

## Lecture 16 : Independence, Covariance and Correlation of Discrete Random Variables

## Definition

Two discrete random variables  $X$  and  $Y$  defined on the same sample space are said to be independent if for any two numbers  $x$  and  $y$  the two events  $(X = x)$  and  $(Y = y)$  are independent  $\Leftrightarrow$

$$P((X = x) \cap (Y = y)) = P(X = x)P(Y = y)$$

$\Leftrightarrow$  and

$\downarrow$

$$P(X = x, Y = y) = P(X = x)P(Y = y)$$

$\Leftrightarrow$

$$P_{X,Y}(x, y) = P_X(x)P_Y(y) \quad (*)$$

Now (\*) say the joint pmf  $P_{X,Y}(x, y)$  is determined by the marginal pmf's  $P_X(x)$  and  $P_Y(y)$  by taking the product.

### Problem

*In case  $X$  and  $Y$  are independent how do you recover the matrix (table) representing  $P_{X,Y}(x, y)$  from its margins?*

Let's examine the table for the standard example

X \ Y	Y				
	0	1	2	3	
0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0	$\frac{1}{2}$
1	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	$\frac{1}{2}$
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

Note that

$X = \#$  of heads on the first toss

$Y =$  total  $\#$  of heads in all three tosses

So we wouldn't expect  $X$  and  $Y$  to be independent (if we know  $X = 1$  that restricts the values of  $Y$ .)

Lets use the formula (\*)

It says the following.

Each position inside the table corresponds to two positions on the margins

- 1 Go to the right
- 2 Go Down

$X \backslash Y$	0	1	2	3	
0					$\frac{1}{2}$
1					$\frac{1}{2}$
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

The table represents the joint probability distribution of two discrete random variables X and Y. The rows correspond to X=0 and X=1, and the columns correspond to Y=0, 1, 2, and 3. The last column shows the marginal probabilities for X, and the last row shows the marginal probabilities for Y. Arrows indicate the path from the cell (0,1) to the marginal probabilities 1/2 and 3/8.

So in the picture

- 1 If we go right we get  $\frac{1}{2}$
- 2 If we go down we get  $\frac{3}{8}$

If  $X$  and  $Y$  are independent then the formula (\*) says the entry inside the table is obtain by multiplying 1 and 2

$X \backslash Y$	0	1	2	3	
0		$\frac{3}{16}$			$\frac{1}{2}$
1					$\frac{1}{2}$
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

Diagram illustrating the calculation of joint probabilities for independent variables  $X$  and  $Y$ . A horizontal arrow points from the entry  $\frac{3}{16}$  in the row  $X=0$  to the marginal probability  $\frac{1}{2}$  for  $Y=1$ . A vertical arrow points from the same entry  $\frac{3}{16}$  to the marginal probability  $\frac{3}{8}$  for  $X=0$ .

So if  $X$  and  $Y$  were independent then we would set

$X \backslash Y$	0	1	2	3	
0	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{2}$
1	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{2}$
	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	

(#)

























































