Lecture 16 : Independence, Covariance and Correlation of Discrete Random Variables

Definition

Two discrete random variables X and Y defined on the same sample space are said to be independent if for nay two numbers x and y the two events $(X = x)$ and $(Y = y)$ are independent $\Leftrightarrow$

$$P((X = x) \cap (Y = y)) = P(X = x)P(Y = y)$$

$$\Leftrightarrow$$

and

$$P(X = x, Y = y) = P(X = x)P(Y = y)$$

$$\Leftrightarrow$$

$$P_{X,y}(x, y) = P_X(x)P_Y(y) \qquad (*)$$

Now (*) say the joint *pmf* $P_{X,Y}(x,y)$ is determined by the marginal *pmf*'s $P_X(x)$ and $P_Y(y)$ by taking the product.

Problem

In case X and Y are independent how do you recover the matrix (table) representing $P_{X,y}(x,y)$ from its margins?

Let's examine the table for the standard example

$X \backslash Y$	0	1	2	3	
0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0	$\frac{1}{2}$
1	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{2}$
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

Note that

$X = \sharp$ of heads on the first toss
Y = total $\sharp$ of heads in all three tosses

So we wouldn't expect X and Y to be independent (if we know $X = 1$ that restricts the values of Y.)

Lets use the formula (*)
It says the following.
Each position inside the table corresponds to two positions on the margins

1. Go to the right
2. Go Down

$X \backslash Y$	0	1	2	3	
0					$\frac{1}{2}$
1					$\frac{1}{2}$
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

So in the picture

1. If we go right we get $\dfrac{1}{2}$
2. If we go down we get $\dfrac{3}{8}$

If X and Y are independent then the formula (*) says the entry inside the table is obtain by multiplying 1 and 2

$X \backslash Y$	0	1	2	3	
0		$\frac{3}{16}$			$\frac{1}{2}$
1					$\frac{1}{2}$
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

So if X and Y wave independent then we would set

$X \backslash Y$	0	1	2	3	
0	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{2}$
1	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{2}$
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

(♯)