B and E are producing the same next state combination for input 0 and 1. So, they are equivalent. E is replaced by B. C and D are producing the same next state combination, and D is replaced by C. The contracted table becomes

	Next State	
$Present State$	$X = 0$	$X = 1$
A	C	B
B	A	B
C	C	B

States A and C are producing the same next state combination for input 0 and 1. So, A and C are equivalent. C is replaced by A. The contracted table becomes

	Next State	
$Present State$	$X = 0$	$X = 1$
A	A	B
B	A	B

Both of the states are producing the same next state combination, and so they are equivalent. B is replaced by A.

	Next State	
$Present State$	$X = 0$	$X = 1$
A	A	A

The machine becomes a single state machine. It is of definite memory. The number of steps required to make it a single state machine is 3. Therefore, the order of definiteness of the machine is 3.

4.10 Information Lossless Machine

The main problem of coding and information transmission theory is to determine the conditions for which it is possible to regenerate the input sequence given to a machine from the achieved output sequence. Let us assume that the information used for a coding device in a machine be the input and the coded message be the output achieved, and let the initial and final states be known. It this case, information losslessness of the machine guarantees that the coded message (output) can always be deciphered.

A machine is called information lossless if its initial state, final state, and output string are sufficient to determine uniquely the input string.

A machine is said to be (information) lossless of order μ $(ILF - \mu)$ if the knowledge of the initial state and the first μ output symbols is sufficient to determine uniquely the first input symbol.

	Next State	
$Present State$	$X = 0$	$X = 1$
A	$A, 1$	$B, 0$
B	$B, 0$	$A, 1$

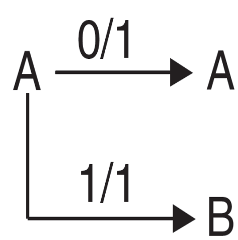
Test for Information Losslessness

Let the initial state of the machine be A, the final state achieved be A and the coded message (output) be 01. We have to find the information given as input. Now we are constructing the machine according to the output achieved.

	PreviousState, I/P	
PresentState	$Z = 0$	$Z = 1$
A	$-$	$A, 0$
		$B, 1$
B	$A, 1$	$-$
	$B, 0$	

An output of 01 means that from the reverse side it is 10. The initial state is A and final state is A. So, the diagram will become

1 picture

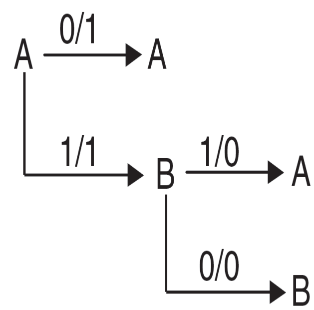


The output 1 with the next state A is produced for two cases—previous state A with input 0 or previous state B with input 1.

The output 0 with next state A is produced for no cases. The output 0 with next state B is produced for two cases—previous state A with input 1 and previous state B with input 0.

The initial state is A. In the previous case, A is produced for input 11.
Therefore, the input is 11.

2 picture



Lossy Machine: A machine which is not lossless is called lossy. Consider the following state table

	$Next State, Z$	
$Present State$	$X = 0$	$X = 1$
A	$A, 1$	$B, 0$
B	$B, 1$	$A, 1$

If the testing table obtained from the given machine contains repeated entry of states, the machine is lossy. As an example, for the previous machine, the testing table is as follows

	$NextState$	
$PresentState$	$Z = 0$	$Z = 1$
A	B	A
B	$-$	AB
AB	$-$	AA, AB

The testing table contains repeated entry of states in the form of AA. So, the machine is lossy.

Two states S_i, S_j of a machine M are said to be output compatible if there exists some state S_p, such that both S_i and S_j are its Ok successor or there exists a compatible pair of states Sa, S_b such that S_i S_j are its Ok successor.

Test for Information Losslessness

	$NextState, Z$	
$PresentState$	$X = 0$	$X = 1$
A	$A, 1$	$B, 0$
B	$B, 1$	$A, 1$

If we construct an output successor table for it, the table will be

	$PreviousState, I/P$	
$PresentState$	$Z = 0$	$Z = 1$
A	$B, 1$	$A, 0$
B	$-$	$B, 0$
		$A, 1$

Let us apply input 1 and 0, respectively, on states B and A. The next state will be A for both the cases, and the output will be 1 for both the cases. Therefore, we can say A and B are output compatible as there exists a state A such that both A and B are its 1 successor.

4.10.1 Test for Information Losslessness

The process of testing whether a machine is information lossless or not is done by constructing a testing table and a testing graph.

The testing table is constructed in the following way.

- The testing table for checking information losslessness is divided into two halves. The upper half contains the present states and its output successors.

- The lower half of the table is constructed in the following way

– Every compatible pair appearing in the output successor table is taken into the present state column. The successors of these pairs are constructed from the original table. Here, if any compatible pair appears, then that pair is called the implied pair for the compatible pair in the present state column.

– That new pair is again taken in the present state column if it is not taken.

– The process terminates when all compatible pairs have been taken in the present state column.

The machine is information lossless if and only if its testing table does not contain any compatible pair consisting of repeated entry.

From the testing table, the testing graph is constructed. The testing graph is constructed in the following way.

- The number of vertices of the testing graph is equal to the number of output compatible pairs taken in the lower half of the testing table. The labels of the vertices are the compatible pairs in the lower half of the testing table.

- A directed arc is drawn from vertex S_i S_j to vertex SpSq $(p \neq q)$ if SpSq is an implied pair for S_i S_j .

The machine is information lossless of finite order if the testing graph is loop-free and its order $\mu = l + 2$, where l is the length of the longest path of the testing graph.

Consider the following examples to clarify the method described previously.

Test for Information Losslessness

Example 4.18 Test whether the following machine is information lossless or not. If lossless, find its order.

Solution:

	$NextState, Z$	
$PresentState$	$X = 0$	$X = 1$
A	$C, 1$	$D, 1$
B	$A, 1$	$B, 1$
C	$B, 0$	$A, 0$
D	$D, 0$	$C, 0$

The first step to test whether a machine is lossless or not is to construct a testing table. The testing table is divided into two halves.

$PresentState$	$z = 0$	$z = 1$
A	–	(AB)
B	(BD)	–
C	D	B
D	–	(CE)
(AB)	–	$(AC)(BC)$
		$(AD)(BD)$
(CD)	$(BD)(BC)$	–
	$(AD)(AC)$	
AC	–	–
AD	–	–
BC	–	–
BD	–	–

The testing graph consists of six vertices.

The testing table does not contain any repeated entry. The machine is a lossless machine. The testing graph as shown in $Fig.$4.25 does not contain any loop. The order of losslessness is $\mu = 1 + 2 = 3$. The length of the longest path of the graph is 1.

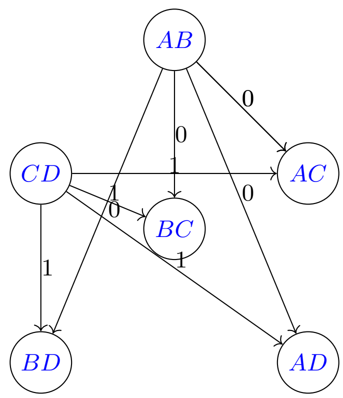


Fig. 4.25 Testing Graph for Information Losslessness

Example 4.18 Find the input string which is applied on state 'A' producing the output string 00011 and the final state 'B' for the following machine.