Exercice 1 7 points

x_i : le nombre de centaine kWh produit par

(1) la centrale $i \,|\, 1, 2, 3, 4$

$$y_i : \begin{cases} 1 & \text{si la centrale } i \,|\, 1, 2, 3, 4 \text{ est utilisée} \\ 0 & \text{si non} \end{cases}$$ (0,5)

$$\text{Min } Z = 9x_1 + 16x_2 + 12x_3 + 15x_4$$ (1,5)

s.l.c

$$\left.\begin{aligned} x_1 &\leq 500\,y_1 \\ x_2 &\leq 1200\,y_2 \\ x_3 &\leq 800\,y_3 \\ x_4 &\leq 900\,y_4 \end{aligned}\right)$$ (1)

$$\left.\begin{aligned} x_1 + x_2 + x_3 + x_4 &\geq 2987 \\ x_3 &\leq x_1 + x_2 \end{aligned}\right)$$ (1)

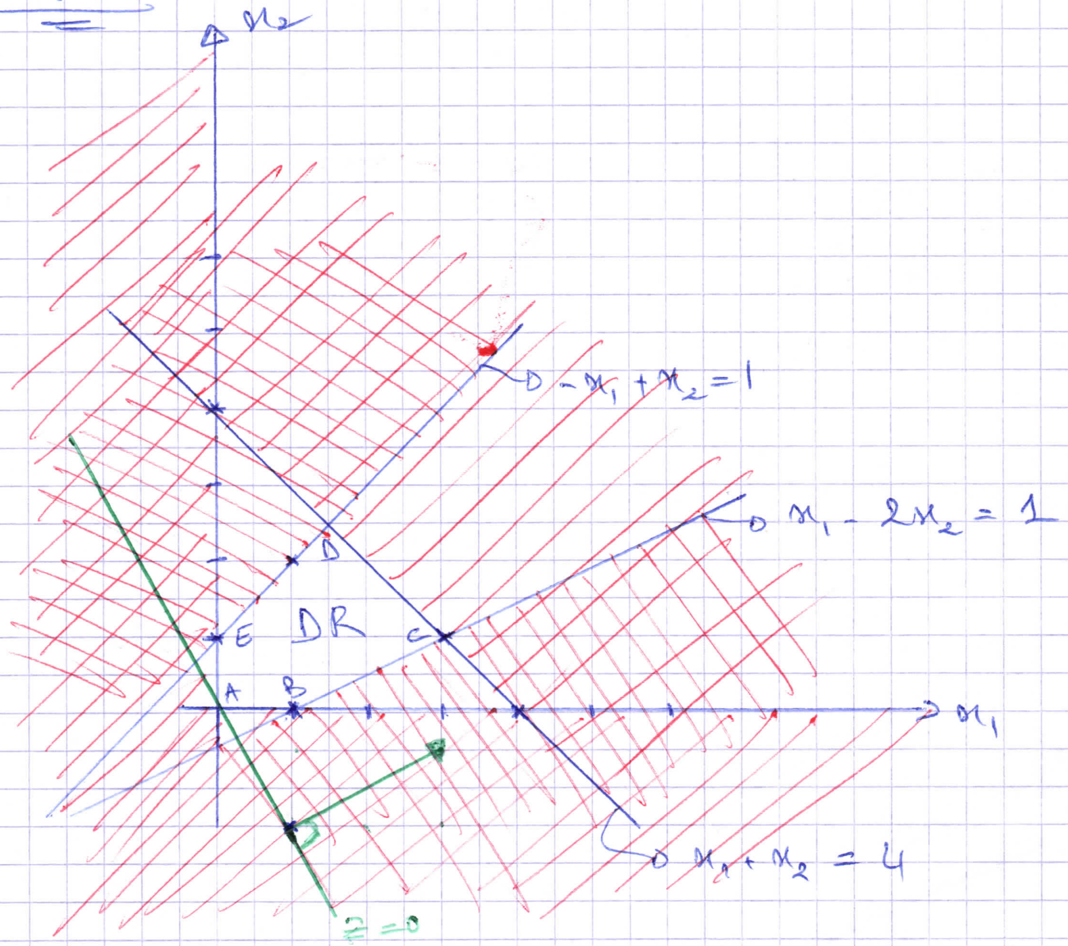
$$\left.\begin{aligned} x_4 &\leq x_1 \\ x_4 &\leq x_2 \\ x_4 &\leq x_3 \end{aligned}\right)$$ (0,5)

$$y_1 + y_3 - 1 \leq y_4$$ (1)

(0,5) $x_i \geq 0 \quad \forall\, i = 1, \ldots, 4 \qquad y_i \in \{0, 1\}$

Exercice 2

1/a



1/b/

A	$(0,0)$	$z = 2x_1 + x_2$
B	$(1,0)$	2
C	$(3,1)$	7
D	$(\frac{3}{2}, \frac{5}{2})$	$\frac{11}{2}$
E	$(0,1)$	1

→ le point C $(3,1)$ est l'unique solution optimale pour $z_c = 7$

2/ si on élimine la contrainte $x_1 + x_2 \leq 4$, on aura un DR qui n'est pas complètement borné. Puisque le vecteur gradient est orienté vers la partie ouverte du DR, donc on aura une solution sans bornes.

3/

$$(P) = \begin{cases} \text{Max } Z = 2x_1 + x_2 \\ \text{s.l.c} \\ x_1 + x_2 \leq 4 \\ -x_1 + x_2 \leq 1 \\ x_1 - 2x_2 \leq 1 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{cases}$$

$$(D) \begin{cases} \text{Min } Z' = 4y_1 + y_2 + y_3 \\ \text{s.l.c} \\ y_1 - y_2 + y_3 \geq 2 \\ y_1 + y_2 - 2y_3 \geq 1 \\ y_1 \geq 0 \\ y_2 \geq 0 \\ y_3 \geq 0 \end{cases}$$

4/

$$P \begin{cases} \text{Max } Z = 2x_1 + x_2 \\ x_1 + x_2 + s_1 = 4 \\ -x_1 + x_2 + s_2 = 1 \\ x_1 - 2x_2 + s_3 = 1 \\ x_1, x_2, s_1, s_2, s_3 \geq 0 \end{cases}$$

$$D \begin{cases} \text{Min } Z' = 4y_1 + y_2 + y_3 \\ y_1 - y_2 + y_3 - r_1 = 2 \\ y_1 + y_2 - 2y_3 - r_2 = 1 \\ y_1, y_2, y_3, r_1, r_2 \geq 0 \end{cases}$$

$x_1 \times r_1 = 0$

$x_2 \times r_2 = 0$

$s_1 \times y_1 = 0$

$s_2 \times y_2 = 0$

$s_3 \times y_3 = 0$

la solution optimale du primal est

$$c \begin{pmatrix} 3 = x_1 \\ 1 = x_2 \\ 0 = s_1 \\ 3 = s_2 \\ 0 = s_3 \end{pmatrix} \Rightarrow \begin{matrix} r_1 = 0 \\ r_2 = 0 \\ y_2 = 0 \end{matrix} = \begin{cases} y_1 + y_3 = 2 \\ y_1 - 2y_3 = 1 \end{cases}$$

$y_3 = 1/3$

$y_1 = 5/3$

$\Rightarrow$ solution optimale du Dual est

$$y^* = \begin{bmatrix} y_1 = 5/3 \\ y_2 = 0 \\ y_3 = 1/3 \\ r_1 = 0 \\ r_2 = 0 \end{bmatrix} \quad Z' = 7$$

5/

Base	x_1	x_2	s_1	s_2	s_3	CD	Ratio
s_1	1	1	1	0	0	4	4
s_2	-1	1	0	1	0	1	—
s_3	1	-2	0	0	1	1	1 ←
Z	-2 ↑	-1	0	0	0	0	

$$A \begin{pmatrix} x_1 = 0 \\ x_2 = 0 \\ s_1 = 4 \\ s_2 = 1 \\ s_3 = 1 \end{pmatrix} \quad Z_A = 0$$

Base	x_1	x_2	s_1	s_2	s_3	CD	Ratio
s_1	0^{0}	3^{1}	$1^{1/3}$	0^{0}	$-1^{-1/3}$	3^{1}	1 ←
s_2	0	-1	0	1	1	2	—
x_1	1	-2	0	0	1	1	—
Z	0	-5 ↑	0	0	2	2	

$$B \begin{pmatrix} x_1 = 1 \\ x_2 = 0 \\ s_1 = 3 \\ s_2 = 2 \\ s_3 = 0 \end{pmatrix} \quad Z_B = 2$$

Base	x_1	x_2	s_1	s_2	s_3	CD
x_2	0	1	$1/3$	0	$-1/3$	1
s_2	0	0	$1/3$	1	$2/3$	3
x_1	1	0	$2/3$	0	$1/3$	3
Z	0	0	$5/3$	0	$1/3$	7

$$C \begin{pmatrix} x_1 = 3 \\ x_2 = 1 \\ s_1 = 0 \\ s_2 = 3 \\ s_3 = 0 \end{pmatrix} \quad Z_C = 7$$

le point C(3,1) est l'unique solution optimale pour $Z_C = 7$

6/ $Z^*_N = Z^*_A + \Delta b_3 \times y^*_3 = 7 + (+1) \times \frac{1}{3} = \frac{22}{3} = \frac{22}{3}$