$$f_Y(y)dy = \frac{1}{\beta^\alpha}\frac{1}{\Gamma(\alpha)}(\beta y)^{\alpha-1}e^{-\frac{\beta y}{\beta}}d(\beta y)$$

$$= \frac{1}{\cancel{\beta^\alpha}}\frac{1}{\Gamma(\alpha)}\cancel{\beta^{\alpha-1}}y^{\alpha-1}e^{-y}\cancel{\beta}dy$$

$$= \underbrace{\frac{1}{\Gamma(\alpha)}y^{\alpha-1}e^{-t}\,dy}_{\text{standard gamma}}$$