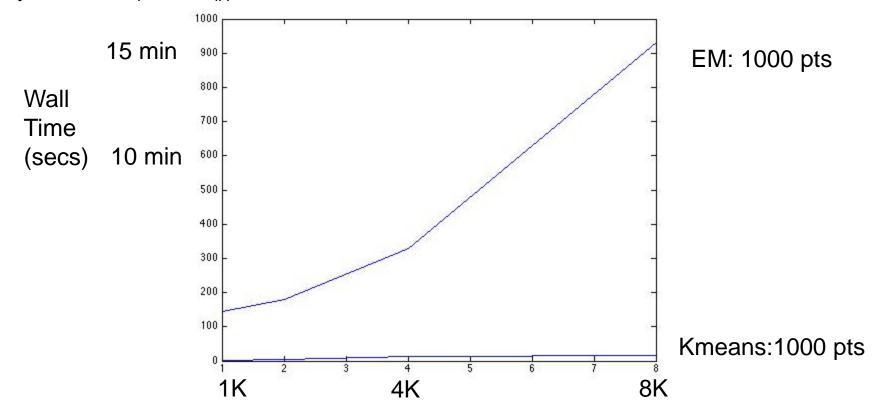


- A probabilistic model using mixture of Gaussians
- Handles unequal variance and/or cluster sizes better than K-means

```
R:
library('mclust')
em_fit=Mclust(x);
plot(em_fit);
```

Kmeans vs EM performance

1 Gordon compute node, normal random matrices R: system.time(Mclust())



Number of Dimensions (i.e. columns in data matrix)



Kmeans big data example

45,000 NYTimes articles, 102,000 unique words

(UCI Machine Learning repository)

Full Data Matrix: 45Kx102K ~ 40Gb

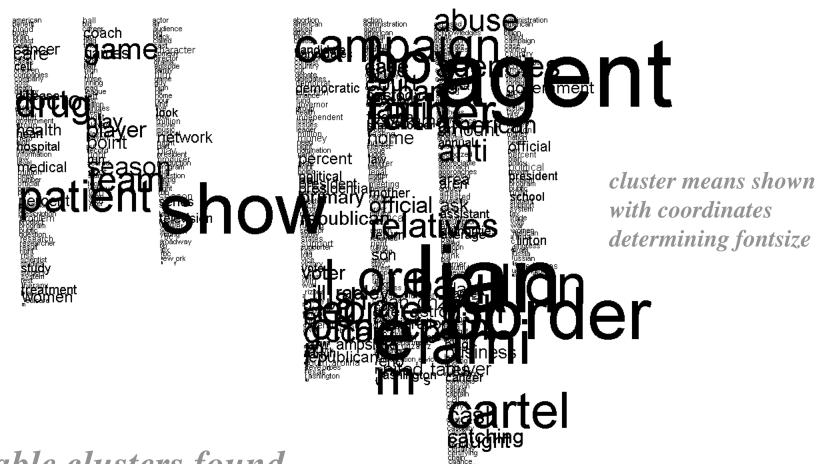
article 1
article 2
article 3
...

article 45K

Cell i,j is count of ith-word in jth-article



Kmeans results



7 viable clusters found



Kmeans for image segmentation

R snippet with K=8

```
install.packages('ripa')
library('ripa')
source("http://bioconductor.org/biocLite.R")
biocLite()
biocLite("EBImage")
library('EBImage')
im=readImage('1a34086v.jpg')
library('ripa')
img=rgb2grey(im, coefs=c(0.30, 0.59, 0.11))
imgx1 =as.vector(img)
numk=8
km_imx1=kmeans(imgx1,numk,50,1);
img km mat =matrix(km imx1$cluster,dim(im)[1],dim(im)[2])
display(img_km_mat/numk)
```







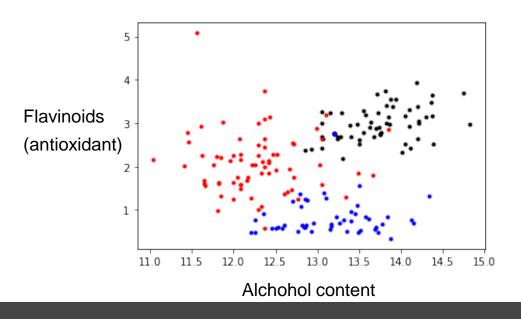
Kmeans with a Winery dataset

(UCI Machine Learning Repository)

178 observations

13 variables of wine characteristics

3 target classes that indicate which winery produced the wine



Black=58 cases Red=71

Blue=48

Will cluster match classes?



Pause



Principle Components vs Clustering

- PCA, SVD reduces dimensions, Clustering reduces to categorical groups
- In some cases, k PCs $\Leftrightarrow k$ clusters
- It is also useful to visualize clusters in PC space



Summary

 Having no label doesn't stop you from finding structure in data

Unsupervised methods are somewhat related

