Discrete Mathematics in Computer Science

M. Helmert, G. Röger
F. Pommerening
Fall Term 2024

University of Basel
Computer Science

Exercise Sheet 1

Due: Monday, September 30, 2024

Note: Submissions that are exclusively created with LaTeX will receive a bonus mark. Please submit only the resulting PDF file.

Exercise 1.1 (2 marks)

(a) Describe $S_1 = \{3x + 1 \mid x \in \mathbb{N}_0\}$ implicitly as a sequence with dots.

(b) Describe $S_2 = \mathcal{P}(\mathcal{P}(\{1, 2\}) \setminus \{\{1\}, \{2\}\})$ with explicit enumeration.

(c) Consider set $S_3 \subseteq \mathbb{N}_0$ that is inductively defined as follows:

- $1 \in S_3$ and
- $2 \in S_3$ and
- $3 \in S_3$ and
- if $x \in S_3$ and $y \in S_3$ then $xy \in S_3$.

Describe S_3 with the set-builder notation.

(d) Describe $S_4 = \{x^2 \mid x \in \mathbb{N}_0, \text{there is no } y \in \mathbb{N}_0 \text{ with } x = 3y\}$ in natural language.

Exercise 1.2 (1 mark)

We consider sets $A, B \subseteq \mathbb{N}_0$. The following formalizations of the described expressions are syntactically wrong. Specify the correct syntax.

(a) The set A consists of all even numbers except n:
$A = \{x = 2k\} \setminus n$

(b) The set B is the smallest set containing $5x$ and $7x$ for all natural numbers x:
$B = \{5x, 7x \mid x \in \mathbb{N}_0\}$

Exercise 1.3 (3 marks)

For this exercise, we consider the set of all objects to be $U = \{1, \dots, 10\}$. In each subtask specify sets $A, B \subseteq U$ that have all the required properties.

(a) $A \subset B$ and $A \cap B = \{1, 2, 3\}$

(b) $A \cup \{7, 2, 4\} = B$ and $\overline{B} = \{1, 6, 8, 1\}$

(c) $A \setminus B \supseteq \overline{\{1, 3, 5, 7, 9\}}$ and $A \cap B \supset \{3, 9\}$

Exercise 1.4 (4 marks)

On ADAM, you will find two documents on writing mathematical proofs. Read § 1 (Minicourse on technical writing) by Knuth, and the remarks on writing mathematical proofs by Lee. Some of Knuth's rules are more relevant for us than others. In particular, pay attention to rules 1, 2, 3, 7, 10, 12, 13, 14, 17, and 26, and the last sentence of 11.

Note that the fragments below do not make sense on their own. This exercise is not about what these proofs say, but on how they say it. In the future when we ask you to prove something on an exercise sheet, we also expect you to write well-written proofs but on this sheet, we only focus on the form of the proof instead of its content.

(a) Which of the following arguments is the better proof for $A \cap B \subseteq A \cup B$? Justify your answer by referring to the documents.

- We show that $A \cap B \subseteq A \cup B$ by considering an arbitrary element $x \in A \cap B$. With the semantics of the intersection we get $x \in A$. Since x is in A, it also has to be in $A \cup B$. We showed that $x \in A \cap B$ implies $x \in A \cup B$ for an arbitrary element x, so this holds for all elements and we have shown $A \cap B \subseteq A \cup B$.
- $\forall x \in A \cap B \Rightarrow x \in A \Rightarrow x \in B \Rightarrow x \in A \cup B \Rightarrow A \cap B \subseteq A \cup B$.

(b) Which of the following derivations of the equality $a(b-c) + ac = b(a/2^0)$ should be used as part of a proof? Justify your answer by referring to the documents.

$$\begin{aligned} a(b-c) + ac &= b(a/2^0) \\ ab - ac + ac &= b(a/1) \\ ab &= ab \\ 0 &= 0 \quad \checkmark \end{aligned}$$

$$\begin{aligned} &a(b-c) + ac \\ &= ab - ac + ac \\ &= ab \\ &= b(a/1) \\ &= b(a/2^0) \end{aligned}$$

$$\begin{aligned} &ab \\ &= ab - ac + ac \\ &= a(b-c) + ac \\ &= b(a/2^0) \\ &= b(a/1) \\ &= ab \end{aligned}$$