Classification- Visual Recognition

1. Introduction
2. Supervised learning
3. Perceptrons
4. **SVM classifiers**
5. Datasets and evaluation

SVM

Notations:

- Image/Patterns $\mathbf{x} \in \mathbf{X}$
- Φ: function transforming the patterns into feature vectors $\Phi(x)$
- $< \cdot, \cdot >$ dot product in the feature space endowed by $\Phi(\cdot)$
- Classes $y = \pm 1$

Early kernel classifiers derived from the perceptron [Rosenblatt58]:

- taking the sign of a linear discriminant function:

$$f(\mathbf{x}) =< \mathbf{w}, \Phi(\mathbf{x}) > +b$$

- Classifiers called Φ-machines

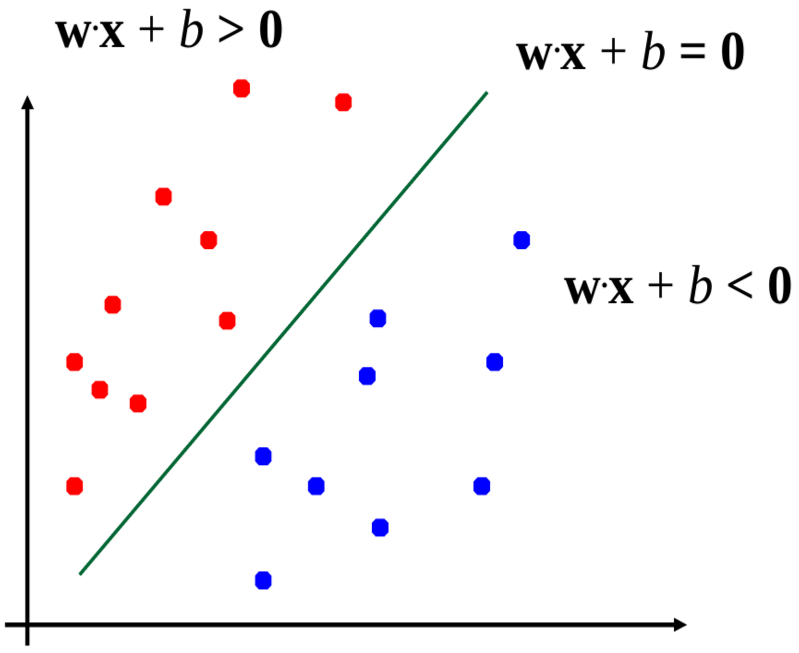
SVM

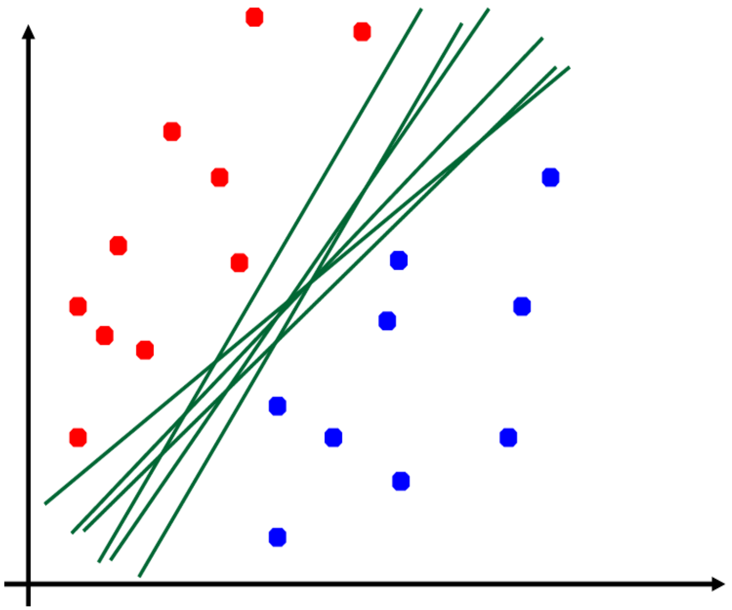
- Question: how to find/estimate f ?
  - Feature function Φ usually hand-chosen for each problem
  - Several Φ for image processing like BoW
  - w and b: parameters to be determined

$$f(x) = \langle w, \Phi(x) \rangle + b$$

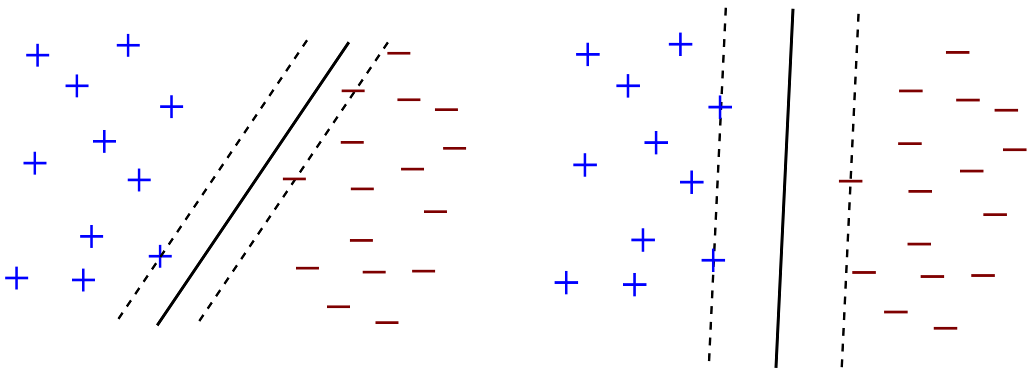
- Learning algorithm on a set of training examples:
  $\mathcal{A} = (x_1, y_1) \cdots (x_n, y_n)$

Which hyperplane ? w? b?





SVM



SVM optimization: maximizing the margin between + and -

Def.: Margin = distance between the hyperplanes $f(x) = 1$and $f(x) = -1$ (dashed lines in Figure).

Intuitively, a classifier with a larger margin is more robust to fluctuations

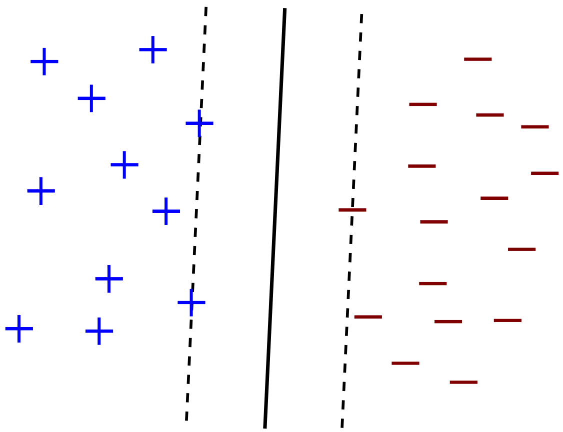
Hard Margin

Final expression for the Hard Margin SVM optimization:

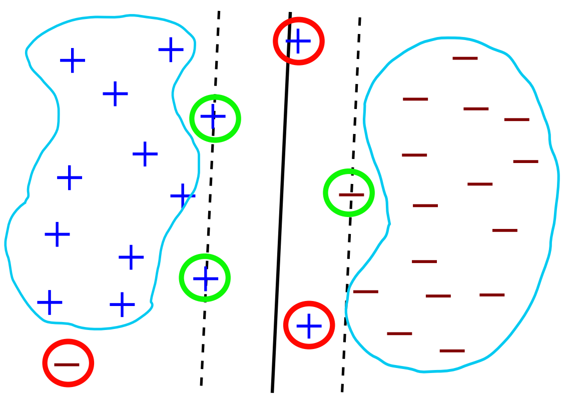
$$\min_{w,b} \; P(w, b) = \frac{1}{2}\|w\|^2 \quad \text{with} \quad \forall\, i \quad y_i\, f(x_i) \geq 1$$

SVM

- Hard Margin: OK if data are linearly separated



- Otherwise: noisy data (in red) disrupt the optim.
- Solution: Soft SVM



SVM: Soft Margin

Introducing the slack variables ξ_i, one usually gets rid of the inconvenient max of the loss and rewrite the problem as

$$\min_{w,b}\ P(w,b) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}\xi_i \quad \text{with} \quad \left\{\begin{array}{ll} \forall\, i & y_i\, f(x_i) \geq 1 - \xi_i \\ \forall\, i & \xi_i \geq 0 \end{array}\right.$$

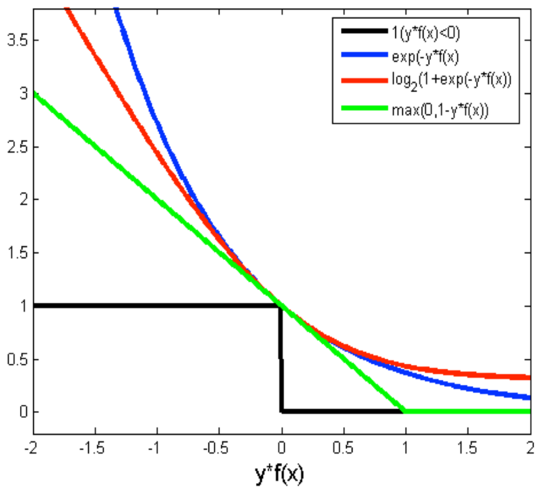
For very large values of the hyper-parameter C, **Hard Margin** case:

- Minimization of $\|w\|$ (ie margin maximization) under the constraint that all training examples are correctly classified with a loss equal to zero.

Smaller values of C relax this constraint: **Soft Margin** case

- SVMs that produces markedly better results on noisy problems.

SVM learning scheme



Equivalently, minimizing the following objective function in feature space with the hinge loss function:

$$\ell(y_i\, f(x_i)) = \max\left(0, 1 - y_i\, f(x_i)\right)$$

$$\min_{w,b}\ P(w,b) = \frac{1}{2}\|w\|^2 + C\sum_{i=1}^{n}\ell(y_i\, f(x_i))$$

Regularization

Margin Maximization

Data fitting

Constraint satisfaction

Learning SVMs: Primal/Dual

- In practice: Convex optimization problem
  - Primal optimization: $f(x) = \langle w, \Phi(x) \rangle + b$
  - Dual optimization: learning SVMs can be achieved by solving the dual of this convex optimization problem
- Dual (using Lagragian and $k(x_i, x_j) = \langle \Phi(x_i), \Phi(x_j) \rangle$):
  - For Hard Margin:

$$\max_{\boldsymbol{\alpha}} \mathcal{L}(\boldsymbol{\alpha}) = \sum_{i=1} \alpha_i - \frac{1}{2} \sum_{i,j} y_i y_j \alpha_i \alpha_j k(x_i, x_j) \quad \text{with} \quad \begin{cases} \sum_i \alpha_i y_i = 0 \\ 0 \leq \alpha_i \end{cases}$$

  - For Soft Margin:

$$\max_{\boldsymbol{\alpha}} \mathcal{L}(\boldsymbol{\alpha}) = \sum_{i=1} \alpha_i - \frac{1}{2} \sum_{i,j} y_i y_j \alpha_i \alpha_j k(x_i, x_j) \quad \text{with} \quad \begin{cases} \sum_i \alpha_i y_i = 0 \\ 0 \leq \alpha_i \leq C \end{cases}$$

SVM optimization

Standard equivalent formulation without enforcing α_i to be positive:

- Optimization on coefficients α_i of the SVM kernel expansion $f(x) = \sum_{i=1}^{n} \alpha_i k(x, x_i) + b$ by defining the dual objective function:

$$D(\boldsymbol{\alpha}) = \sum_i \alpha_i y_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j k(x_i, x_j)$$

- and solving the SVM dual *Quadratic Programming* (QP) problem.

$$\max_{\boldsymbol{\alpha}} \; D(\boldsymbol{\alpha}) \quad \text{with} \quad \left\{ \begin{array}{l} \sum_i \alpha_i = 0 \\ A_i \leq \alpha_i \leq B_i \\ \quad A_i = \min(0, C y_i) \\ \quad B_i = \max(0, C y_i) \end{array} \right.$$

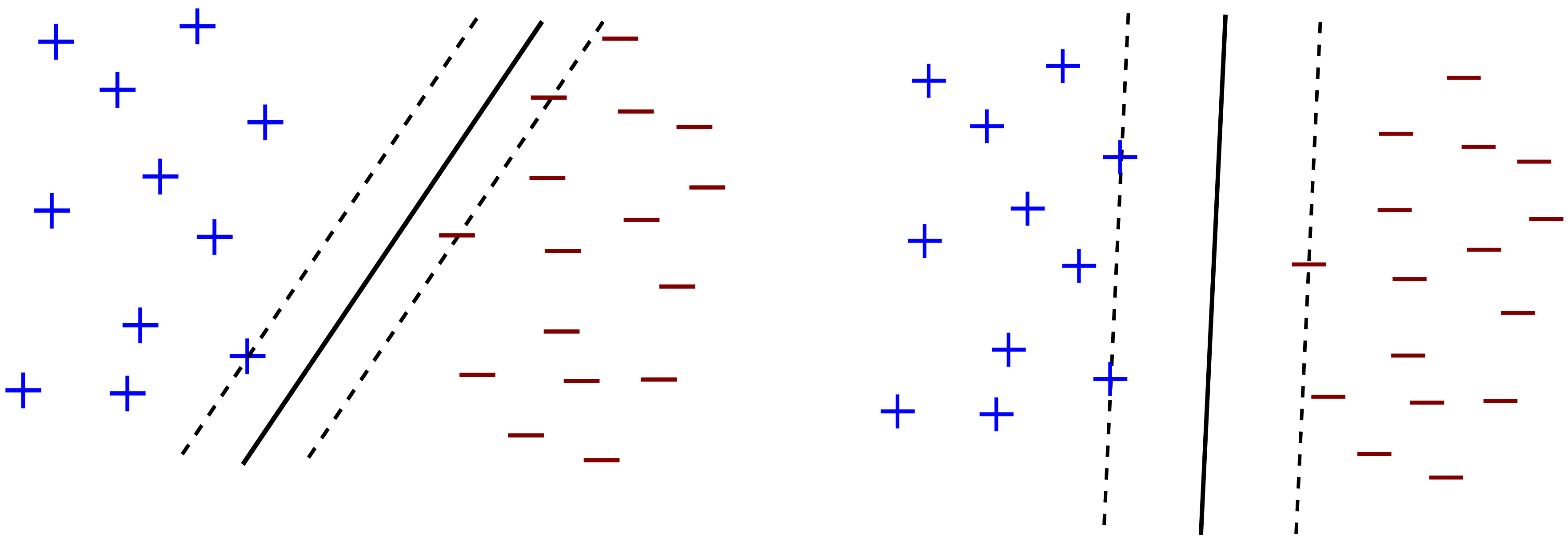
Classification pipeline

To summarize on SVM :

Solving equation: SVM

Support Vector Machines (SVM) defined by three incremental steps:

1. [Vapnik63]: linear classifier / separates the training examples with the **widest margin** => Optimal Hyperplane



Solving equation: SVM

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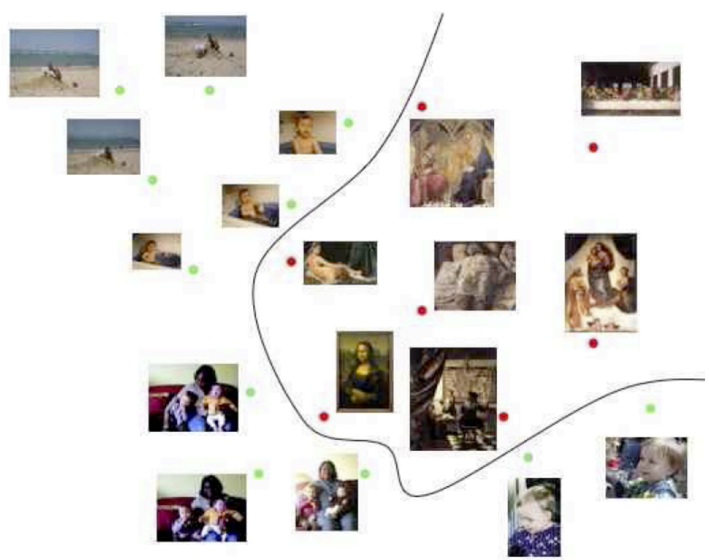
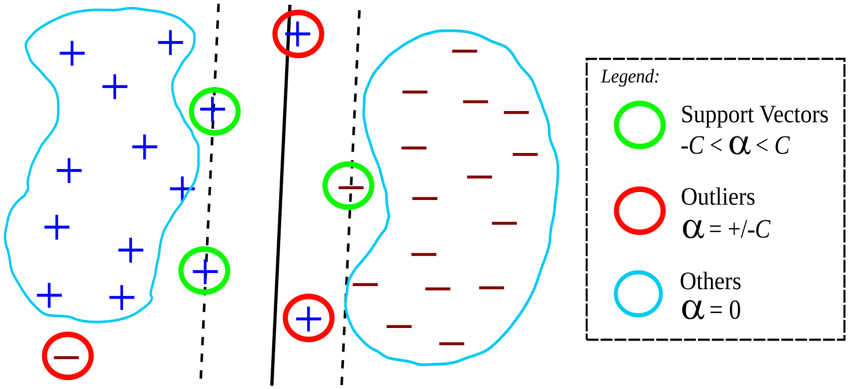
2. [Guyon93] Optimal Hyperplane built in the feature space induced by a kernel function



Solving equation: SVM

Support Vector Machines (SVM) defined by three incremental steps:

1. [Vapnik63]: linear classifier / separates the training examples with the widest margin =>Optimal Hyperplane
2. [Guyon93] Optimal Hyperplane built in the feature space induced by a kernel function
3. [Cortes95] soft version: noisy problems addressed by allowing some examples to violate the margin constraint



Appendix: Solving SVM

- Min P or Max D
  => QP (Quadratic programing) family optimization
- Good news: efficient batch numerical algorithms have been developed to solve the specific SVM QP problem (hinge loss, convex objective,…)

- Some strategies (exploiting specif.):
  – Conjugate Gradient method [Vapnik]
  – Sequential Minimal Optimization (SMO) [platt].
- In both methods successive searches along well chosen directions

- Some famous SVM solvers like SVMLight [Joachims] or SVMTorch propose to use *decomposition* algorithms to define such directions
- SVMstruct (for structured outputs)
- State-of-the-art implementation of SMO: [libsvm] => used in tutorials
- LibLinear bib for primal optim (with MATLAB)

SMO algo for SVM optimization

1. Set $\boldsymbol{\alpha} \leftarrow \mathbf{0}$ and compute the initial gradient $\boldsymbol{g}$ of $D(\boldsymbol{\alpha})$

2. Choose a τ-violating pair(*) (i, j) Stop if no such pair exists

3. $\lambda \leftarrow \min \left\{ \dfrac{g_i - g_j}{k_{ii} + k_{jj} - 2k_{ij}},\ B_i - \alpha_i,\ \alpha_j - A_j \right\}$

4. $\alpha_i \leftarrow \alpha_i + \lambda\ ,\ \ \alpha_j \leftarrow \alpha_j - \lambda$

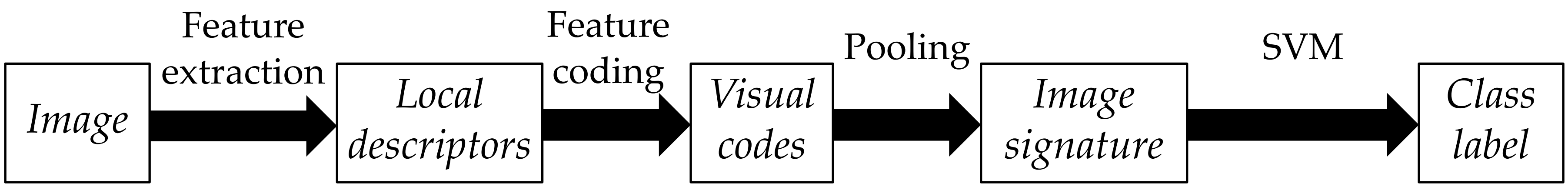
5. $g_s \leftarrow g_s - \lambda(k_{is} - k_{js}) \quad \forall\, s \in \{1 \dots n\}$

6. Return to step 2

(*) pairs in +1/-1 with significant diff of gradients

A ways to easily satisfy the null sum coeff constraint

Classification pipeline



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