

$$\begin{aligned} P_{X,Y}(x,y) &= \sum_{\text{all } z} P_X(x)P_Y(y)P_Z(z) \\ &= P_X(x)P_Y(y) \left( \sum_{\text{all } z} P_Z(z) \right) \\ &= P_X(x)P_Y(y) \quad \text{"1"} \end{aligned}$$