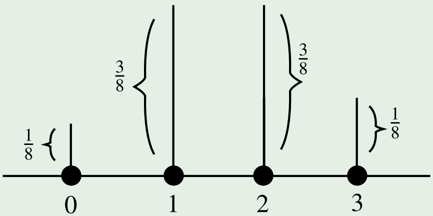
Lecture 9 : Change of discrete random variable

You have already seen (I hope) that whenever you have "variables" you need to consider change of variables. Random variables are no different.
The notion of "change of random variable" is handle too briefly on page 103 of the text. This is something I will test you on.

Example 1

Suppose $X \sim Bin\left(3, \dfrac{1}{2}\right)$.

line graph



table

X	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

(b)

Suppose what to define a new random variable $Y = 2X - 1$.
How do we do it?
So how do we define $P(Y = k)$?
Answer - express Y in terms of X and compute so

$$\begin{aligned} P(Y = k) &= P(2X - 1 = k) \\ &= P\left(X = \frac{k+1}{2}\right) \qquad (*) \end{aligned}$$

The right-hand site is the logical <u>definition</u> of the left-hand side.
But as is often the case in probability it is easier to pretend we know what $P(Y = k)$ means already and then the next two steps are a computation.

So let's compute the *pmf* of Y.
What are the possible values of Y?
From (*) k is a possible value of $Y \Leftrightarrow \dfrac{k+1}{2}$ is a possible values of X.

$$\Longleftrightarrow \quad \frac{k+1}{2} = \begin{cases} 0 \\ 1 \\ 2 \\ 3 \end{cases} \quad \Longleftrightarrow \quad Y = \begin{cases} -1 \\ 1 \\ 3 \\ 5 \end{cases}$$

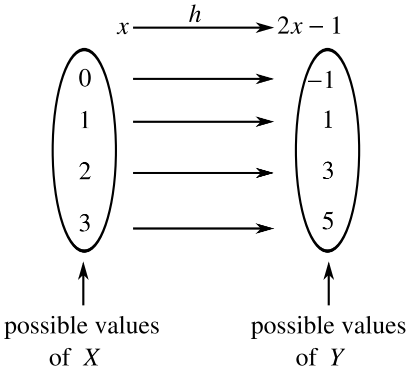
Note

$$\frac{k+1}{2} = x \Leftrightarrow k = 2x - 1$$

possible value of X

possible value of Y

So the possible values of Y are obtained by applying the function $h(x) = 2x - 1$ to the possible values of X.
(note $Y = f(X)$).



Just "push forward" the values of X.

Example (Cont.)

Now we have computed the possible values of Y we need to compute their probabilities. Just repeat what we did

$$\begin{aligned} P(Y=-1) &= P(2X-1=-1) \\ &= P(X=0) = \frac{1}{8} \\ P(Y=1) &= P(2X-1=1) \\ &= P(X=1) = \frac{3}{8} \end{aligned}$$

Similarly

$$P(Y=3) = \frac{3}{8} \quad \textit{and} \quad P(Y=5) = \frac{1}{8}$$

X	-1	1	3	5
$P(Y=y)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

So we have the “same probabilities” as before namely $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$ it is just then are pushed-forward to new locations