GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LESSON 6: PROBABILITY & LOGISTIC REGRESSION

I. LINEAR REGRESSIONS

II. REGULARIZATION

QUESTIONS?

I. REVIEW OF REGULARIZATION
II. PROBABILITY
III. LOGISTIC REGRESSION

LAB:
IV. USING AND ANALYZING LOGISTIC REGRESSIONS

QUESTIONS?

I. REVIEW OF REGULARIZATION

These regularization problems can also be expressed as:

OLS: $min(\|y - x\beta\|^2)$

L1 regularization: $min(\|y - x\beta\|^2 + \lambda\|x\|)$

L2 regularization: $min(\|y - x\beta\|^2 + \lambda\|x\|^2)$

We are no longer just minimizing error but also an additional term.

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When do we use L1?

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Common case: Text Classification

X = [animal = 1, ..., carnival = 0, ..., xylophone =0, ...zebra = 0]
Y = Topic or Y = Important/Not Important or Y = Positive/Negative

REVIEW

Power Stance!!

II. INTRO TO PROBABILITY

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The probability of an event A is denoted P(A)

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The probability of the sample space $P(\Omega)$ *is 1.*

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A distribution can be discrete or continuous

Ex:

Discrete – Uniform distribution

$X \sim \{1, \ldots, N\}$ – $P(X = x) = 1/N$

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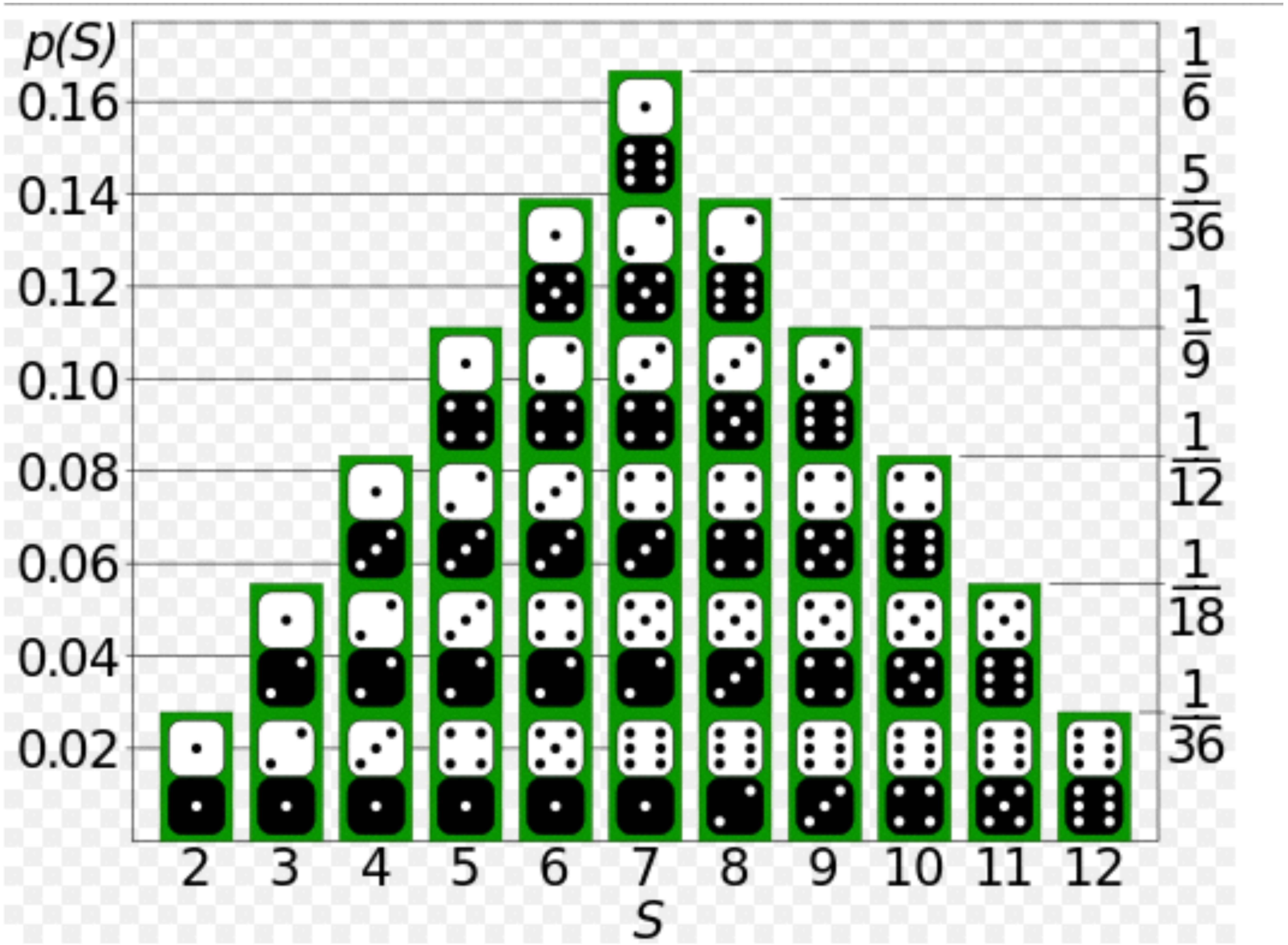
A distribution can be discrete or continuous

Ex:

Continuous – Normal distribution – N(u, o)

$X \sim N(0, 1)$ - $P(X = x) = 0$

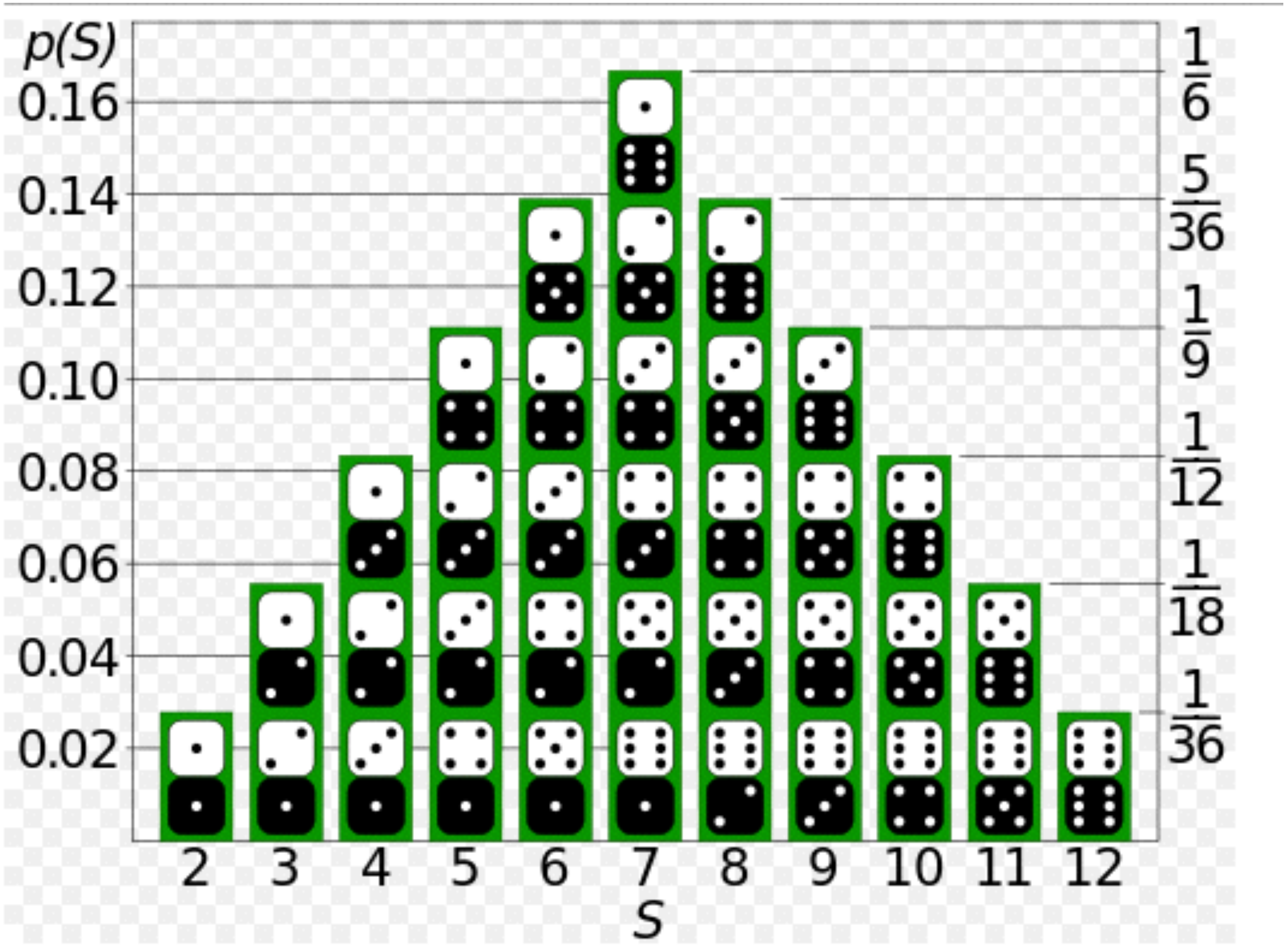
Q: Is this a discrete or continuous distribution?



from Probability Distribution on Wiki

Q: Is this a discrete or continuous distribution?

A: Discrete



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For discrete distributions

$$E(X) = \Sigma\ x * p(x)$$

For continuous distributions

$$E(X) = \text{integral}\ (\ x * p(x)\ )$$

Linda is 31 years old, single, outspoken and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1) *Linda is a bank teller.*
2) *Linda is a bank teller and active in the feminist movement.*

Q: Consider two events A & B. How can we characterize the **intersection** *of these events?*

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A: With the **joint probability** *of* A *and* B, *written* P(AB).

Q: Suppose event B has occurred. What quantity represents the probability of A **given** *this information about B?*

A: The intersection of A & B divided by region B.

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Notice, with this we can also write $P(AB) = P(A|B) * P(B)$.

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A: Information about one does not affect the probability of the other.

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Using the definition of the conditional probability, we can also write:

$$P(A|B) = P(AB) \,/\, P(B) = P(A) \;\rightarrow\; P(AB) = P(A) * P(B)$$

This result is called **Bayes' theorem.** *Here it is again:*

$$P(A|B) = P(B|A) * P(A) \,/\, P(B)$$

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Some facts:

- This is a simple algebraic relationship using elementary definitions.

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Some facts:

– This is a simple algebraic relationship using elementary definitions.

– It's a very powerful computational tool.

– It's interesting because it's kind of a "wormhole" between two different "interpretations" of probability.

Each term in this relationship has a name, and each plays a distinct role in any probability calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

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$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **likelihood function**. *It represents the joint probability of observing features* $\{x_i\}$ *given that that record belongs to class* C.

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$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **prior probability** *of C. It represents the probability of a record belonging to class C before the data is taken into account.*

Each term in this relationship has a name, and each plays a distinct role in any probability calculation (including ours).

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

This term is the **normalization constant.** *It doesn't depend on* C*, and is generally ignored until the end of the computation.*

REVIEW

What's the difference between discrete and continuous distributions?

REVIEW

Define the three terms in this equation

$$P(\text{class } C \mid \{x_i\}) = \frac{P(\{x_i\} \mid \text{class } C) \cdot P(\text{class } C)}{P(\{x_i\})}$$

III. LOGISTIC REGRESSION

	Continuous	Categorical
Supervised	regression	classification
Unsupervised	dimension reduction	clustering

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Unsupervised	dimension reduction	clustering

Q: What is **logistic regression***?*

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A: A generalization of the linear regression model to classification problems.

In linear regression, we used a set of covariates to predict the value of a (continuous) outcome variable.

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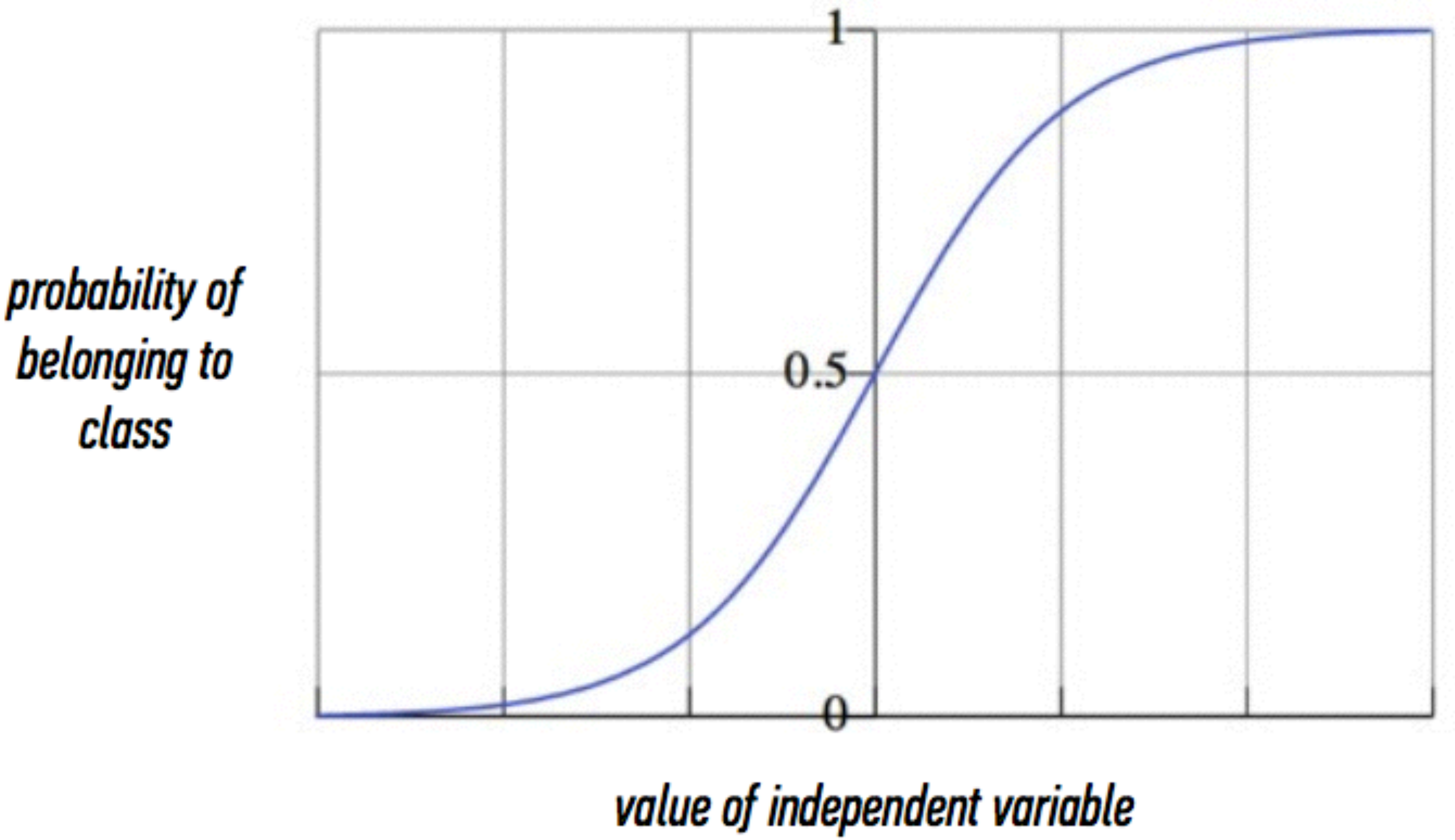
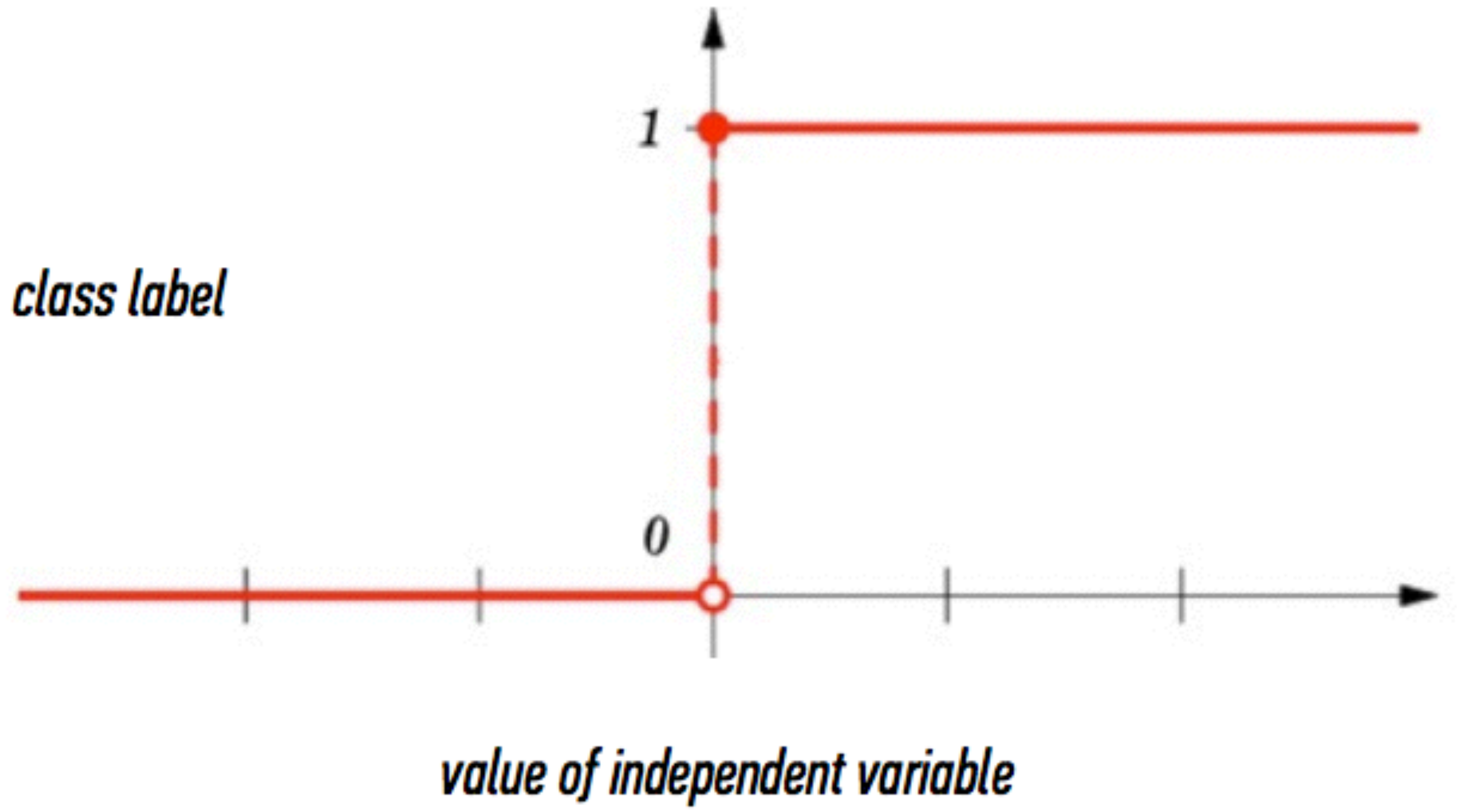
In logistic regression, we use a set of covariates to predict probabilities of (binary) class membership.

These probabilities are then mapped to class labels, thus solving the classification problem.



NOTE

Probability predictions look like this.



NOTE

Probabilities are "snapped" to class labels (eg by threshholding at 50%).

The logistic regression model is an extension of the linear regression model, with a couple of important differences.

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The main difference is in the outcome variable.

The key variable in any regression problem is the **response type** *of the outcome variable y given the value of the covariate x:*

$$E(y|x)$$

The key variable in any regression problem is the **conditional mean** *of the outcome variable y given the value of the covariate x:*

$$E(y|x)$$

In linear regression, we assume that this conditional mean is a linear function taking values in $(-\infty, +\infty)$:

$$E(y|x) = \alpha + \beta x$$

In logistic regression, we've seen that the conditional mean of the outcome variable takes values only in the unit interval $[0, 1]$.

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The first step in extending the linear regression model to logistic regression is to map the outcome variable $\mathbb{E}(y \mid x)$ *into the unit interval.*

Q: How do we do this?

A: By using a transformation called the **logistic function:**

$$E(y|x) = \pi(x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

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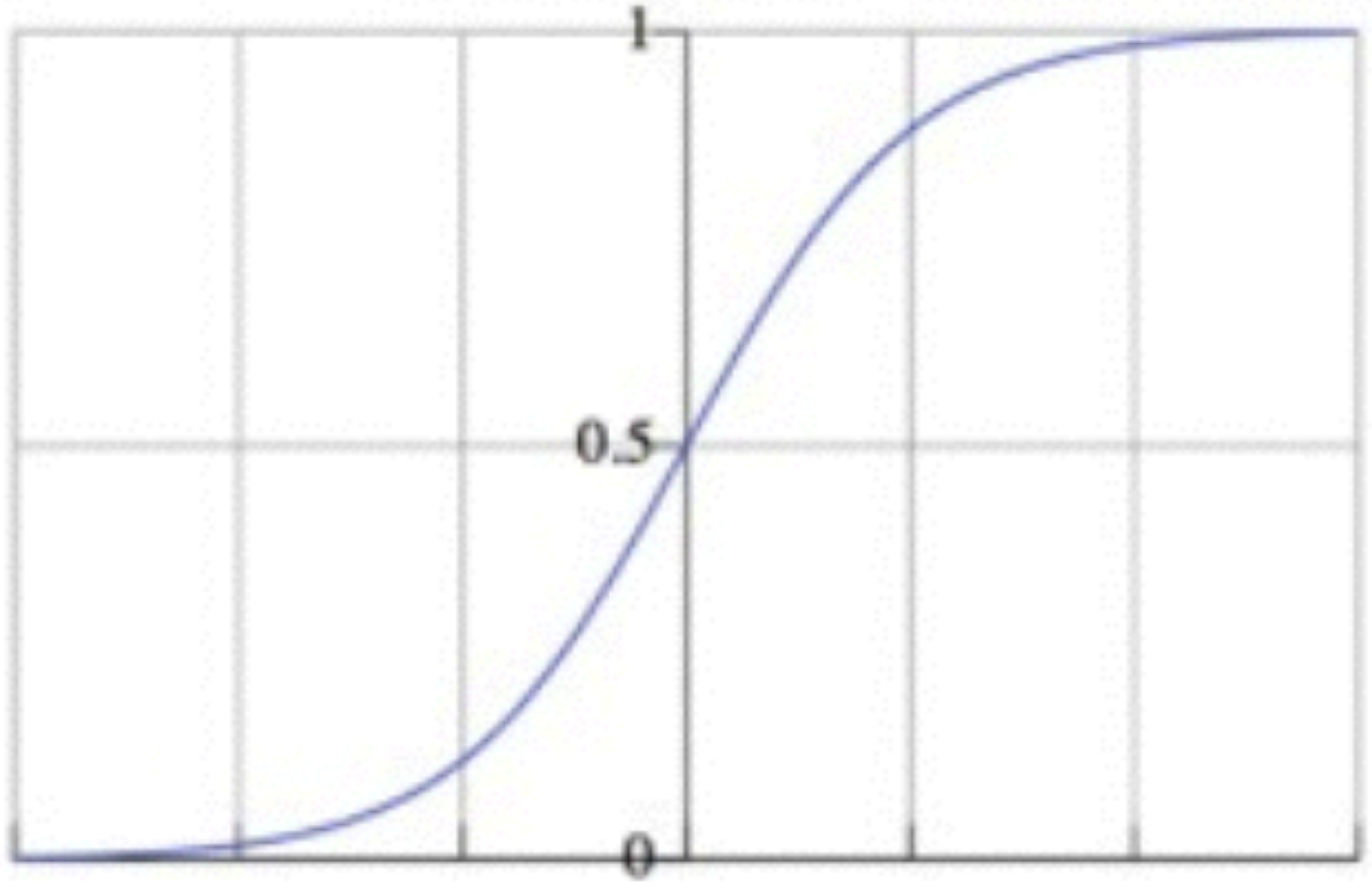
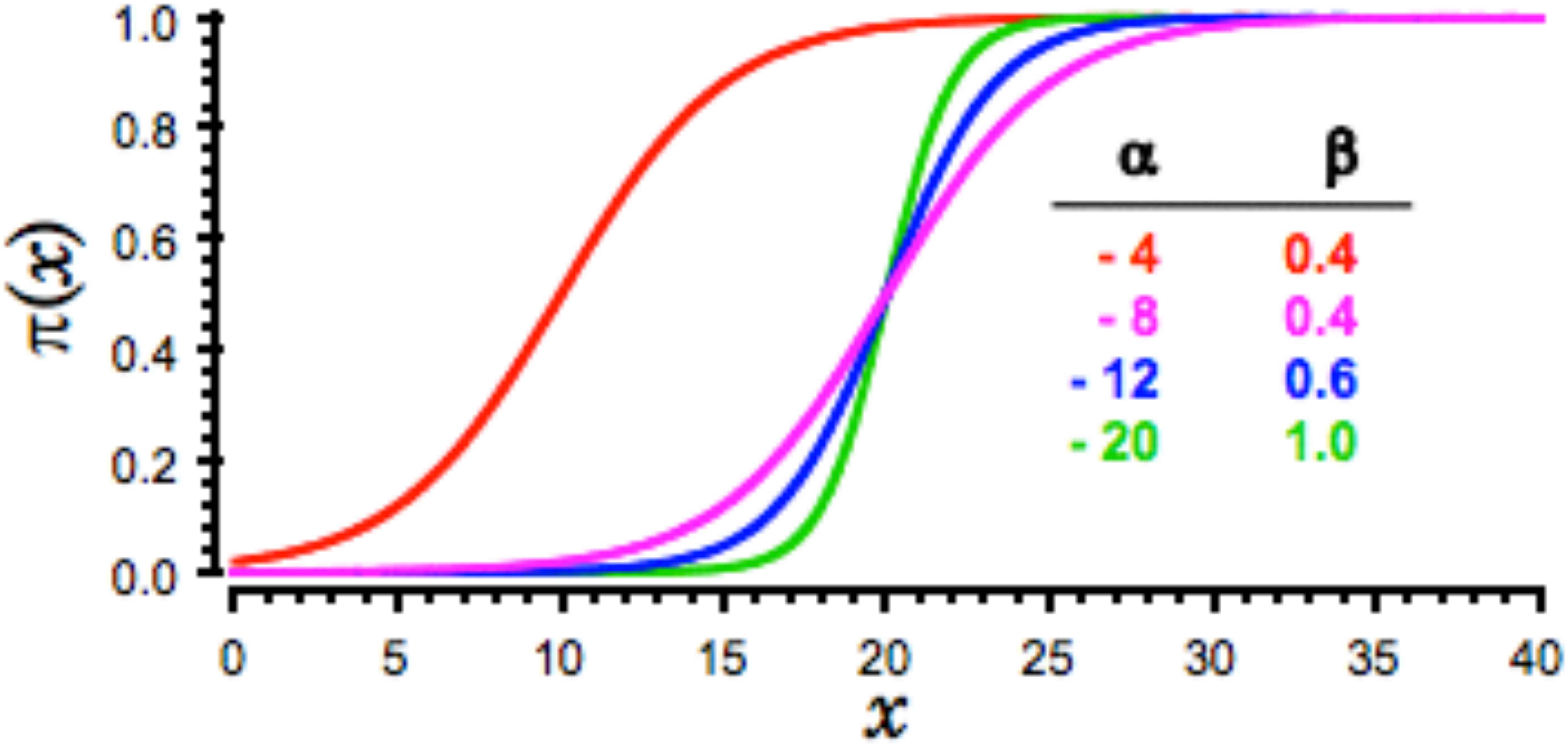
$$E(y|x) = \pi(x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

We've already seen what this looks like:



$$\pi(x) = \exp(\alpha + \beta x) / (1 + \exp(\alpha + \beta x))$$



The **logit function** *is an important transformation of the logistic function. Notice that it returns the linear model!*

$$g(x) = ln(\frac{\pi(x)}{1-\pi(x)}) = \alpha + \beta x$$

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The logit function is also called the **log-odds function.**

IV. LAB

The code for the lab is inside the ipynb file in dropbox. The wiki page also has several links with logistic regression packages and a more in depth explanation of the probability used in logistic regressions