Marginal Utility Shocks and the Precautionary Saving Puzzle

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Abstract

Keywords: ...

1 Background and Motivation

When markets are complete, no incentives exist for precautionary saving ((Ljungqvist and Sargent 2012)). Perfect consumption smoothing is possible and since people born alike, these models cannot generate heterogeneity. Therefore wealth inequality is absent in these models.

Bewley (1977), Aiyagari (1994), and Huggett (1993) (BAH) introduce market incompleteness, while individual's wealth depends on entire history of idiosyncratic shocks. In these models, lack of full risk-sharing creates precautionary saving motive against uninsurable idiosyncratic income risk, which gives rise to heterogeneity in wealth. BAH models collectively became workhorse models of inequality research in modern macro (aka "income fluctuations models").

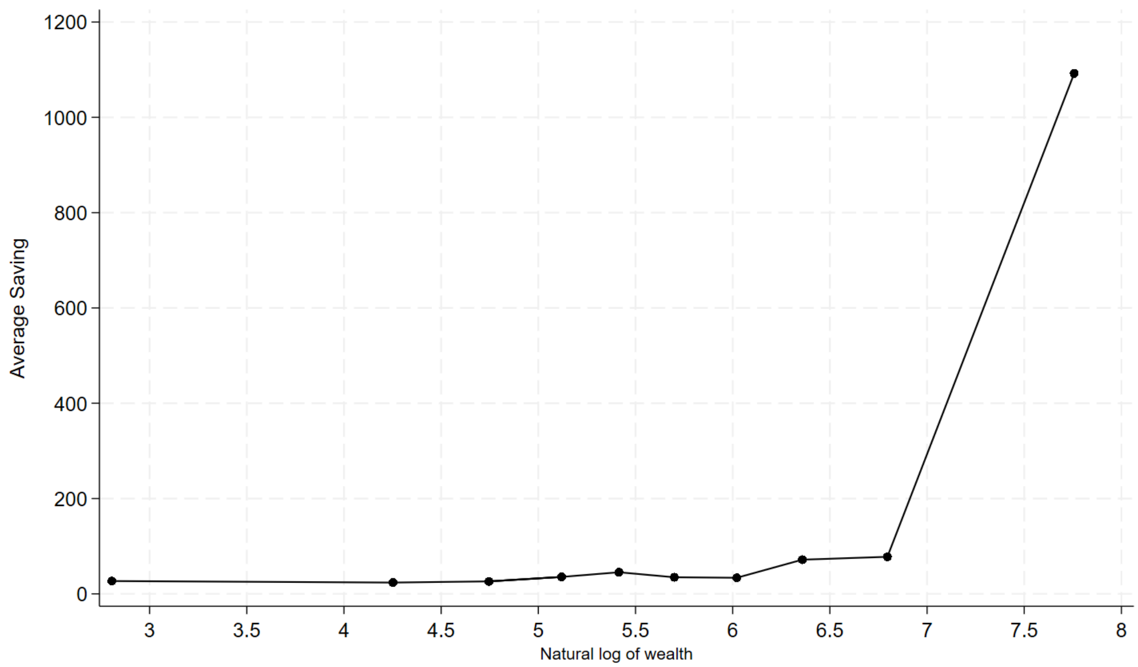
But BAH models cannot account for top inequality. Wealth distribution is Pareto in US, Canada, Europe. Even for extreme distributions of idiosyncratic risk, BAH models predict a thin-tailed wealth distribution. Underlying reason here is even without Arrow securities, it becomes possible to smooth consumption "almost perfectly" with "large enough" wealth. Therefore, BAH models predict a threshold after which individuals cease saving—saving becomes negative.

Empirically, we do not observe BAH predicted saving behaviour. 1 reports saving increases by wealth decile. Where saving is defined by the first difference of wealth. The first seven wealth groups report savings between \$27,000 and \$45,000, with these figures rising to \$72,000 and \$78,000 for the eighth and ninth wealth groups, respectively. A substantial increase in savings is observed for the wealthiest couples, with households in the top group accumulating approximately \$1,092,000 in net worth on average.

To address the non-monotonic behavior, we need a new framework in which inequality at the top arises from precautionary saving of wealthy individuals, and not, *e.g.*, because of idiosyncratic capital income risk, *á la* Benhabib, Bisin, and Luo (2015, 2017).

In our mechanism, shock affects marginal continuation value, in a way that keeping marginal utility from dropping becomes "luxury" compared to consumption (*á la* (Jones et al. 2019)). Therefore, it is optimal to maintain marginal utility, instead of smoothing consumption. Hence, Saving can be bounded away from zero, for certain parameter values.

Figure 1. Average Saving



This figure represents average savings of couple households in different wealth brackets among the US elderly from 1994 to 1996. Average savings are in thousands of dollars.

2 Saving at the Tail

To understand the reason our framework can give rise to a saving function that is increasing in wealth, after some threshold—in contrast to the Bewley (1977), Aiyagari (1994), and Huggett (1993) (BAH)'s framework where saving becomes negative after some threshold—we consider two stylized environments with uninsurable idiosyncratic shocks in this section: An income fluctuations problem, *a lá* BAH, where individuals have to self-insure against fluctuations in their income, and a modified framework where income shocks are replaced by longevity shocks—or, more accurately, shocks to continuation marginal utility.

Consider the problem of an individual in a standard income fluctuations framework, where income can take one of two values, $\left\{\underline{y}, \bar{y}\right\}$, where $\underline{y} < \bar{y}$. If we refer to these two as the "low" and "high states," and denoting the value and policy functions in each state by means of lower and upper bars, respectively, we can write the individual's problem in high state as

$$\begin{aligned} \bar{V}(b) = \max_{c,b'} \quad & \left\{u(c) + \rho\left[\pi \bar{V}(b') + (1-\pi)\underline{V}(b')\right]\right\} \\ s.t. \quad & b' = (1+r)\,b + \bar{y} - c, \\ & b' \geq \underline{b}. \end{aligned} \tag{BAH}$$

In this problem, π is the conditional probability of staying in the high state in the next period, conditioned on being in the high state in the current period. The problem in the low state is rather similar to (BAH), and we omit it here.

In our framework, we replace income shocks by longevity shocks. To this end, we denote an individual's underlying health by h, and assume it can take two values, , $\left\{\underline{h}, \bar{h}\right\}$, where $\underline{h} < \bar{h}$. With some abuse of terminology, we will refer to these two as the "low" and "high states," and keep denoting them via lower and upper bars as in problem (BAH).[1]

To make sure the proceeding arguments deal explicitly and entirely with these two different mechanisms, in this section, we assume that the low state is an absorbing state in this latter

1. This choice of notation serves a simplifying purpose, as it becomes apparent when writing the first order conditions. It should be clear which problem a function refers to from the context. We make the reference explicit if not.

framework: Once an individual enters the low state, he remains there forever. The probability of an individual transitioning to this state from the high state is denoted by $1-\pi$, as in problem (BAH). In addition, we assume that an individual does not face any chance of mortality, whereas this probability in the low state is determined by the *health production function*, $\chi(m)$, where m captures what the individual spends on his health.

As such, the individual's problem in the high state is quite similar to that in (BAH) in this alternative setting:

$$\begin{aligned} \bar{V}(b) = \max_{c,b'} \quad & \left\{u(c) + \rho\left[\pi\bar{V}(b') + (1-\pi)\underline{V}(b')\right]\right\} \\ s.t. \quad & b' = (1+r)b + y - c, \\ & b' \geq \underline{b}, \end{aligned} \tag{BL-H}$$

where y denotes the individuals' income, which is independent of their health state (so that we can focus on marginal utility shocks, as states earlier). In the low state, this problem becomes:

$$\begin{aligned} \underline{V}(b) = \max_{c,m,b'} \quad & \left\{u(c) + \rho\cdot\chi(m)\cdot\underline{V}(b')\right\} \\ s.t. \quad & b' = (1+r)b + y - c - m, \\ & b' \geq \underline{b}. \end{aligned} \tag{BL-L}$$

Note that, in this problem, we have implicitly normalized the value upon death to zero.

In all that follows, we are going to assume that the Bernoulli utility takes the CRRA functional form with a constant term representing the value of being alive:

$$u(c) = \nu + \frac{c^{1-\sigma}}{(1-\sigma)},$$

for some $\sigma > 0$ and $\nu > 0$ such that, for some minimal sustainable level of consumption, $\underline{c}$, we have $u(\underline{c}) > 0$.[2] Moreover, we assume the following form for the health production

2. Otherwise, (BL-L) turns into a trivial problem where individuals prefer death to being alive. The CRRA

function for the sake of tractability:

$$\chi(m) = 1 - \left(\frac{1}{\alpha \cdot m + \underline{h}}\right)^{\beta}, \tag{1}$$

for some $\alpha > 0$ and $\beta \leq 1$.

Consider the Euler equation for an *interior solution* to problems (BAH) and (BL-H):

$$\bar{V}_b(b) = \rho \cdot (1+r) \cdot \left[\pi \bar{V}_b(b') + (1-\pi) \underline{V}_b(b')\right], \tag{HEE}$$

where subscripts show first order derivatives, and we have used the envelope condition to replace for marginal utility of consumption:

$$\bar{V}_b(b) = (1+r) \cdot u_c(c). \tag{HEC}$$

Under the assumption that the value function is concave in wealth in both states, the Euler equation determines the direction of saving at any current level of wealth. To see this more clearly, let us momentarily assume that $\pi = 1$. When so, when $\rho(1+r) = 1$, we must have $b = b'$ and saving will be zero: $c = c' = rb + \bar{y}$ (or $c = c' = rb + y$, when income is constant). In other words, if time discount rate and the rate of return cancel out, individual would "smooth consumption" perfectly. (When $\rho(1+r) < 1$, individual front loads consumption: In this case, for (HEE) to hold, $\bar{V}_b(b)$ must be greater than $\bar{V}_b(b')$. Assuming that the value function is concave, we must have $b' < b$ for this to be the case. The inverse of this argument holds when $\rho(1+r) > 1$.)

In the presence of uninsurable risk, saving might not be negative at lower levels of wealth even when $\rho(1+r) < 1$, as long as the arrival of the shock (whether it is to income or underlying health) significantly increases the marginal effect of assets on future value. We will refer to this—*i.e.* $V_b(b')$—as the *marginal continuation value*. To see this, let us write (HEE) as

$$\pi \left[\frac{\bar{V}_b(b')}{\bar{V}_b(b)}\right] + (1-\pi)\left[\frac{\underline{V}_b(b')}{\bar{V}_b(b)}\right] = \frac{1}{\rho \cdot (1+r)}. \tag{2}$$

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utility form is indispensable to Huggett (1993)'s proof of negative saving above some level.

When value function is concave, the first term in brackets in this equation will be less than one when $b' > b$. Therefore, the only possibility for this equality to hold when $b' > b$ and $\rho(1+r) < 1$ is for $\underline{V}_b(b')$ to greater than $\bar{V}_b(b)$ by a "large enough" margin, and this is the logic behind the terminology "precautionary saving": If the value of having one more unit of wealth is high enough *when an unfavourable shock is realized*, individual tends to save.

One can summarize this argument as follows: Under the concavity assumption for the value function, saving is positive at any current level of wealth b if, and only if,

$$\pi\left[\frac{\bar{V}_b(b)}{\bar{V}_b(b)}\right] + (1-\pi)\left[\frac{\underline{V}_b(b)}{\bar{V}_b(b)}\right] > \frac{1}{\rho\cdot(1+r)}.$$ [3]

This inequality can be rearranged as

$$(1-\pi)\left[\frac{\underline{V}_b(b) - \bar{V}_b(b)}{\bar{V}_b(b)}\right] > \frac{1}{\rho\cdot(1+r)} - 1. \tag{3}$$

The right-hand side of this equality is a positive constant when $\rho(1+r) < 1$. As such, the shape of optimal saving as a function of current wealth, b, is determined by the marginal value of wealth in the low state *relative to that in the high state* (and not by the difference between the two).

This is rather intuitive: Individual saves as a "precaution" against the possibility of low shocks, and as long as the marginal benefits of one additional unit of saving is high enough should the low shock arrives. Saving is a precaution against fluctuations in marginal continuation value when value function is concave. On the other hand, when the ratio of marginal continuation value in the low state to that in the high state tends to one, this precautionary saving motive vanishes: When there are not much fluctuations in the marginal continuation value in the first place, there are no incentives to insure against such fluctuations either.[4]

3. Similarly, saving will be zero or negative, respectively, when wealth is equal to b if, and only if, this inequality holds as strict equality or the left-hand side is strictly less than the constant term on the right-hand side.

4. This is not to say that the only incentive for saving is precautionary motive. The ratio of time discount rate to the risk-free rate is still a strong determinant of saving behaviour. Moreover, the precautionary saving motive is determined by the probability of the realization of unfavourable shocks.

One can use condition (HEC) to write (3) as

$$(1-\pi)\left[\frac{u_c\left(\underline{c}\left(b\right)\right)-u_c\left(\bar{c}\left(b\right)\right)}{u_c\left(\bar{c}\left(b\right)\right)}\right]>\frac{1}{\rho\cdot(1+r)}-1, \tag{4}$$

where $\underline{c}\left(\cdot\right)$ and $\bar{c}\left(\cdot\right)$ are the policy functions in low and high states, respectively. If we replace the utility function with its CRRA form, we can write this inequality as:

$$\left[\frac{\bar{c}\left(b\right)}{\underline{c}\left(b\right)}\right]^{\sigma}>\left[\frac{\rho^{-1}(1+r)^{-1}-\pi}{1-\pi}\right]. \tag{5}$$

Note that the first term of the numerator on the right-hand side of this inequality is greater than one. As such, the right-hand side is a constant, greater than one.

To see the intuition behind this inequality, we can use a linear expansion around $\bar{c}\left(b\right)$ to write the numerator in (4) in terms of consumption in two states:

$$(1-\pi)\left[\frac{u_{cc}\left(\bar{c}\left(b\right)\right)\left[\underline{c}\left(b\right)-\bar{c}\left(b\right)\right]}{u_c\left(\bar{c}\left(b\right)\right)}\right]>\frac{1}{\rho\cdot(1+r)}-1.$$

Using the CRRA utility form, the ratio on the left-hand side of this inequality can be written as:

$$1-\frac{\underline{c}\left(b\right)}{\bar{c}\left(b\right)}>\frac{1-\rho\cdot(1+r)}{\sigma\cdot(1-\pi)\cdot\rho\cdot(1+r)}.\text{[5]} \tag{6}$$

As such, the shape of saving function depends critically on σ, π, and $\rho\left(1+r\right)$, as well as the ratio $\underline{c}\left(b\right)/\bar{c}\left(b\right)$ given these parameters. As this inequality suggests, if $1-\pi$ or σ are too small, it is well possible for saving to remain negative at all levels of wealth. This is rather intuitive: When the possibility of a negative shock is "too small," or when individuals are "too risk neutral," precautionary saving motive would also be too small. In such cases, if the incentive to front-load consumption is strong enough—when $\rho\left(1+r\right)$ is far from unity—individual might not have any incentives to save against unlikely adverse shocks in the future.

5. Note that, since $u_c\left(\cdot\right)$ itself is a convex function, this inequality is the sufficient condition for the saving rate to be positive and not a necessary condition, at least unless consumption is not "too large."

On the other hand, when $1-\pi$ and σ are large enough, it is possible for the precautionary saving motive to dominate the motive to front-load consumption, leading to positive saving. When this is the case, we'd expect the ratio of consumption in two states to remain far from one. The same forces that determine how this ratio changes as wealth increases (in the optimal solution) also determine the shape of optimal saving as a function of wealth.

The aforementioned arguments are summarize in lemma 2.1, under the assumption that the solutions to the functional equations in (BL-H) and (BL-L) are concave. This assumption, however, cannot be proved analytically: The product of two concave functions on the right-hand side of functional equation (BL-L) is not necessarily concave. As a result, the usual inheritance arguments in dynamic programming do not carry over to this case. As such, throughout this section, we maintain the following assumption:

Assumption 2.1 *The solution to the functional equation* (BL-L), $\underline{V}\left(\cdot\right)$, *is concave.*

Note that, if $\underline{V}\left(\cdot\right)$ is concave, $\bar{V}\left(\cdot\right)$ will necessarily be concave. Moreover, we know from the envelope condition that assumption 2.1 is equivalent to the policy function, $\underline{c}\left(\cdot\right)$, being increasing. In our numerical experiments, we have not managed to come up with a set of parameter values that violate this latter condition.

Lemma 2.1 *Under assumption 2.1, optimal saving in* (BAH) *or* (BL-H) *is positive when wealth is b if, and only if, the ratio of optimal consumption in high state to that in low state satisfies* (5).

What about non-interior solutions to (BL-H)? Euler equation in high vs low states, binding of borrowing constraint

The preceding argument suggests that, to determine the shape of saving as a function of current wealth in either problem, one can focus on the ratio $\bar{c}\left(b\right)/\underline{c}\left(b\right)$ and how it changes with wealth. One can show that, in problem (BAH), this ratio converges to one as $b\rightarrow\infty$, and, as such, there exists some $\bar{b}$ above which saving inevitably becomes negative.[6]

6. This claim, stated by Huggett (1993) in Theorem 2, enables him to show that an individual's wealth does not exceed $\bar{b}$. As such, the state-space is bounded and a stationary distribution of wealth can emerge as a result. We postpone the proof of this claim for problem (BAH)—which is different from Huggett's proof—to appendix A.

In our environment with health shocks, on the other hand, the same mechanics but in the opposite direction indicate the exact inverse regarding the saving behaviour: that optimal saving may have to be positive above some threshold $\bar{b}$ (while it can be negative below $\bar{b}$). The reason that the ratio $\bar{c}(b)/\underline{c}(b)$ may be bounded away from one in problem (BL-H) is due to the role of health spending in (BL-L).

The intuition is as follows: If health spending is a luxury good, its share in total expenditures must increase with wealth compared to that of consumption. In other words, in the presence of this luxury good, the marginal value of saving remains high in the low state compared to that in the high state, even as individual becomes wealthier. As such, the precautionary saving motive does not vanish with wealth, as is the case in the income fluctuation problem in (BAH).

To formally derive the conditions under which inequality (5) holds, at least above some wealth threshold $\bar{b}$, we start by characterizing the optimal medical spending in the low-state, in (BL-L). When interior, the optimal health spending is given by the following first order condition:

$$\rho \cdot \chi_m(m) \cdot \underline{V}(b') = u_c(c).$$

Replacing the utility and health production functions by their functional forms, we can rearrange this equation to find optimal health spending as a function of optimal consumption in the low state. This is given by

$$\underline{m}(b) = \left[\frac{\rho \cdot \beta \cdot \underline{V}(\underline{b}'(b))}{\alpha^{\beta}}\right]^{\left(\frac{1}{1+\beta}\right)} \cdot [\underline{c}(b)]^{\left(\frac{\sigma}{1+\beta}\right)} - \left(\frac{h}{\alpha}\right), \tag{7}$$

when the right-hand side of (7) is positive, and zero otherwise. In this equation, $\underline{m}(\cdot)$, $\underline{c}(\cdot)$ and $\underline{b}'(\cdot)$ are the policy functions for medical spending, consumption and future wealth in the low state, respectively.

We are now in a position to prove the following theorem, the central result of this paper:

Theorem 2.2 *Under assumption 2.1, saving rate in problem* (BL-H) *cannot be negative above some threshold $\bar{b}$, when $\sigma > 1 + \beta$.*

Proof. We accept, without proof, that the value function in (BL-L) is non-decreasing in current wealth. As such, we can use (7) to conclude:

$$
\begin{aligned}
\underline{m}\left(b\right) = \left[\frac{\rho\cdot\beta\cdot\underline{V}\left(\underline{b}'\left(b\right)\right)}{\alpha^{\beta}}\right]^{\left(\frac{1}{1+\beta}\right)}\cdot\left[\underline{c}\left(b\right)\right]^{\left(\frac{\sigma}{1+\beta}\right)}-\left(\frac{\underline{h}}{\alpha}\right) \\
\geq\left[\frac{\rho\cdot\beta\cdot\underline{V}\left(\underline{b}\right)}{\alpha^{\beta}}\right]^{\left(\frac{1}{1+\beta}\right)}\cdot\left[\underline{c}\left(b\right)\right]^{\left(\frac{\sigma}{1+\beta}\right)}-\left(\frac{\underline{h}}{\alpha}\right) \\
= A\cdot\left[\underline{c}\left(b\right)\right]^{\left(\frac{\sigma}{1+\beta}\right)}-\left(\frac{\underline{h}}{\alpha}\right),
\end{aligned}
\tag{8}
$$

where

$$
A := \left[\frac{\rho\cdot\beta\cdot\underline{V}\left(\underline{b}\right)}{\alpha^{\beta}}\right]^{\left(\frac{1}{1+\beta}\right)} > 0.
$$

The claim that $A > 0$ follows from the assumption that, even at $b = \underline{b}$, individual can maintain a sustenance level of consumption, delivering a utility that is strictly greater than zero—*i.e.* the utility upon death. When $\sigma > 1$, the value of being alive, $\nu > 0$, guarantees that this is the case. Since such a stream of consumption is sustainable as long as the individual lives, value function must remain strictly positive for all b.

Let us assume, for the sake of contradiction, that saving is non-positive above some arbitrary threshold $\tilde{b}$. Then, we must have

$$
\bar{c}\left(b\right) \geq r\cdot b + y, \qquad \forall b \geq \tilde{b}.
$$

If we let

$$
\gamma := \liminf_{b\to\infty}\frac{\bar{c}\left(b\right)}{\underline{c}\left(b\right)},
$$

condition (5) implies that

$$
\gamma \leq \left[\frac{\rho^{-1}(1+r)^{-1}-\pi}{1-\pi}\right]^{\frac{1}{\sigma}}, \tag{9}
$$

above some threshold of wealth. Therefore, we can assume without any loss of generality that, for all $b \geq \tilde{b}$,

$$
\underline{c}\left(b\right) \geq \gamma\cdot\bar{c}\left(b\right) = \gamma\left(r\cdot b + y\right), \tag{10}
$$

and—from (8),

$$\underline{m}(b) \geq A\left[\gamma \cdot \bar{c}(b)\right]^{\left(\frac{\sigma}{1+\beta}\right)} - \left(\frac{\underline{h}}{\alpha}\right) \geq A\left[\gamma\left(r \cdot b + y\right)\right]^{\left(\frac{\sigma}{1+\beta}\right)} - \left(\frac{\underline{h}}{\alpha}\right). \tag{11}$$

If we replace these in individual's budget constraint, we get:

$$\begin{aligned}\underline{b}'(b) &= (1+r)\,b + y - \underline{c}(b) - \underline{m}(b)\\ &\leq (1+r)\,b + y + \left(\frac{\underline{h}}{\alpha}\right) - \gamma\left(r \cdot b + y\right) - A\left[\gamma\left(r \cdot b + y\right)\right]^{\left(\frac{\sigma}{1+\beta}\right)}\\ &= -A\left[\gamma\left(r \cdot b + y\right)\right]^{\left(\frac{\sigma}{1+\beta}\right)} + \left(1 + r - \gamma \cdot r\right) b + \left(1-\gamma\right) y + \left(\frac{\underline{h}}{\alpha}\right).\end{aligned} \tag{12}$$

When $\sigma > 1 + \beta$, the polynomial on the right-hand side of this inequality is concave and, as such, decreases without bound with b. As a result, the borrowing constraint must bind for all $b \geq \hat{b}$, for some large enough $\hat{b}$. We let $\bar{b}$ denote the maximum of this threshold and $\tilde{b}$:

$$\bar{b} := \max\left\{\tilde{b}, \hat{b}\right\}.$$

The Kuhn-Tucker conditions for problem (BL-H) when the borrowing constraint binds imply that

$$u_c(c) = \rho \cdot \chi(m) \cdot \underline{V}_b(b') + \mu, \tag{13}$$

where $\mu \geq 0$ is the Lagrange multiplier on the borrowing constraint, which is strictly positive when this constraint binds. Thus, for all $b \geq \bar{b}$, we should have:

$$u_c(c) = \chi(m) \cdot \rho \cdot \underline{V}_b(\underline{b}) + \mu. \tag{14}$$

As noted earlier, the term $\rho \cdot \underline{V}_b(\underline{b})$ in this equality is a positive constant, independent of b. Moreover, from (10) and (11), we know that $\underline{c}(b)$ and $\underline{m}(b)$ both increase without bound with b. Therefore, $u_c(\underline{c}(b)) \searrow 0$ and $\chi(\underline{m}(b)) \nearrow 1$, as $b \to \infty$. Therefore, as long as μ is to remain positive, equation (14) is bound to get violated for some $b \geq \bar{b}$. This contradicts the optimality of $\underline{c}(b)$ and $\underline{m}(b)$, and completes our proof. □

In the proof of theorem 2.2, what causes a contradiction is not whether or not $\bar{c}(b)/\underline{c}(b)$

converges to a constant term *per se*. In fact, this ratio can theoretically converge to a constant even when saving remains positive for all b above some threshold $\bar{b}$, as long as this ratio is above the threshold prescribed by (5). What leads to the contradiction is the assumption that saving becomes negative for large enough b. As such, $\bar{c}(b)$ must increase with $rb + y$ to keep the saving negative. Moreover, negative saving implies that $\bar{c}(b)$ and $\underline{c}(b)$ cannot diverge. It is the combination of these two that lead to a contradiction. For saving to remain positive, then, we need the ratio $\bar{c}(b)/\underline{c}(b)$ to remain above

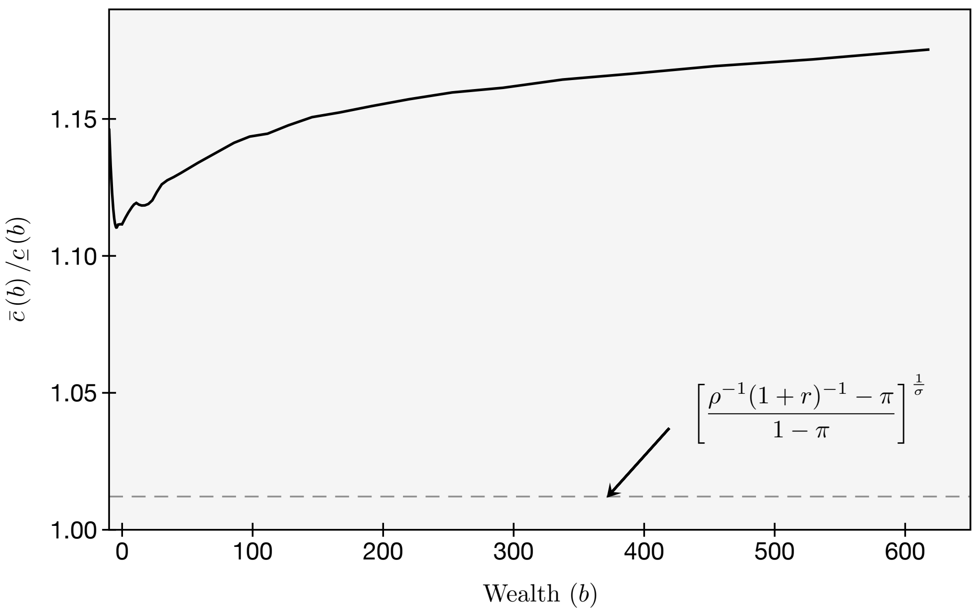
$$\left[\frac{\rho^{-1}(1+r)^{-1} - \pi}{1-\pi}\right]^{\frac{1}{\sigma}}.$$

As figure 2 demonstrates how, for our sample choice of parameter values in ..., consumption in the high state diverges from total income, leading to excessive saving. Panel (b) in this graph suggests that the ratio $\bar{c}(b)/\underline{c}(b)$ remains consistently above the right-hand side of (5). (In the numerical solutions to the problem in (BL-H) and (BL-L), this ratio seems to be exploding, though at a slow rate, as can be seen in figure 2 too.)

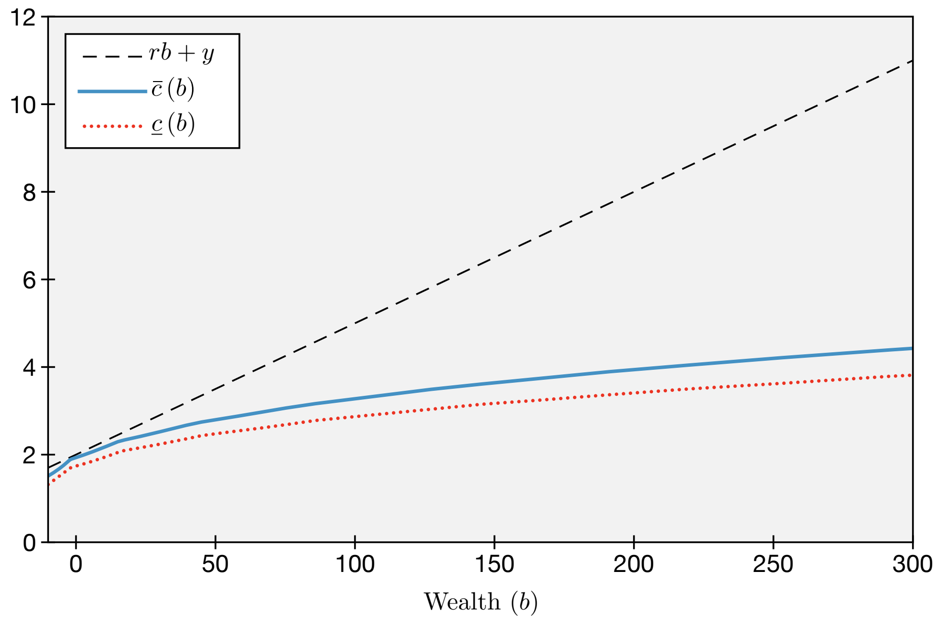
The condition $\sigma > 1 + \beta$ in theorem 2.2 has an intuitive and straightforward interpretation: As long as the *curvature* of the marginal utility (and not the marginal utility itself) is greater than the *curvature* of the marginal product of health spending (in the health production function), then health is a luxury good. It is important to note that it is not the result that health spending is a luxury that causes precautionary saving to increase without bound, *per se*; But It is the that such spending is necessitated by an adverse health shock.

It is also worth mentioning that $\sigma > 1+\beta$ is not a necessary condition for health spending to increase faster than consumption with wealth, but a sufficient one: Any assumption implied by equation (7) that causes health spending to increase faster than consumption with b suffices for the proof of theorem 2.2 to go through. This can be a high enough value of being alive (ν) or a high enough share of health spending (α) in the health production function, as confirmed by our numerical results.

Figure 2. Consumption in Low and High States as Function of Wealth



Panel (a). Consumption in high state to low state



Panel (b). Consumption in two states vs income

Note: For $\bar{c}(b)/\underline{c}(b)$, centred moving averages are drawn to smooth out high-frequency fluctuations at lower wealth levels due to numerical error.

Stationary Distribution

In this section, we show how the combined effect of saving at the tail and random separation (i.e., mortality) can lead to a stationary Pareto distribution of wealth in a partial equilibrium setting.

To this end, consider an economy in which, in each period t, a new cohort of individuals—of measure ψ_0—enters the economy with initial wealth distribution $\varphi_0(\cdot)$ (where $\varphi_0(\cdot)$ is a probability measure, defined on $(S, \mathcal{S})$, in which S is a Borel subset of $\mathbb{R}$), in high health.

An individual's problem is the same as before, with healthy individuals facing a probability $1 - \pi$ of transition to the low health state, and individuals in low health with medical spending m facing probability $1 - \chi(m)$ of mortality.

The distribution of wealth in this economy can be written as the sum of the conditional distribution of wealth for individuals in high and low health states, weighted by the relative measure of each. If one of these distributions follows a power law—e.g., is Pareto—then the sum of the two will also follow a power law, even if the other one does not. As noted by (Gabaix 2, 2009), *Power Laws in Economics and Finance*:

> "Power laws also have excellent aggregation properties. [...] The general rule is that, when combining two PL variables, the fattest (i.e., the one with the smallest exponent) PL dominates. [...] Hence, multiplying by normal variables, adding nonfat tail noise, or summing over independent and identically distributed (i.i.d.) variables preserves the exponent."

As such, in what follows, we focus on the tail behaviour of the conditional distribution of wealth for high-health individuals, and show that this distribution follows a power law with a power exponent equal to one—i.e., it follows Zipf's law at the tail.

Lemma 2.3 *Any discrete-time, heterogeneous-agent economy with random separation has a stationary state-distribution.*

Proof. Consider an economy in which individuals are heterogeneous in some dimension—say b, where b can be any vector of states. Suppose an initial measure ψ_0 of individuals enter the economy in every period, with initial distribution $\varphi_0(\cdot)$. Of these individuals, fraction $1-\pi$ separate in each subsequent period.

Assuming P is the economy's transition function and T^* the adjoint operator associated with it, the distribution of individuals who enter in t, in $t+n$ would be

$$T^{*n}\varphi_0(\cdot) = \varphi_n(\cdot).$$

Then, for any $s \in S$, we must have:

$$\left(\frac{\psi_0}{1-\pi}\right) \cdot \varphi(s) = \psi_0 \big[\, \varphi_0(s) + \pi \cdot \varphi_1(s) + \pi^2 \cdot \varphi_2(s) + \pi^3 \cdot \varphi_3(s) + \ldots \big].$$

Since $\varphi_n(\cdot) < 1$ for all n (and $\pi < 1$), this infinite sum is bounded and converges for all s. $\square$

Theorem 2.4 *When $b' \geq (1+\gamma)b$, for some $\gamma > 0$, the stationary distribution of wealth follows Zipf's law at the upper tail.*

Proof. Let saving be positive for some $x > \underline{b}$, and assume $b' = (1+\alpha)b$ for all $b \geq x$. Because we are only interested in the tail behaviour of the wealth distribution, we assume—without any loss of generality—that $S = [x, +\infty)$.

Since a stationary distribution exists, the above-mentioned assumption on saving implies:

$$\Pr(b \geq x) = \pi \cdot \Pr\left(b \geq \frac{x}{1+\alpha}\right).$$

Also, for x, we must have $\Pr(x \geq x) = 1$. If n is the smallest integer for which $x \leq (1+\alpha)^n \cdot x$, then

$$\Pr(b \geq x) \geq \pi^n \cdot \Pr(x \geq x) = \pi^n.$$

Writing n as

$$n \geq \frac{\ln(x/x)}{\ln(1+\alpha)},$$

we have

$$\ln \Pr(b \geq x) \;\geq\; n \ln(1-\pi) \;\geq\; \frac{\ln(1-\pi)}{\ln(1+\alpha)} \cdot \ln\left(\frac{x}{x}\right).$$

Define

$$\gamma = -\frac{\ln(1-\pi)}{\ln(1+\alpha)} > 0.$$

Thus,

$$\ln \Pr(b \geq x) \;\geq\; \gamma \cdot \ln\left(\frac{x}{x}\right),$$

or equivalently,

$$\Pr(b \geq x) \;\geq\; \left(\frac{e^{\gamma} \cdot x}{x}\right).$$

This shows that the distribution of wealth in this economy follows Zipf's law in its tail, regardless of what the conditional distribution of wealth for low-health individuals looks like. □

3 High Utilization State and Realized Health Spending

Life expectancy in low state:

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Appendices

A Huggett (1993): Theorem 2

Lemma A.1 *For all* b, $b_l'(b) \leq b$.

Proof. The Euler equation for the low-type with wealth b is

$$u'\left(c_l\left(b\right)\right) = \frac{\rho}{\left(1+r\right)}\left[\pi_l u'\left(c_l\left(b_l'\left(b\right)\right)\right) + \left(1-\pi_l\right)u'\left(c_h\left(b_l'\left(b\right)\right)\right)\right]$$
$$< \pi_l u'\left(c_l\left(b_l'\left(b\right)\right)\right) + \left(1-\pi_h\right)u'\left(c_h\left(b_l'\left(b\right)\right)\right). \quad (15)$$

Since $c_l\left(b\right) \leq c_h\left(b\right)$, this last inequality holds only if

$$u'\left(c_l\left(b_l'\left(b\right)\right)\right) > u'\left(c_l\left(b\right)\right),$$

or $c_l\left(b_l'\left(b\right)\right) < c_l\left(b\right)$. Since $c_l\left(b\right)$ is non-decreasing in b, this is true only if $b_l'\left(b\right) < b$. □

Note that $b_l'\left(-\underline{b}\right) \leq -\underline{b}$ can only hold as a strict equality. As a result, $b_l'\left(b\right)$ must pass through the point $\left(-\underline{b}, -\underline{b}\right)$ and lie under the 45-degree line for all values of b.

Lemma A.2 *When $b_h'\left(b\right) > b$, then $c_h\left(b\right) - c_l\left(b\right) < e_h - e_l$.*

Proof. By lemma A.1, we know that $rb + e_l - c_l\left(b\right) \leq 0$, or

$$c_l\left(b\right) - e_l \geq rb.$$

On the other hand, for $b_h'\left(b\right) > b$, we need to have $c_h\left(b\right) - e_h < rb$, or

$$-\left[c_h\left(b\right) - e_h\right] > -rb.$$

If we add these two inequalities, we get the intended result. □

A corollary of this is that

$$0 < b_h'\left(b\right) - b_l'\left(b\right) = e_h - e_l - \left[c_h\left(b\right) - c_l\left(b\right)\right] < e_h - e_l. \quad (16)$$

Therefore,

$$b_l'\left(b\right) \geq b - \left(e_h - e_l\right). \quad (17)$$

If this were the case, then

If we let $\kappa := (e_h - e_l)$, from lemma A.2, we can write $c_h(b) - \kappa < c_l(b) \leq c_h(b)$, where the second inequality follows from the monotonicity of optimal consumption. Therefore,

$$u'(c_l(b)) < u'(c_h(b) - \kappa) \approx u'(c_h(b)) - \kappa u''(c_h(b)). \tag{18}$$

Let us plug this into the Euler equation for the high-types:

$$\begin{aligned} u'(c_h(b)) &= \frac{\rho}{(1+r)}\left[\pi_h u'(c_h(b_h'(b))) + (1-\pi_h) u'(c_l(b_h'(b)))\right] \\ &< \pi_h u'(c_h(b_h'(b))) + (1-\pi_h) u'(c_l(b_h'(b))) \\ &< \pi_h u'(c_h(b_h'(b))) + (1-\pi_h)\left[u'(c_h(b_h'(b))) - \kappa u''(c_h(b_h'(b)))\right]. \end{aligned} \tag{19}$$

The last inequality can be rearranged and written as

$$u'(c_h(b)) - u'(c_h(b_h'(b))) < -(1-\pi_h)\kappa u''(c_h(b_h'(b))). \tag{20}$$

If we replace for the first and second derivatives of the utility function, assuming a CRRA felicity, we have

$$\frac{1}{[c_h(b)]^\sigma} - \frac{1}{[c_h(b_h'(b))]^\sigma} < \frac{(1-\pi_h)\kappa}{[c_h(b_h'(b))]^{\sigma+1}}. \tag{21}$$

The next theorem uses this inequality to show that the individual's saving rate must eventually become negative, regardless of her level of current income.

Theorem A.3 *There must exist some $\underline{b} < \bar{b} < \infty$ for which, for all $b \geq \bar{b}$, we must have $rb + e_h - c_h(b) \leq 0$.*

Proof. Suppose, for the sake of contradiction, that this were not the case, and, for all $\underline{b} < b$, optimal policy satisfies $rb + e_h - c_h(b) > 0$. Then, by the monotonicity of the optimal consumption in income, and by lemma A.1, we have

$$rb + e_l < c_l(b) \leq c_h(b) < rb + e_h. \tag{22}$$

In other words, as b □

B Data Appendix

B.1 Data Description and Sample Selection

Data on asset holdings of elderly residents in the United States are available from the Rand HRS dataset, a panel survey of Americans over the age of 50. The Health and Retirement Study (**HRS**) is sponsored by the National Institute on Aging (grant number NIA U01AG009740) and conducted by the University of Michigan. Surveys are collected every two years at the household level at the end of each survey year, beginning in 1992.

The selection criteria are based on age and the year of the interview. To reduce the impact of age-related variations in the analysis, a specific age group is selected. Table 1 shows the distribution of couple households across different age groups, with the highest number of observations in wave 3 for couples aged 55 to 64. Similarly, Table 2 displays the corresponding data for single households, which generally have enough observations to form an age-wave cohort. This is because when a member of a couple household dies, they transition into single households, thereby increasing their numbers.

The chosen age-wave cohort for analysis consists of wave 3 households with members aged 55 to 64. This group was selected because it contains the largest number of observed couples, totaling 1,555 couple households, along with 1,610 single households. This ensures a robust sample size for examining household wealth within this demographic.

B.2 Wealth Variables

The data on total wealth include the net value of real estate—such as primary and secondary residences—as well as vehicles, businesses, IRA and Keogh accounts, stocks, mutual funds, investment trusts, checking, savings, and money market accounts, certificates of deposit, government savings bonds, Treasury bills, bonds and bond funds, and other financial components. The Rand HRS then deducts the net value of mortgages, land contracts, home loans, and other debts from this total to compute net worth.

Figure 3 illustrates the distribution of total wealth across different wealth deciles, calculated as the proportion of total wealth held by households in each decile, with results shown

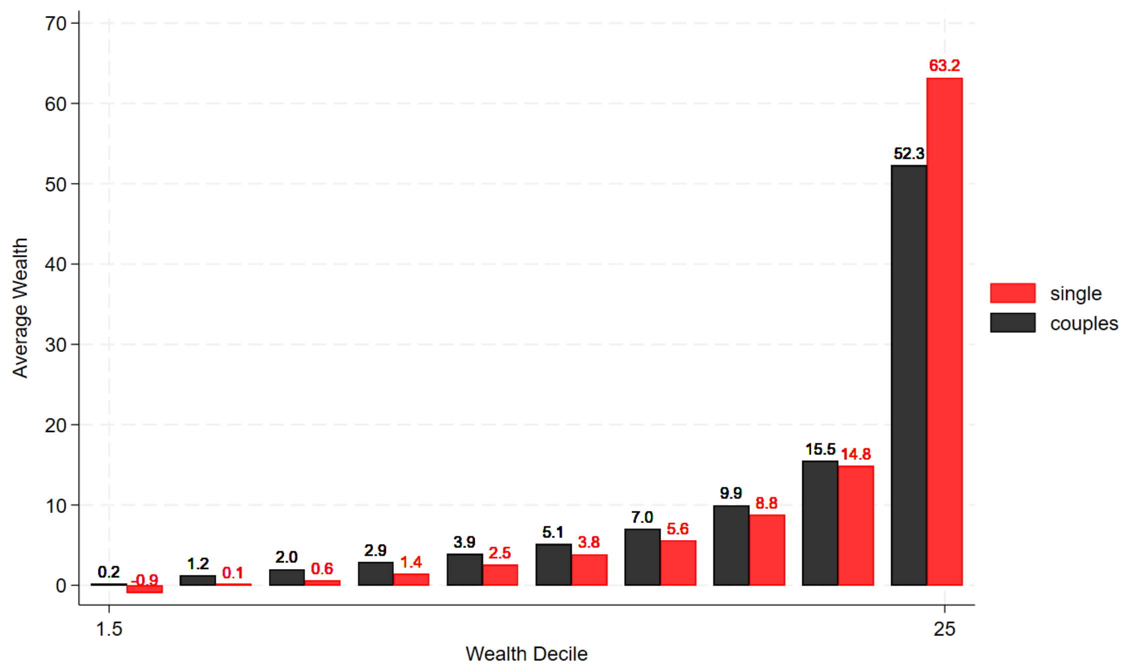
Table 1. Age Groups

Couples					Singles				
Wave / Age	55-64	65-74	75-84	85-94	Wave / Age	55-64	65-74	75-84	85-94
1992	1468	0	0	0	1992	1341	7	0	0
1994	1507	741	485	70	1994	1480	1077	1896	690
1996	1555	582	504	79	1996	1610	705	1886	876
1998	1552	1171	507	77	1998	1402	1365	1778	989
2000	1416	1124	498	64	2000	1122	1277	1528	961
2002	1265	1142	552	61	2002	936	1271	1318	925
2004	1091	1120	564	74	2004	858	1194	1071	838
2006	880	1115	573	81	2006	772	1183	897	774
2008	833	1107	615	93	2008	815	1018	807	716
2010	1125	1017	599	89	2010	1685	818	824	617
2012	1213	917	612	114	2012	1823	674	792	478
2014	1176	791	570	107	2014	1839	692	763	392
2016	1170	664	542	106	2016	2180	822	719	346
2018	944	613	510	94	2018	1961	839	544	284
Total	17195	12104	7131	1109	Total	19824	12942	14823	8886

This table presents the number of households in different age groups and years.

separately for couples and singles. These findings align with existing wealth distribution reports in the United States.

Figure 3. Share of Wealth (1996) couple vs single - 55 to 65 years old



This figure displays the distribution of wealth across different wealth deciles among the US elderly from 1994 to 1996. The red bars indicate the share of wealth held by single households, while the black bars represent the share held by couples. The share is calculated by dividing the total value of assets owned by households in each decile by the overall total value of assets.

As reported by the Fed, Americans held a total wealth of $29.13 trillion in 1996. The top 0.1% of the wealth distribution accounted for $3.37 trillion, the 99th to 99.9th percentiles held $4.63 trillion, and the 90th to 99th percentiles held $9.94 trillion. This indicates that the top 10 percent of the wealth distribution controlled 61.% of the total wealth.

Similarly, the Rand HRS data show that a substantial share of total wealth is concentrated within the top decile, specifically among individuals aged 55 to 65. Within this group, single households in the top decile hold 63.2% of total wealth, while couple households hold 52.24%, which is comparatively lower than singles.

Table 2 presents the average and median wealth of households. For single individuals, the mean and median wealth are similar, except for those in the highest and lowest deciles. Households in the lowest wealth category have an average net liability of approximately \$21,000, whereas the median household in this category has a net debt of \$3,000. Conversely, households in the highest wealth category have an average net worth of \$1,442,000, while the median household possesses assets totaling \$833,000.

Table 2. Average and Median Wealth (thousand dollars)

	Singles		Couples	
Wealth Group	Mean Wealth	Median Wealth	Mean Wealth	Median Wealth
P0 - P10	-21	-3	12	19
P10-P20	2	1	72	72
P20 - P30	13	13	116	115
P30 - P40	32	32	168	169
P40 - P50	57	57	226	224
P50 - P60	87	87	299	299
P60 - P70	127	126	413	410
P70 - P80	197	194	580	565
P80 - P90	336	332	903	871
P90 - P100	1442	833	3070	1820

This table presents the average and median wealth for different brackets within the US elderly population from 1994 to 1996. Columns (1) and (2) display the average and median net worth at the beginning of 1996 for singles, while columns (3) and (4) provide the same information for couples. A higher mean compared to the median indicates that wealth distribution within each group is skewed.

The \$609,000 net worth gap at the top and the \$18,000 gap at the bottom of the wealth distribution indicate outliers. Some households have significantly lower or higher net worths. Specifically, the household with the lowest net worth owes \$647,000 in liabilities, which is below the \$21,000 average liability in the bottom wealth group, while the household with the highest net worth owns \$19,478,000 in assets, above the \$1,442,000 average net worth in the top group.

Wealth distribution among couples in the bottom wealth group is less skewed than among singles. The gap between median and mean net worth is larger in the top two wealth groups, at \$32,000 and \$1,250,000, compared to \$7,000 in the bottom group. The poorest households owe \$309,000, while the richest households have a net worth of \$22,629,000.

There are stark differences in asset holdings between couples and singles. Couples generally hold significantly more assets than singles, likely due to having two individuals per household. Even when halving a couple's net worth, the disparity remains. For instance, at the bottom, couples have an average net worth of $12,000, half of which is $6,000—still higher than singles' negative $21,000. At the top, couples' average net worth is $3,070,000, with half being $1,535,000, exceeding singles' $1,442,000 by $93,000.

The top panel of Table 3 outlines the composition of wealth among single households. For those in the first two deciles, primary housing represents only a small proportion of total wealth, indicating that these households generally do not own homes. Households in the lowest wealth decile primarily hold financial assets and have liabilities related to pensions and transportation. In the second decile, financial and transportation-related assets make up most of their wealth. The distribution of wealth sources differs notably between the higher and these lower wealth deciles, which may be attributed to differences in home and vehicle ownership. Households in the first decile typically lack sufficient resources for home or vehicle ownership and thus concentrate their assets in financial holdings. Those in the second decile also tend not to own homes but may have enough income to acquire vehicles, making transportation and financial wealth their primary asset categories.

The distribution of wealth among the middle six wealth deciles demonstrates a consistent pattern. Housing constitutes the primary component of wealth, accounting for 52 percent in the third decile, increasing to 68 percent in the fifth decile, and then declining to 51 percent in the eighth decile. The proportion of wealth attributed to transportation assets steadily decreases from 43 percent in the third decile to 6 percent in the eighth decile. Conversely, the share of financial wealth rises significantly, moving from -8 percent up to 28 percent across these deciles. Pension and real estate holdings also represent an increasing share of total wealth among these households. Finally, unlike those in the first two deciles, businesses account for approximately 1 to 2 percent of total wealth within the middle deciles.

Households in the top two wealth deciles display diversified asset portfolios. For those in the ninth decile, housing wealth accounts for 38 percent of total wealth, while financial assets comprise 28 percent. Pension, business, and real estate holdings collectively represent 30 percent of their assets. In the highest decile, primary housing decreases in relative importance, constituting only 23 percent of total wealth, whereas financial assets increase to 36 percent and become the predominant component. The combined share of pension,

Table 3. Wealth Categories

a. Singles

Wealth Decile	Housing	Financial	Transportation	Pension	Real State	Businesses
P0 - P10	1%	104%	-4%	-1%	0%	0%
P10-P20	2	32	67	0	0	0
P20 - P30	52	-8	43	7	5	1
P30 - P40	64	8	16	7	4	1
P40 - P50	68	10	13	4	4	1
P50 - P60	56	15	8	8	10	2
P60 - P70	60	14	9	8	9	1
P70 - P80	51	21	6	11	8	2
P80 - P90	38	28	4	13	14	3
P90-P100	23	36	2	13	19	6
Total	42	26	16	7	7	2

b. Couples

Wealth Decile	Housing	Financial	Transportation	Pension	Real State	Businesses
P0 - P10	39%	32%	32%	1%	-3%	-1%
P10-P20	69	3	16	6	4	2
P20 - P30	68	9	13	3	5	1
P30 - P40	53	15	11	9	9	3
P40 - P50	49	14	11	13	9	3
P50 - P60	43	21	9	14	9	5
P60 - P70	34	22	7	18	15	4
P70 - P80	28	27	5	19	14	7
P80 - P90	23	27	4	19	19	9
P90 - P100	14	27	2	12	27	17
Total	42	20	11	11	11	5

This table describes the distribution of wealth components among households in different brackets within the US elderly population from 1994 to 1996. Housing wealth is defined as the net value of primary residence. Financial wealth includes the net value of stocks, mutual funds, money market accounts, certificates of deposit, bonds, and other financial assets. Transportation refers to the net value of items used for transportation, such as cars, trucks, trailers, motor homes, boats, or airplanes. Pension indicates the net value of IRAs and Keogh accounts. Real estate signifies the net value of all non-primary housing properties. Businesses represent the net value of owned enterprises.
**The figures are expressed as percentages.*
** Wealth shares are calculated for each individual and then averaged for each decile.*

business, and real estate assets further expands to 38 percent within this group.

The bottom panel of Table 3 presents the wealth composition for couple households. Similar to single households, with the exception of the lowest decile, housing wealth constitutes the main component among lower wealth deciles, and its share declines as household wealth increases. As household wealth grows, assets become distributed across additional sources. Financial, pension, real estate, and business assets become increasingly significant; in the highest decile, these collectively account for 85 percent of total wealth, compared to 15 percent in the second decile. When considering all households, the composition of wealth can be compared between singles and couples. Housing assets comprise 42 percent of total wealth for both groups. In other categories, the assets of single households are primarily concentrated in the financial and transportation sectors, whereas couples generally hold a more diversified portfolio.

B.3 Savings

We use the commonly referenced definition of total saving from the literature:

$$S_{it+1} = b_{it+1} - b_{it} \tag{23}$$

Where $b_{i,t}$ denotes the wealth held by household i at time t, and $S_{i,t}$ denotes the total saving held by individual i at time t. One can also think consider saving out of income. In our model budget constraint is defined as:

$$b_{it+1} = (rb_{it} + y_{it}) - (c_{it} + m_{it}) + b_{it}$$

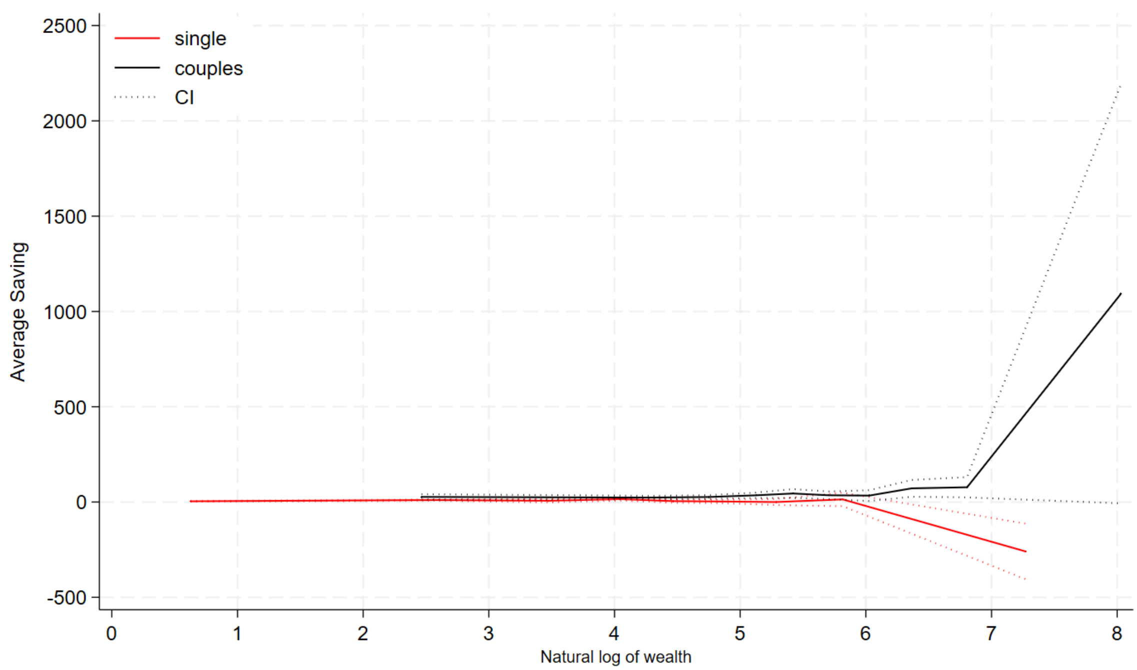
On the right-hand side, the first term in the bracket refers to the sum of investment and labour income, while the second term indicates total spending on consumption and survival. The term b_{it} represents all assets individual i has at time t. Note that saving could also be thought of as:

$$S_{it} = (rb_{it} + y_{it}) - (c_{it} + m_{it}) \tag{24}$$

Although in theory these two should be equivalent, practical issues—mainly measurement error in wealth, as noted by (Juster, Smith, and Stafford 1999) matters. Since our data aligns better with the equation 23 definition, we use that approach.

Figure 4 presents the saving patterns of both couple and single households across different segments of the wealth distribution. Among single households, those in the lowest wealth group exhibit higher savings levels than other groups, with an average savings amount of $21,000. In contrast, individuals in the highest wealth group demonstrate the greatest level of dissaving, reducing their assets by an average of $260,000. Households in the remaining wealth groups typically save amounts ranging from zero to $15,000.

Figure 4. Average Saving



This figure represents average savings of couple and single households in different wealth brackets among the US elderly from 1994 to 1996. The red line represents single households while the black line represents the couple household. The numbers are thousands of dollars. Note that wealth deciles are defined differently for couples and singles as illustrated in table 2.

The saving behaviours among couple households differ notably. The first seven wealth groups report savings between $27,000 and $45,000, with these figures rising to $72,000 and $78,000 for the eighth and ninth wealth groups, respectively. A substantial increase in savings is observed for the wealthiest couples, with households in the top group accumulating approximately $1,092,000 in net worth on average.

Couple households generally exhibit greater wealth and higher savings than single households throughout the wealth distribution. In the lowest wealth category, the average couple household holds $33,000 more in assets and saves $6,000 more annually compared to the average single household. These disparities become more pronounced at higher wealth tiers; within the top group, couple households own $1,628,000 more in assets and save $1,352,000 more than their single counterparts. This pattern diverges from classical economic theories predicting that affluent households would draw down their wealth to support increased consumption. While consumption data are not presented here, it is plausible that these households maintain elevated levels of consumption even as they continue to build net worth. To mitigate the influence of differing wealth levels on savings comparisons, we subsequently report the saving rate, defined as saving scaled by net worth.

B.4 Saving rates

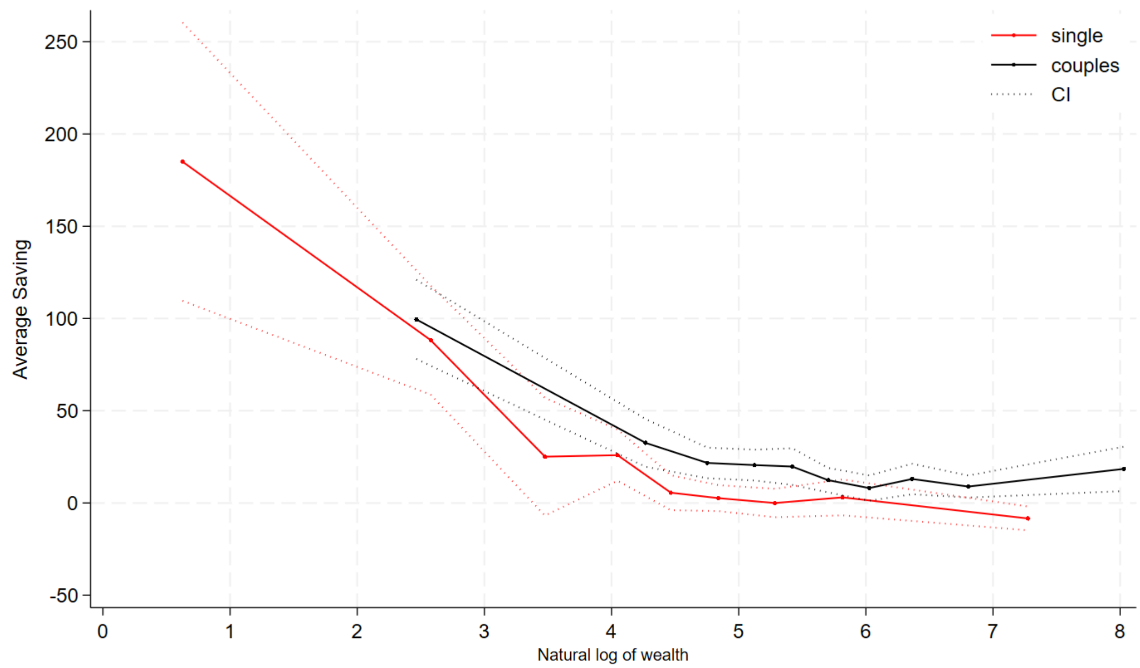
Examining saving levels can provide insights into the saving behaviours of various wealth cohorts. However, as our primary interest lies in saving intensity rather than absolute saving levels across the wealth distribution, our analysis will centre on the total saving rate:

$$s_{it} = \frac{S_{it}}{W_{it}}$$

There are some technical considerations with this definition. Since denominators cannot be zero, cases with zero wealth in the denominator must be omitted. Individuals with negative wealth are included by considering the share of debt repaid as part of savings; for example, if an individual borrows $100 in one period and repays $50 in the next, their saving rate is calculated as one-half. For couple households, omitting zero-wealth cases removes 56 observations from the sample, while for single households, 539 are excluded. Alternative approaches to include zero-wealth households are possible, but were not used here to minimize the potential impact of outlier observations.

Figure 5 presents the average saving rate for single and couple households across various

Figure 5. Average Saving Rates



This figure represents average saving rates of couple and single households of different net worth brackets among the US elderly from 1994 to 1996. The red line represents single households while the black line represents the couple household. Saving rate is defined as saving relative to total net worth.

wealth groups. Among single households, the lowest wealth group exhibits a saving rate of 172 percent, which increases to 185 percent for those in the second wealth decile. These elevated saving rates can be attributed to the minimal wealth levels in these groups, resulting in a small denominator in the wealth growth calculation. From the third wealth decile onward, the average saving rate among single households steadily declines, reaching zero percent in the eighth decile and remaining approximately at that level thereafter. Specifically, the second richest and the richest single household wealth groups have saving rates of 3 percent and -8 percent, respectively.

For couples, saving rate patterns exhibit notable variation across wealth deciles. Households in the first decile save 99 percent of their wealth, which remains relatively elevated, similar to single households. This rate decreases significantly to 32 percent in the second wealth decile, then stabilizes at approximately 20 percent for the third, fourth, and fifth deciles.

A further decline occurs among households in the sixth through ninth deciles, where the average saving rate is around 10 percent. Notably, among the wealthiest households, the saving rate increases once more, reaching 20 percent.

Table 4.a outlines the components of average savings to net worth for single-person households. Households in the first five wealth deciles exhibit the highest savings rates, predominantly driven by increases in their housing, financial, and, in some cases, transportation assets. Specifically, of the 172 percent asset growth observed in households within the first decile, 166 percent is attributable to gains in housing, financial, and transportation wealth. For those in the second wealth decile, 184 percent of the total 185 percent increase in wealth is likewise explained by growth in these asset categories. Similarly, households in the third wealth decile saw an 88 percent rise in housing, transportation, and financial wealth, out of a total 89 percent increase in net worth.

In households within the top five wealth deciles, two primary trends are observed in the composition of savings relative to total net worth. Both primary housing and non-primary real estate assets show a reduction, while most savings are allocated to financial accounts, except among the highest wealth group. This reflects a reallocation of assets from real estate to financial market instruments. For example, households in the eighth decile experience an average reduction of 1 percent in their total assets, attributed to a 5 percent decline in total real estate assets, including primary residences, as a proportion of net worth, while their financial wealth as a share of net worth increases by 4 percent.

Table 4.b presents the average saving rate composition for couple households. With the exception of the first decile, pension and financial savings constitute the primary sources of savings, whereas business-related savings begin to play a significant role from the seventh decile onward. In contrast to single-person households, couples do not exhibit dissaving patterns in primary housing assets, although this does not apply to their other real estate holdings. Overall, apart from the real estate category, couples generally demonstrate a tendency to save across different asset classes rather than to dissave.

Table 4. Saving Categories

a. Singles

Wealth Decile	Housing	Financial	Transportation	Pension	Real State	Businesses	Total
P0 - P10	37%	87%	42%	6%	0%	0%	172%
P10 - P20	50	50	84	0	0	0	185
P20 - P30	76	14	-2	-0	-3	4	89
P30 - P40	13	1	-2	2	13	-1	26
P40 - P50	18	4	2	1	-1	1	25
P50 - P60	-1	4	2	-0	-2	3	6
P60 - P70	0	3	-1	7	-5	-0	4
P70 - P80	-3	4	-0	1	-2	-0	-1
P80 - P90	-2	5	1	1	-5	3	3
P90 - P100	-2	-4	1	1	-3	-1	-8
Total	19	17	13	2	-1	1	50

b. Couples

Wealth Decile	Housing	Financial	Transportation	Pension	Real State	Businesses	Total
P0 - P10	42%	40%	18%	1%	0%	-1%	100%
P10 - P20	7	11	1	11	4	-0	32
P20 - P30	1	9	2	3	2	4	22
P30 - P40	5	8	1	4	1	-1	20
P40 - P50	1	16	-1	8	-3	-0	20
P50 - P60	2	5	-0	8	-2	-0	12
P60 - P70	0	4	-0	6	-3	1	8
P70 - P80	0	6	-0	6	-2	4	13
P80 - P90	-0	3	1	7	-2	1	9
P90 - P100	3	10	0	3	-1	3	18
Total	6	11	2	6	-1	1	26

This table reports the composition saving rate for couples in different brackets of the net wealth among the US elderly from 1994 to 1996. We consider: the ratio of saving into primary housing wealth over net worth, the ratio of saving into financial wealth over net worth, the ratio of saving into transportation over net worth, the ratio of saving into IRA and Keogh accounts over net worth, the ratio of saving into real estate over net worth, the ratio of saving into businesses over net worth. The last column depicts saving over net worth.
**The figures are expressed as percentages.*
** Wealth shares are calculated for each individual and then averaged for each decile.*

B.5 Symmetry and dispersion of saving rates

In this section we discuss measures of dispersion and symmetry in saving rates across wealth brackets for single and couple households. We examine three key statistics: the standard deviation (SD), the interdecile range (IDR), and quantile-based skewness.

The interdecile range (IDR) – the difference between the 90th and 10th percentiles – is used alongside the standard deviation to mitigate the influence of outliers. In a standard normal distribution, the SD is 1, and the IDR is 2.56 (the theoretical distance between the 90th and 10th percentiles). A useful heuristic is that if the IDR exceeds 2.5 times the SD, the distribution likely exhibits heavy tails or outliers.

For assessing symmetry, we employ quantile-based skewness, a robust measure that accounts for outliers. It is defined as:

$$Skewness = \frac{P90 + P10 - 2 \cdot P50}{P90 - P10}$$

This metric helps determine whether the distribution is symmetric or skewed, even in the presence of extreme values.

Table 5 examines the dispersion and symmetry of saving rates. The results indicate that saving rates exhibit less noise as net worth increases, evidenced by reductions in both the standard deviation and interdecile range. For single households, the standard deviation of the saving rate is 430 percent in the first wealth decile—more than eight times higher than the 52 percent observed in the top decile. Similarly, the interdecile range for households in the lowest wealth bracket is 640 percent, which is 4.7 times larger than the range of 136 percent for those in the highest bracket. The ratio of the standard deviation to the interdecile range is generally below 2.5, suggesting the presence of outliers. Across nearly all deciles, the skewness is consistently positive, indicating right-skewed distributions and a departure from symmetry.

The saving rate of couple households follow a pattern like that of single households. As wealth brackets increase, there is a general decline in noise, though the magnitude of this reduction is less pronounced compared to single households. Notably, the standard deviation of saving rates doubles when moving from the second highest to the highest net

Table 5. Dispersion of Saving Rate

	Standard Deviation		Interdecile Range		Skewness	
	Singles (1)	Couples (2)	Singles (3)	Couples (4)	Singles (5)	Couples (6)
P0 - P10	430%	174%	641%	448%	0.38	0.47
P10 - P20	610	104	886	176	0.80	0.35
P20 - P30	239	68	521	133	0.55	0.19
P30 - P40	258	67	238	139	0.17	0.28
P40 - P50	112	81	192	139	0.30	0.22
P50 - P60	76	54	171	111	-0.02	0.16
P60 - P70	57	55	134	124	0.03	0.29
P70 - P80	62	67	132	117	0.17	0.19
P80 - P90	79	48	155	111	0.22	0.12
P90 - P100	52	97	136	150	0.23	0.26
Total	198	82	320	165	0.28	0.25

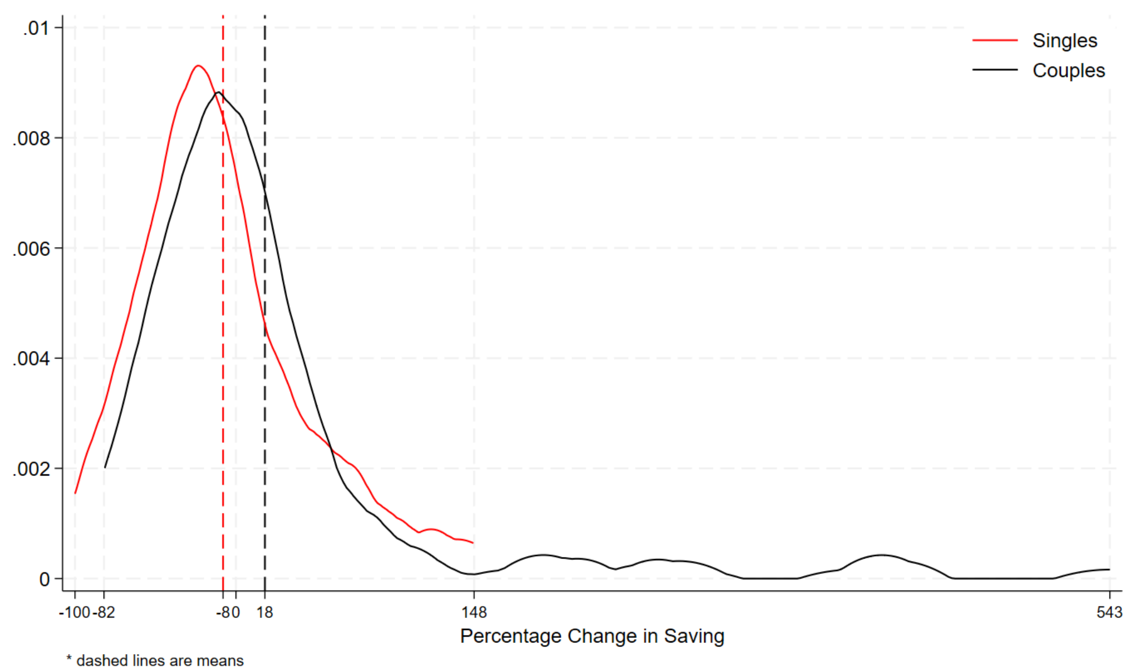
This table represents the dispersion and symmetry of saving rate distribution among the US elderly from 1994 to 1996. Standard deviation and interdecile range are used for analyzing the dispersion and skewness is useful for understanding symmetry.

worth bracket—a pattern not observed in the interdecile range. This discrepancy suggests the presence of outliers in the top wealth bracket. These findings align with recent studies documenting high saving rates among the highest wealth brackets.

Figure 6 demonstrates the disparity between couples and singles within the top wealth decile. As previously noted, the distribution is right skewed for both groups. Additionally, the majority of households exhibit negative saving rates, regardless of household composition. The positive mean saving rate for couples stems from more substantial positive outliers relative to singles. In particular, the highest saving rate among single households in the top wealth bracket reaches 148 percent, whereas the corresponding figure for couple households is 543 percent. Between these extremes, the density plot for couples shows multiple observations—visible as distinct bumps—which account for the higher average.

Figure 7 compares the kernel density estimates of saving rates against a normal distribution. The top panel displays singles, while the bottom panel shows couples. In both groups, the interdecile range is narrower than that of a normal distribution, and the tails are heavy—though this effect is more pronounced among singles. Couples exhibit higher

Figure 6. Kernel density estimate of percentage change in savings – Top bracket



This figure illustrates the kernel density estimates of saving rates among the US elderly from 1994 to 1996 in the tenth wealth decile using Epanechnikov method with the bandwidth of 15.33. Numbers are in percent, for example, for singles, the highest increase in wealth were about 148 percent.

positive saving rates overall, yet their right tail remains less extreme than that of singles. Both distributions are right-skewed, with the higher average saving rates driven by positive outliers—a common feature in saving rate data.

B.6 Correlation with Income

To evaluate income as a determinant of savings dispersion, Table 6 reports the correlation between saving rates and the natural log of income. The key finding is the lack of a strong monotonic relationship between income and saving rates, suggesting that variations in saving behavior across deciles are not primarily driven by income levels. Among single households, the correlation between saving rates and total income remains below 0.2 for the first seven wealth deciles, with the exception of the sixth decile. This correlation rises to above 0.25 for households in the top three wealth brackets, except for the highest net worth decile, where it is 0.24.

TABLE 6. Income and saving rate correlation

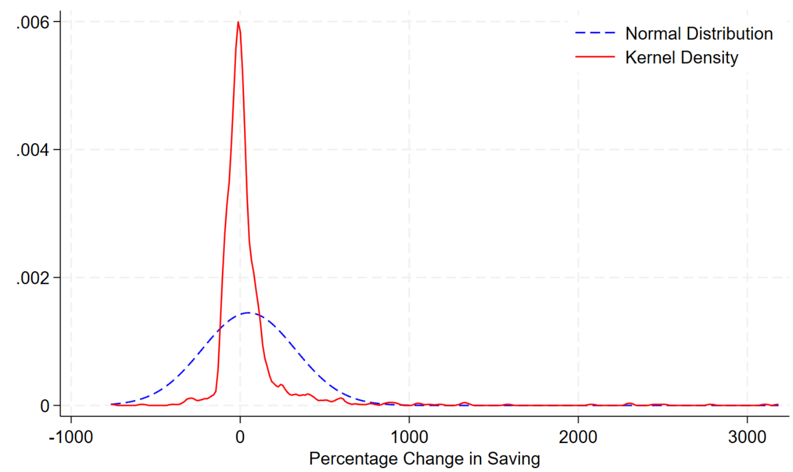
	Total Income		Earnings		Wealth Returns	
	Singles	Couples	Singles	Couples	Singles	Couples
P0-P10	-0.01	0.14	-0.08	0.16	0.20	0.08
P10 - P20	0.11	0.16	0.13	0.09	0.23	0.35
P20 - P30	0.01	0.19	0.10	0.22	0.02	0.18
P30 - P40	0.17	0.29	-0.03	0.13	0.28	0.34
P40 - P50	0.10	0.25	0.10	0.11	0.08	0.31
P50 - P60	0.30	0.08	0.16	0	0.34	0.19
P60 - P70	0.12	0.21	0.07	0.06	0.12	0.17
P70 - P80	0.27	0.23	0.15	0.09	0.30	0.20
P80 - P90	0.29	0.25	-0.10	0	0.30	0.41
P90 - P100	0.24	0.34	-0.01	0.10	0.25	0.28

This table represents the correlation of income and saving rate distribution among the US elderly from 1994 to 1996. The first two columns illustrate the correlation with total income. The second two columns represent correlation with earnings and the last two represent correlation with wealth return.

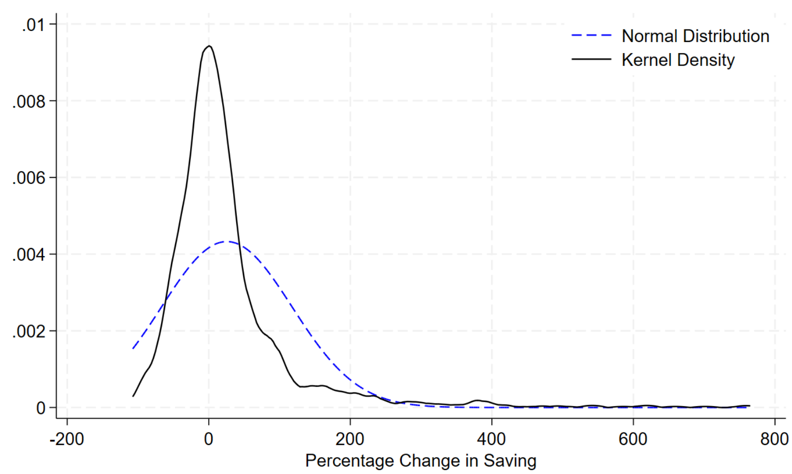
The correlation with earnings is consistently below 0.2 in nearly all cases, reinforcing that earnings levels do not significantly influence the overall correlations. In contrast, corre-

Figure 7. Kernel density estimate of percentage change in savings – All bracket

a. Singles



b. Couples



This figure illustrates the kernel density estimates of saving rates among the US elderly from 1994 to 1996 using Epanechnikov method with the bandwidth of 16.70 for singles and 9.49 for couples.

lations with wealth returns follow a pattern similar to that of total income: they remain below 0.3 for the first seven deciles (excluding the sixth decile) but increase to above 0.3 for the top three deciles (except the highest net worth bracket, where it is 0.25). This pattern underscores the role of capital income in shaping saving behavior.

For couples, the correlation between saving rates and total income is below 0.2 in the bottom three deciles but exceeds this threshold in the top seven deciles, except for the sixth decile. In contrast, the correlation with earnings is at most 0.1 for the top six deciles but higher in the bottom four deciles. This suggests that earnings play a more significant role in savings for households at the lower end of the wealth distribution, while their importance diminishes among wealthier households. Meanwhile, the correlation between saving rates and capital income is consistently stronger than those observed for earnings and total income, highlighting its greater influence on saving decisions across all wealth brackets—except for the bottom wealth group.

An alternative approach for measuring changes in wealth is through the active saving framework. This methodology posits that, in calculating total savings, re-invested dividend income is excluded, as these reinvestments do not reflect discretionary actions by individuals. The formal representation is as follows:

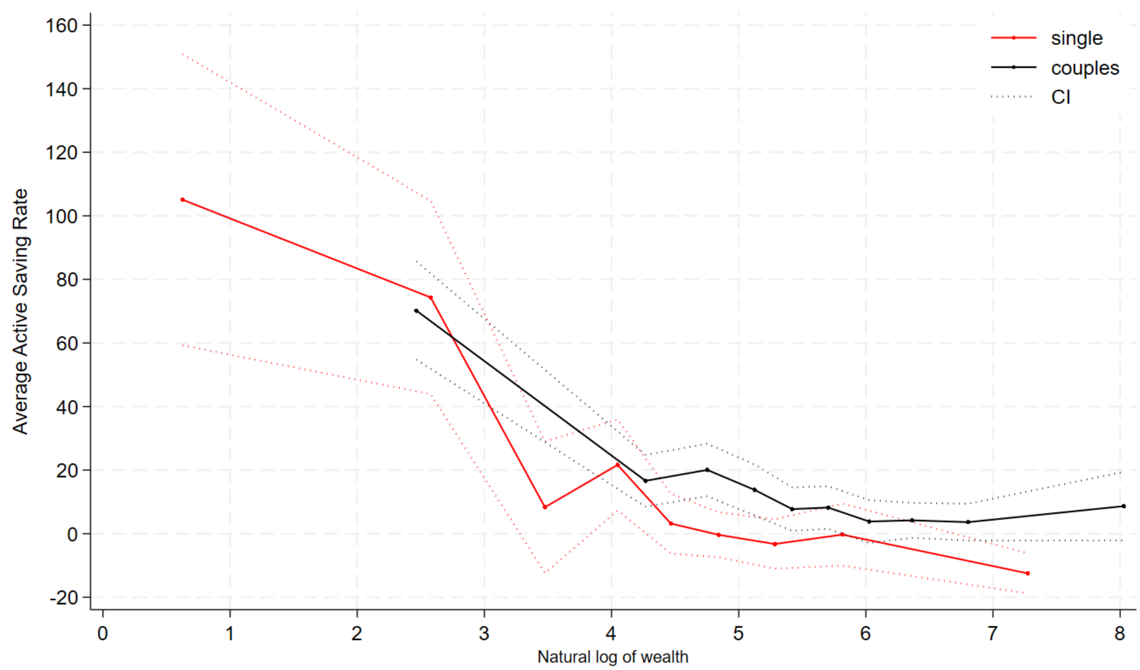
$$S_{it}^{act} = S_{it}^{tot} - r_{it+1} b_{it}$$

Where the second term on the right-hand side is capital income. As discussed in the previous section, the correlation with capital income is relatively more prominent compared to other income sources. Since in many cases, capital income is automatically re-invested, the distinction lies in the fact that total saving rate does not exclude reinvested wealth from the savings decision. Active saving rate is then defined as:

$$s_{it}^{act} = \frac{S_{it}^{tot} - r_{it+1} b_{it}}{W_{it}}$$

Figure 8 presents the active saving rates for couples and single households across different wealth brackets, alongside their total saving rates. Panel (a) displays the saving behavior of single households, while Panel (b) illustrates that of couples. Overall, the trends in active and total saving rates are broadly similar, though the active saving rate is consistently lower across all deciles.

Figure 8. Average Active Saving Rates

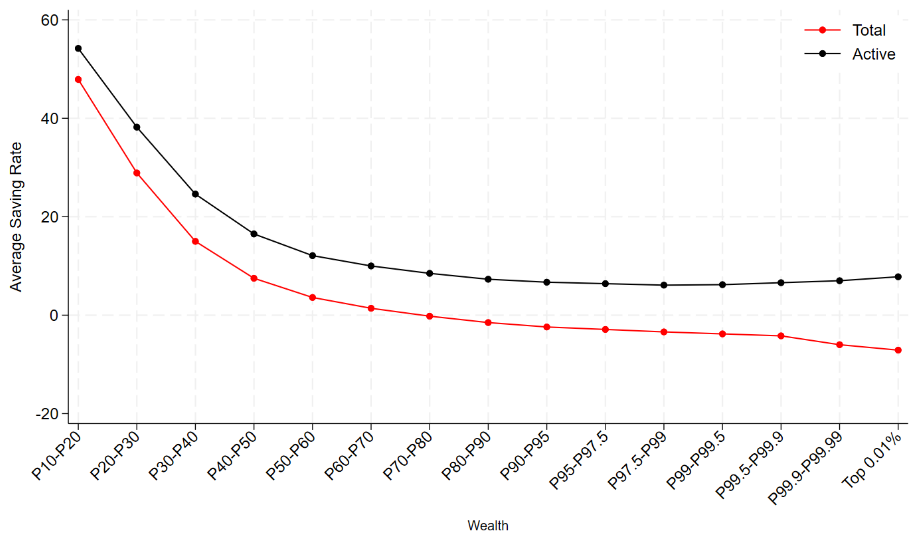


This figure represents average saving rates of couple and single households of different net worth brackets among the US elderly from 1994 to 1996. The red line represents single households while the black line represents the couple household. Saving rate is defined as saving relative to total net worth.

This difference is particularly pronounced among households at the bottom of the wealth distribution, consistent with the earlier finding that their savings are primarily driven by financial saving. For single households, the gap between active and total saving rates is substantial in the bottom two deciles—approximately 50% and 78%, respectively—whereas for couples, the difference is smaller (30% and 16%).

In contrast, among higher wealth deciles, the disparity between active and passive saving rates is more marked for couples. Additionally, the active saving rate declines more smoothly for couples (except for the top decile) and is roughly 10% lower than the total saving rate in the highest wealth bracket. This further underscores the significance of wealth returns for households at the top of the wealth distribution.

Figure 9. Robustness: Positive Saving at the Upper Tail



Source: Figure 4 in (Bach, Calvet, and Sodini 2017), using administrative data from Sweden

Our findings are consistent with recent literature focusing on saving rate distribution in different wealth deciles. Figure 9 reports the results from Bach, Calvet, and Sodini 2017. The general pattern seems to be consistent with our findings. Specially, when considering active saving rate, we observe that saving at the top does not tend to become decreasing. Interestingly, they use Swedish administrative data, which is very different to our US elderly data, but they find the same pattern. Our data is also consistent with Fagereng et al. 2019. In their paper they use Norwegian data to find the a similar pattern.