Week 1

Key Objective:

Be familiar with the basic notions Of probability including: definition of a stochastic process, distributions of discrete and continuous random formula variables, calculation of expectation values, convolution formula for distribution of a sum.

Background:

Please use this week to recap basic probability concepts which you have covered in Probability II (or a equivalent course elsewhere). Use the lecture notes as a guide to what you need to know and read as necessary the relevant parts Of Chapters 1 and 2 of Taylor and Karlin, "An Introduction to Stochastic Modeling" [TA-Kl. Next week, amongst other things, we will utilize convolution formulae (cf. [T+KI Sec. 1.2.5) and the 'trick" with tail probabilities (see [T+KI Sec. 1.5.1). The 'Exercises" in [T+KI (answers at the back of the book) are short questions which provide a good diagnostic tool to check you understand things - I recommend you do a selection of those in Chapters 1 and 2 corresponding to the to topics you feel less confident about. The problems overleaf are of increasing difficulty; note that on these sheets indicates a particularly challenging (sub) question.

Problems:

1. **Random heating**

   Let B be the the number of working boilers discrete in the Mathematics Building. Assume B is a random variable having possible values 0, 1, and 2 and probability mass function

$$p(0) = \frac{1}{10},$$
$$p(1) = \frac{3}{5},$$
$$p(2) = \frac{3}{10}.$$

(a) Plot the corresponding distribution function.

(b) Determine the mean and variance of B.

2. **Gamma distribution:**

   Consider a random variable X with the probability density function

$$f_X(x) = a^2 x e^{-ax} \quad \text{for } x > 0$$

   where a is a positive parameter. (This is a case of the Gamma distribution and will also be important later in the course.)

   (a) Sketch $f_X(x)$, Where is its maximum?

   (b) Find the mean and variance of X.

   (c) Calculate the Laplace transform

$$\hat{F}_X(\lambda) = E[e^{-\lambda X}].$$

   (d) Now consider a second random variable Y which is drawn independently from the same distribution (i.e, $f_Y(y) = a^2 y e^{-ay} \quad \text{for } y > 0$). Use convolution to find the probability density function of the sum $Z = X + Y$.

3. **More practice with Laplace Transforms:**

   (a) Find $\hat{F}_X(\lambda)$ for the case

$$dF_X(x) = e^{ax} dx.$$