Lecture 16: Independence, Covariance and Correlation of Discrete Random Variables

Definition

Two discrete random variables X and Y defined on the same sample space are said to be independent if for nay two numbers x and y the two events (X = x) and (Y = y) are independent \Leftrightarrow

$$P((X = x) \cap (Y = y)) = P(X = x)P(Y = y)$$

$$\Leftrightarrow \quad \text{and}$$

$$P(X = x, Y = y) = P(X = x)P(Y = y)$$

$$\Leftrightarrow \quad P_{X,y}(x, y) = P_X(x)P_Y(y)$$
(*)

Now (*) say the joint $pmf\ P_{X,Y}(x,y)$ is determined by the <u>marginal</u> pmf's $P_X(x)$ and $P_Y(y)$ by taking the product.

Problem

In case X and Y are independent how do you recover the matrix (table) representing $P_{X,y}(x,y)$ from its margins?

Let's examine the table for the standard example

X	0	1	2	3	
0	<u>1</u> 8	<u>2</u> 8	1 8	0	1/2
1	0	<u>1</u> 8		<u>1</u> 8	1/2
	<u>1</u> 8	1 8 3 8	218 318	1 8	

Note that

 $X = \sharp$ of heads on the first toss

 $Y = \text{total} \ \sharp \ \text{of heads in all three tosses}$

So we wouldn't expect X and Y to be independent (if we know X = 1 that restricts the values of Y.)

Lets use the formula (*)

It says the following.

Each position inside the table corresponds to two positions on the margins

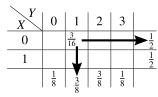
- Go to the right
- 2 Go Down

X^{Y}	0]	l	2	3	
0						$\rightarrow \frac{1}{2}$
			-			1
1		1	,			$\frac{1}{2}$
	$\frac{1}{8}$	2.0	3 3	$\frac{3}{8}$	$\frac{1}{8}$	

So in the picture

- 1 If we go right we get $\frac{1}{2}$ 2 If we go down we get $\frac{3}{8}$

If X and Y are independent then the formula (*) says the entry inside the table is obtain by multiplying 1 and 2



So if X and Y wave independent then we would set

•	X	0	1	2	3			
	0	1 16	<u>3</u> 16	<u>3</u> 16	<u>1</u>	1/2		(#
	1	<u>1</u> 16	<u>3</u> 16	<u>3</u> 16	<u>1</u> 16	1/2		
		1 0	3	3	1 8			