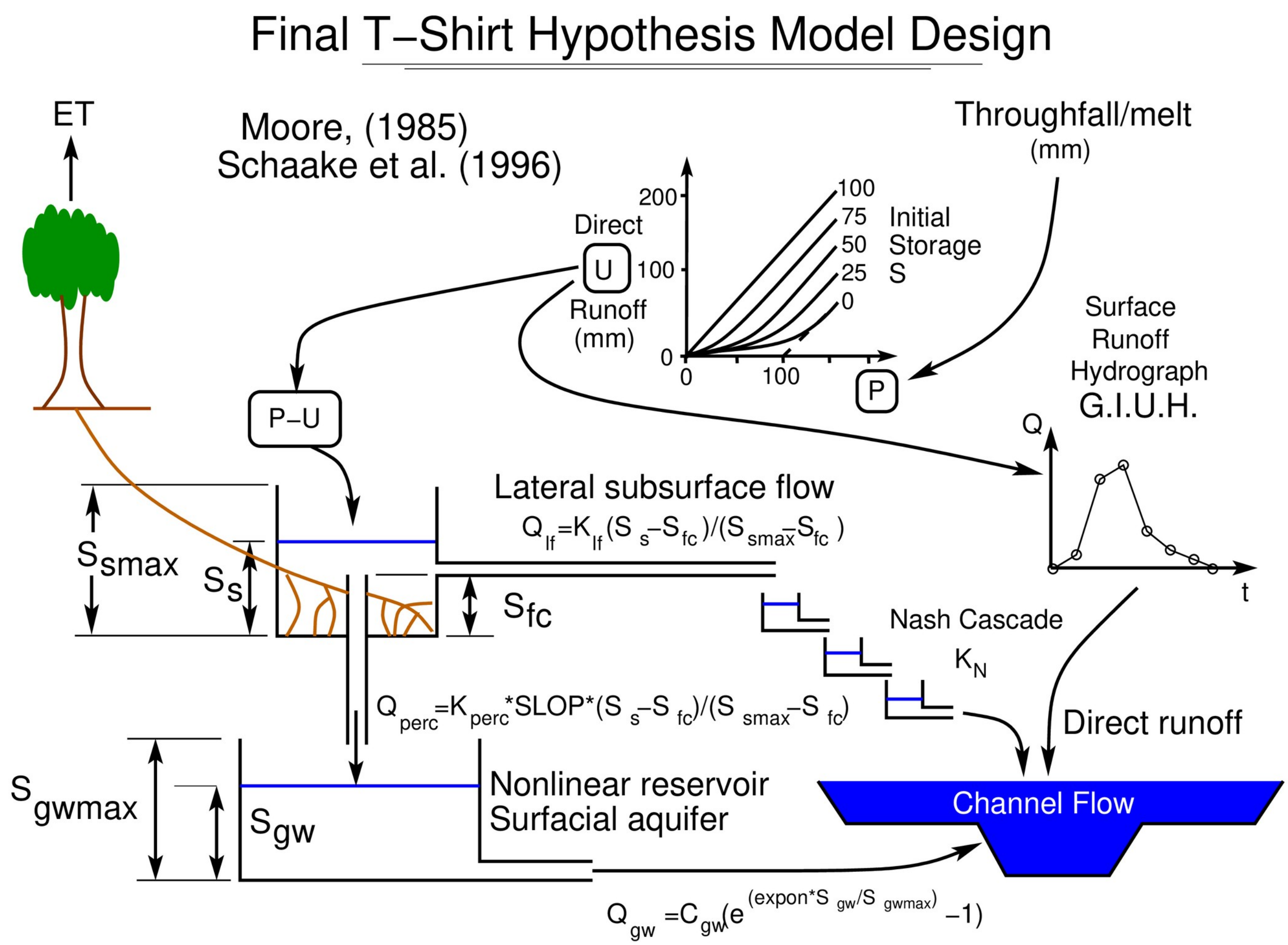

Figure 1: T-shirt model conceptual diagram. For description of parameter calculation or estimation, see summary at end of this document.

Given soil parameters from the NWM, with definitions from Noah-MP SOILPARM.TBL,:

`maxsmc` saturated soil moisture content [-]
`wltsmc` wilting point soil moisture content [-]
`satdk` saturated hydraulic conductivity [m s^{-1}]
`satpsi` saturated capillary head [m]
`bb` 'b' exponent on Clapp-Hornberger (1978) soil water relations [-]
`mult`, the multiplier applied to satdk to route water rapidly downslope in subsurface
Four fixed soil discretizations Δz of thickness from top to bottom of: 0.1 m, 0.3 m, 0.6 m, and 1.0 m.
Total soil column depth D=2.0 m.

The inputs for the Schaake (Schaake et al. 1996) partitioning scheme are:

```
void Schaake_partitioning_scheme
      (double timestep_s,
       double Schaake_adjusted_magic_constant_by_soil_type,
       double column_total_soil_moisture_deficit_m,
```

```
double water_input_depth_m
```

The value of the Schaake_adjusted_magic_constant_by_soil_type, abbreviated as $C_{schaake}$ is:

$C_{schaake}$=3.0* satdk/2.0E-06.

The total soil water storage is Ss_{max} = maxsmc * D, where

D = the total thickness of the soil column [2.0 m]

Therefore, if the current storage in the soil column nonlinear reservoir is S_s, with $0 <= S <= S_{max}$, then the column_total_soil_moisture_deficit = $S_{max} - S$.

Lateral Subsurface Flow

In the NWM implementation of the WRF-Hydro model lateral flow is assumed to occur any time the water content in any soil discretization (incorrectly referred to as a "layer" in WRF-Hydro) exceeds the water content associated with "field capacity", which is defined as the water content where free drainage stops. In finer textured soils, the field capacity water content coincides with a soil suction pressure of approximately 1/3 Atm. Here we will assume that this represents a suction head of 3.44 m. In general, we can assume that for a given soil there is a constant $\alpha<1$ that describes the field capacity so that the suction head above the water table is given by $H_{wt} = \alpha H_{atm}$. Here $H_{atm} = P_{atm} / \gamma$, where P_{atm}=101,300 [Pa] and γ = being the weight of water per unit volume which is very nearly 9,810 [N m^{-3}].

The WRF-Hydro code uses the Clapp-Hornberger (1978) relation to describe soil water retention. This is the relationship between soil water content θ and soil suction head ψ. Clapp and Hornberger neglected the residual water content by assuming that it is equal to zero. Other parameters in this relation are the water content at effective saturation θ_e , which is called SMCMAX in WRF-Hydro, the saturated soil suction head parameter ψ_s , which is called SATPSI in WRF-Hydro, and the Clapp-Hornberger exponent parameter b, which is called bexp in WRF-Hydro. That soil water retention function is:

$$\psi = \psi_s \left(\frac{\theta}{\theta_e} \right)^b \tag{1}$$

With reference to the geometry shown in Figure 2, the parameters needed to simulate the lateral flow response include four storage thresholds, and four linear reservoir constants. Because we assume that groundwater flow is dominated by Darcy's law, linear reservoirs are appropriate for calculation of these fluxes, because under the Boussinesq assumptions, the flux varies approximately linearly with saturated thickness.

Assuming that the soil is in hydrostatic equilibrium, the activation moisture content at the center of each soil discretization coincides with value that results in a soil suction head equal to some fraction of atmospheric pressure head as calculated using $H_{wt} = \alpha H_{atm}$, we can calculate the total storage of moisture in the soil in that condition.

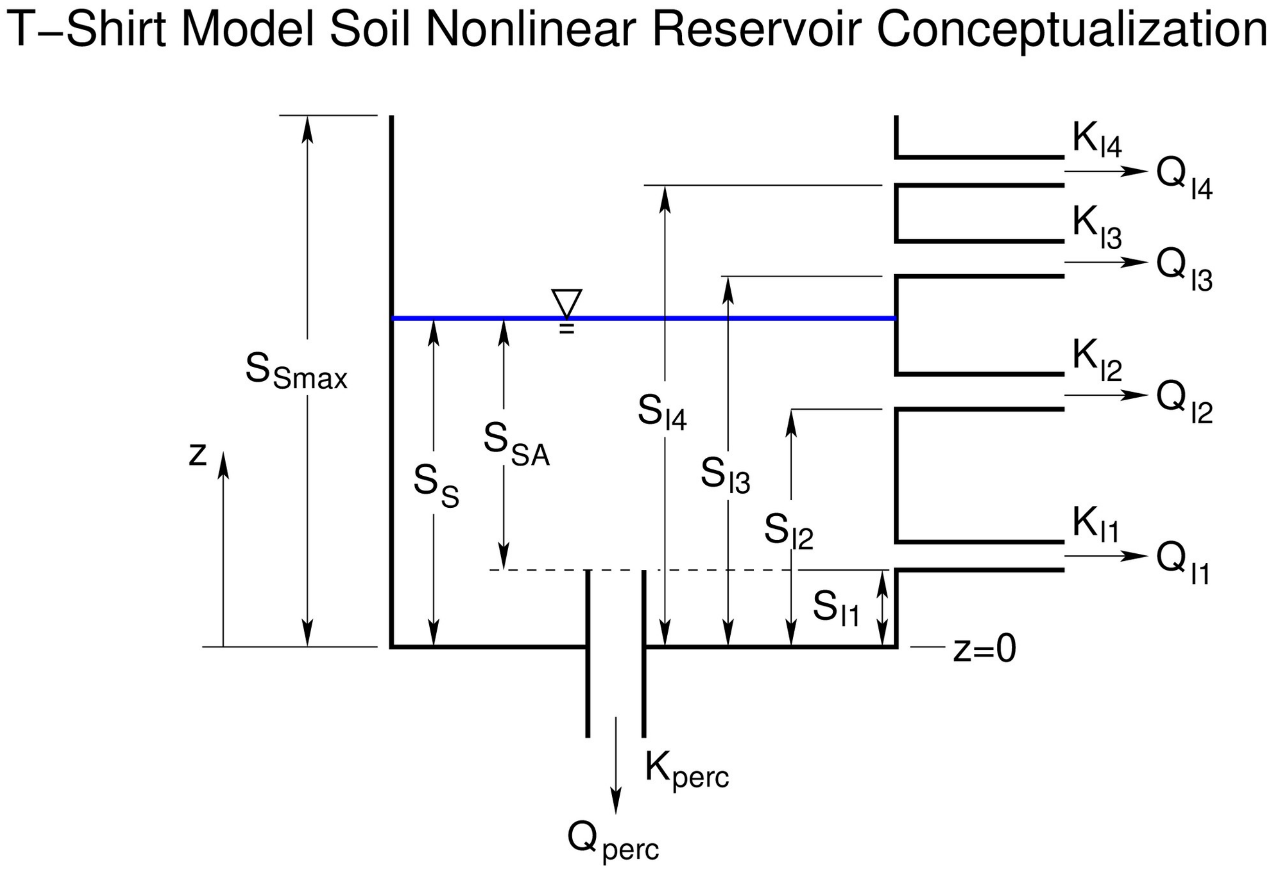

Figure 2: Definition sketch of variables and fluxes for calculation from the soil moisture reservoir, assuming that the flux in each soil discretization is calculated separately. It turns out that this level of detail is not necessary as described below.

If the soil is in hydrostatic equilibrium, then the suction head at the center of each soil discretization relative to the bottom of the soil column will equal the vertical distance z upward from the water table to the center of each soil discretization. Solving Eqn. 1 for water content gives:

$$\theta = \theta_e \left(\frac{z}{\psi_s} \right)^{1/b} \tag{2}$$

The means to estimate a field capacity water content for a particular soil is uncertain. Its value depends on soil structure, heterogeneity, and other difficult to measure/observe factors. In the absence of data, approximate field capacity moisture contents are used. In lieu of data on field capacity, the common approach is to assume that the field capacity occurs at a water content that results in a soil suction head equal to some fraction $\alpha < 1$ of the atmospheric pressure head H_{atm}. In equation form, $H_{wt} = \alpha\, H_{atm}$. In the case of fine textured soils, α is taken as 1/3, while in sandy soils it is nearer to 1/10. The wilting point water content can be solved using:

$$\theta_{fc} = \theta_e \left(\frac{H_{wt}}{\psi_s} \right)^{1/b} = \theta_e \left(\frac{\alpha P_{atm}}{\gamma_w \psi_s} \right)^{1/b} \tag{3}$$

In the case of the four discretizations used in the NWM, numbered from bottom up, the distances are z_i={0.5, 1.3, 1.75, and 1.95 m}.

The linear reservoir requires that we convert this water table into an equivalent storage of water in the control volume. When the water content is equal to the value given by Eqn. 2, then the water stored in the soil is given by the following equation for the i[th] discretization z_i :

$$S_{fci} = \theta_e \left(\frac{1}{\psi_s}\right)^{-1/b} \int_{z_i=\Omega_i}^{z=\Omega_i+2} \zeta^{-1/b} d\zeta \tag{4}$$

where ζ is the variable of integration, and Ω is a vertical offset [m] that depends on the sought after activation storage for a particular discretization {1-4}. The offset Ω varies by soil discretization or outlet number, which are the same in Figure 2. Assuming that α = 1/3, then the suction head at the wilting point water content, H_{wt} = 3.44 m, then for the first outlet, Ω_i= (H_{wt}- z_i) = (3.44-0.5) m, where z_i = 0.5 m is the height from the bottom of the 2.0 m thick soil column up to the center of the bottom (first) discretization.

Integration of Eqn. 4 yields the following solution:

$$S_{fci} = \theta_e \left(\frac{1}{\psi_s}\right)^{-1/b} \left.\frac{z^{1-1/b}}{1-1/b}\right|_{\Omega_i}^{\Omega_i+2} \tag{5}$$

S_i calculated using Eqn. 5 is the storage threshold for activation of the i[th] lateral flowpath. They are close together because the field capacity definition of lateral flow path activation occurs on a portion of the water retention curve where there is not a lot of change of storage with in response to changes in pressure that coincide with changes in elevation for hydrostatic equilibrium. The use of the field capacity as a threshold is dubious because macropores that convey large quanitities of lateral subsurface flow do not activate when the soil is under suction.

For example, given a loam soil with the following parameters: θ_e=0.439, ψ_s = 0.355 m, b = 4.05, we calculate the following lateral flow activation threshold storage values for S_i given in Table 1.

Table 1. Example calculated storage threshold values for preferential flow path activation S_i

i	z_i	Ω_i [m]	Ω_i+2 [m]	Upper limit	Lower limit	S_i [m]
1	0.5	2.942	4.942	1.504	1.018	0.486
2	1.3	2.142	4.142	1.316	0.801	0.515
3	1.75	1.692	3.692	1.207	0.671	0.536
4	1.95	1.492	3.492	1.158	0.610	0.547

Note that the maximum water storage in a 2 m thick soil column with the given effective porosity is 0.878 m, so these storage activation thresholds are only a bit more than about half of the available storage at most. This further supports the notion that the field capacity is not a particularly realistic lateral flow activation threshold from a soil physics standpoint.

In the case of a sand-dominated soil where α =0.1, the hypothetical water table can appear in the soil column when calculating the storage at field capacity for the reservoir outlets that are higher in the soil column. When this occurs, the lower limit used to evaluate the integral in Eqn. 5, Ω_i becomes invalid because it is negative. When Ω_i is negative, then this indicates that the water table appears in the soil column. In this case use 0.0 for the lower limit of evaluation and the upper limit of integration equal to ($2.0- z_i +H_{wt}$), and the water stored in the saturated soil below the water table must be added to the integral. This situation is mathematically represented in Eqn. 6.

$$S_{fc} = \theta_e \left(\frac{1}{\psi_s}\right)^{-1/b} \left[\frac{z_i^{1-1/b}}{1-1/b} \,\Big|\, {}_{0}^{2-z_i+H_{wt}}\right] + \theta_e * (z_i - H_{wt}) \tag{6}$$

The fact that the activation thresholds in Table 1 are nearly the same, ranging from 55-62% of the total storage suggests that they could be replaced with a single value that is either 55% or perhaps the mid-point of this range, 58.5%. Also not that S_l is the activation threshold for percolation to deep groundwater.

Table 2. Relative Soil Moisture Storage Threshold for Lateral Flow Activation [m]

				RELATIVE STORAGE [-] Distance to center of discretization			
Soil Texture	Assumed field capacity suction head fraction α	Distance to w.t. H_{wt} (m)	Eqn. 4 constant $\theta_s(1/\psi_s)^{-1/b}$	z_1 0.5	z_2 1.3	z_3 1.75	z_4 1.95
Sand	0.1	1.03	0.130	0.158	0.257	0.326	0.355
Loamy sand	0.15	1.55	0.193	0.278	0.321	0.404	0.446
Sandy loam	0.18	1.86	0.287	0.419	0.463	0.514	0.560
Silt loam	0.22	2.27	0.452	0.714	0.766	0.811	0.839
Silt	0.28	2.89	0.452	0.687	0.724	0.753	0.768
Loam	0.33	3.41	0.360	0.489	0.512	0.528	0.537
Sandy clay loam	0.33	3.41	0.299	0.395	0.409	0.419	0.424
Silty clay loam	0.33	3.41	0.439	0.698	0.717	0.731	0.738
Clay loam	0.33	3.41	0.395	0.623	0.641	0.654	0.661
Sandy clay	0.33	3.41	0.327	0.468	0.479	0.486	0.490
Silty clay	0.33	3.41	0.420	0.690	0.706	0.717	0.723
Clay	0.33	3.41	0.438	0.730	0.745	0.756	0.761

Notes on Table 2: These relative storage values are dimensionless and calculated by dividing the calculated threshold storage using Eqns 3 and 4 by the maximum storage in the 2.0 m deep soil column thickness, which is equal to $2.0*\theta_e$. Soil parameters taken from Noah-MP SOILPARM.TBL file.

While the spread between the heights of the centers of the z_1 and z_4 is 73% of the total soil column thickness, the spread between the activation storage contents for the four soil discretizations is less than 20% for the four coarsest soil textures and less than 10% for the remainder. This observation supports the notion that using multiple linear reservoir outlets to calculate the lateral subsurface flow does not provide much benefit. The relative storage for activation of lateral flow and the percolation flow path should both be the value at the center of the first discretization, z_1.

Darcy's law together with some assumptions about watershed geometry (slope, flow area) allow calculation of the lateral flow constants K_{li} in Figure 2. In the NWM, the saturated hydraulic conductivity values associated with saturated flow in are taken as a scalar multiple of the soil vertical saturated hydraulic conductivity, `SATDK` [m s^{-1}], which is more commonly described using the notation K_s. This multiplier is a calibration parameter called `LKSATFAC`. Calibrated values of this parameter are bounded between 10 and 10,000 during the calibration process. This means that in the case of the loam soil used in the example calculations for Table 1, with `SATDK`=3.4E-06 m s^{-1} the lateral hydraulic conductivity in the calibrated NWM can vary from 3.4E-05 to 3.4E-02 m s^{-1}, or 3.4E-03 to 3.4 cm s^{-1}. The upper limit of this range is 122 m h^{-1}!

The maximum transmissivity determining the flux of lateral subsurface flow through the i^{th} discretization, $T_i = K_s$ * `LKSATFAC`* Δz_i, where Δz_i is the thickness of the i^{th} soil discretization [m]. The flux through each soil discretization in the NWM relies on the Prickett-Lonnquist Aquifer Simulation Model (PLASM) (Prickett and Lonnquist, 1971) to route water once it is placed into the lateral flow routine. The NWM implementation neglects capillary effects and assumes a constant specific yield, which is generally a poor assumption for groundwater tables near the land surface.

Darcy's Law in the context of one-dimensional saturated groundwater flow in an unconfined aquifer in the principal flow direction, q_s is:

$$q_s = -T \frac{dH}{ds} \tag{6}$$

In the Boussinesq approximation assumes that vertical velocity components are negligible and that groundwater flow is driven by topography in the case of catchment models. Because the T-Shirt model does not include an explicit representation of topography which is necessary to calculate the land surface slope, we cannot use Darcy's law to calculate the nonlinear reservoir K_l.

The findings of this deep analysis of the lateral zone flow code in the NWM and design of its conceptual equivalent suggest two findings. First, the relatively small range of values of the activation storage for the proposed four lateral flow linear reservoir outlets suggests that they can be replaced with a single reservoir outlet. Secondly, the lack of topographical data and linearity of the Darcy's law (Eqn. 6) suggests that the linear reservoir K factor for this single outlet be treated like a calibration factor, just like the NWM treats the `LKSATFAC` parameter.

Deep Groundwater Reservoir

Given the following nonlinear reservoir parameters as applied in the NWM using an exponential function, with the same parameters as defined in the NWM code `C`, `expon`, and z_{max} :

$$Q = C\left[\exp\left(expon * z / z_{max}\right) - 1\right] \tag{7}$$

writing this in the notation used in Figure 1:

$$Q = K_{gw}\left[\exp\left(expon_{gw} S_{gw} / S_{gwmax}\right) - 1\right] \tag{8}$$

from this we can see that there is a one-to-one equivalence between the T-shirt model and NWM parameters: K_{gw} = C, $Expo_{gw}$ = *expon*, and $Sgw_{max} = z_{max}$.

Summary- Estimating T-Shirt Model Parameters

The conceptual model structure shown in Fig 1. is proposed as a functional equivalent of the National Water Model implementation of NOAH-MP with the routing functionality of the WRF-Hydro code. This document goes through the steps of estimating equivalent parameters for the so-called T-Shirt model.

Considering the differences between the representation of the soil reservoir in Figs 1 and 2, and example calculations of activation storage levels for the standard Noah-MP soil parameter values, the differences between the activation storage levels for the soil discretizations are small. Given that the lateral flow calculation in the NWM is entirely conceptual due to the use of the LKSATFAC parameter which is allowed to vary over three orders of magnitude in calibration, an equivalent conceptual model can use one lateral flow outlet. Its activation storage should be the same as that for the percolation flux, based on field capacity.

Here is a summary of how to calculate the parameters for the T-Shirt model from NWM parameters. With reference to the parameters in Figure 1:

Schaake Partitioning Function

This function is identical to the one used in the NWM. Three parameters are required. Note `satdk` is the calibrated saturated hydraulic conductivity value from the NWM, using variable names ftom Fig. 1) where applicable. NWM parameter names are listed in courier font:

1. S_{smax} = 2.0 m * `smcmax`, where `smcmax` is the soil porosity used in the NWM which is either calibrated or from SOILPARM.TBL.

2. Schaake_adjusted_magic_constant_by_soil_type = $C_{schaake}$=`3.0* satdk/2.0E-06`, with `satdk` being the soil saturated hydraulic conductivity in [m s^{-1}].

3. column_total_soil_moisture_deficit_m = $S_{smax} - S_s$ (using variable names form Fig. 1)

Lateral Flow Function

This function requires four parameters, S_{smax}, S_{fc}, K_{lf}, and K_n as defined in Figure 1. As before, NWM parameters are listed in courier font. S_{fc} is calculated using the following steps:

1. Calculate the field capacity water content using Eq. 3. Assume α =0.1 for Sand soil texture, increasing linearly to α =0.33 for a Loam soil texture. For soil textures from Loam to Clay, use α =0.33.

2. Calculate the height above a water table $H_{wt} = \alpha\, H_{atm}$.

3. H_{wt} should be greater than 0.5 m for all soils. Calculate the soil water storage in the soil column when the lowest soil discretization is equal to field capacity S_{fc} using Eqn. 5, with the limits of integration being: $\Omega_1 = (H_{wt} - 0.5)$ m, and $\Omega_2 = \Omega_1 + 2$ m.

4. If a different discretization or height in the soil column is used to trigger the lateral flow function, then it is possible that Eqn. 5 will have a negative lower limit of integration. In this case follow the procedure outlined to use Eqn. 6.

5. The third lateral flow function parameter, K_{lf}, is purely a calibration parameter with no strictly definable relationship to the NWM parameters.

6. The fourth lateral flow function parameter, Kn, is the Nash cascade discharge linear reservoir coefficient. Adjust to improve hydrograph timing.

<u>Percolation Function</u>

This function becomes active whenever the storage in the soil S_s becomes greater than the threshold given by S_{fc}. The flow rate per unit area is given by the following equation:

$$Q_{perc} = K_{perc} * SLOP * (S_s - S_{fc}) / (S_{smax} - S_{fc}) \quad [\mathrm{m\ s^{-1}}] \qquad (9)$$

where:
`Kperc=` the calibrated saturated hydraulic conductivity of the soil, `satdk`
`SLOP` = calibrated NWM `SLOPE` parameter

<u>Deep Groundwater Response Nonlinear Reservoir</u>

The deep groundwater response nonlinear reservoir is identical to that used in the NWM v2.0. It has two parameters, *C*, and *expon*.

$$Q_{gw} = C\left(e^{(expon * S_{gw} / S_{gwmax})} - 1\right) \quad [\mathrm{m\ s^{-1}}] \qquad (10)$$

Use calibrated NWM values of *C* and *expon* parameters, with S_{gw} and S_{gwmax} defined as in Figure 1.

<u>Cited Literature</u>

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