Maths Club

Brighter future

Chapter 1

GCSE Revision - Algebraic Proof and Algebra in Context

1. Use algebra to prove that the sum of three consecutive whole numbers is always divisible by 3.

2. Prove that $(2n + 3)^2 - (2n - 3)^2$ is a multiple of 8 for all positive integer values of n.

3. The diagram shows a trapezium.

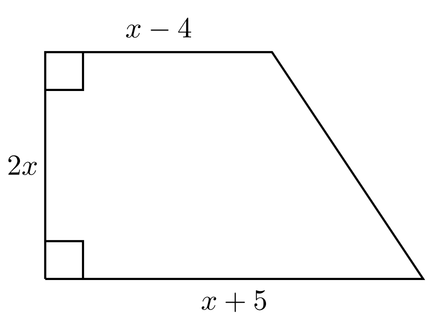


Diagram **NOT** accurately drawn

All the measurements are in centimetres.

The area of the trapezium is $351cm^2$.

(a) Show that $2x^2 + x - 351 = 0$. **(2)**

(b) Work out the value of x. **(3)**

4. Here are two triangles $\mathbf{T_1}$ and $\mathbf{T_2}$.

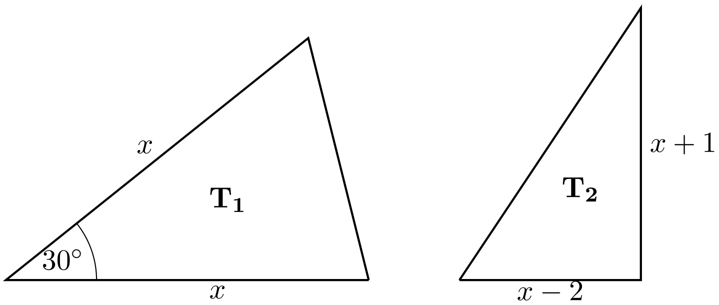


Diagram **NOT** accurately drawn

The lengths of the sides are in centimetres.

The area of triangle $\mathbf{T_1}$ is equal to the area of triangle $\mathbf{T_2}$.

Work out the value of x, giving your answer in the form $a+\sqrt{b}$ where a and b are integers.

5. Prove algebraically that the difference between the squares of any two consecutive integers is equal to the sum of these two integers.

6.

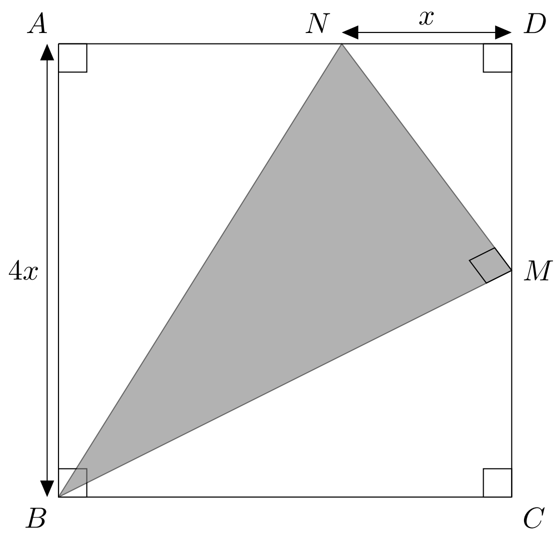


Diagram **NOT** accurately drawn

$ABCD$ is a square with a side length of $4x$.

M is the midpoint of DC.

N is the point on AD where $ND = x$.

BMN is a right-angled triangle.

Find an expression, in terms of x, for the area of triangle BMN.

Give your expression in its simplest form.

7.

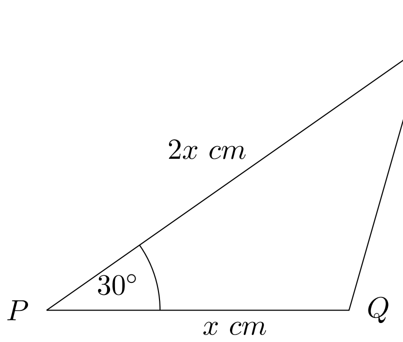


Diagram **NOT** accurately drawn

$PQ = x\ cm$

$PR = 2x\ cm$

Angle $QPR = 30°$

The area of triangle $PQR = A\ cm^2$

Show that $x = \sqrt{2A}$.

8.

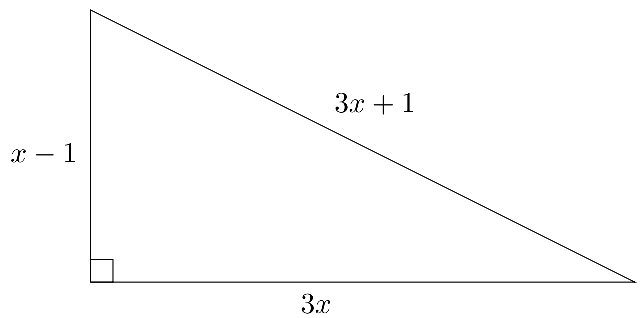


Diagram **NOT** accurately drawn

In the diagram, all the measurements are in metres.

The perimeter of the triangle is 56 m.

The area of the triangle is A m^2.

Work out the value of A.

9.

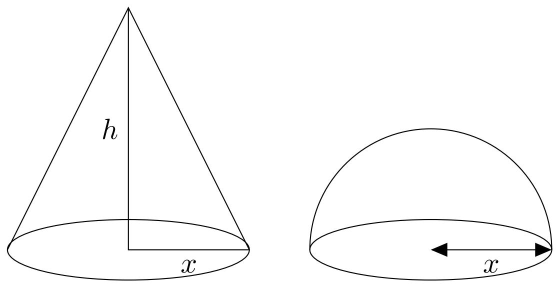


Diagram **NOT** accurately drawn

The diagram shows a solid cone and a solid hemisphere.

The cone has a base of radius x cm and a height of h cm.

The hemisphere has a base of radius x cm.

The surface area of the cone is equal to the surface area of the hemisphere.

Find an expression for h in terms of x.

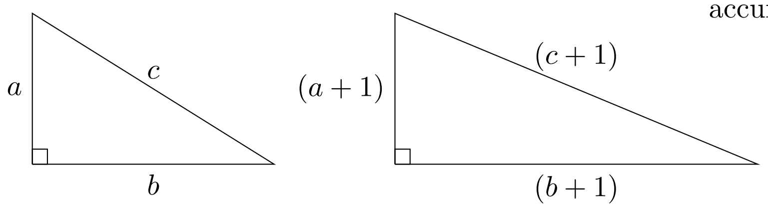
10. Umar thinks $(a+1)^2 = a^2 + 1$ for all values of a.

(a) Show that Umar is wrong. **(2)**

(b) Here are two right-angled triangles.

All the measurements are in centimetres

Diagram **NOT** accurately drawn



Show that $2a + 2b + 1 = 2c$. $a,$ b and c cannot all be integers. **(3)**

(c) Explain why. **(1)**

11. The diagram below shows a large rectangle of length $(2x + 6)$ cm and width x cm. A smaller rectangle of length x cm and width 3 cm is cut out and removed.

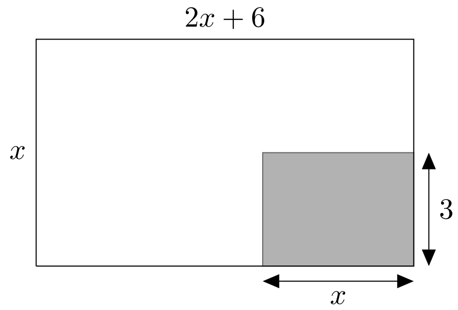


Diagram **NOT** accurately drawn

The area of the shape that is left is 100 cm^2

(a) Show that $2x^2 + 3x - 100 = 0$. **(3)**

(b) Calculate the length of the smaller rectangle. Give your answer correct to 3 significant figures. **(4)**

12.

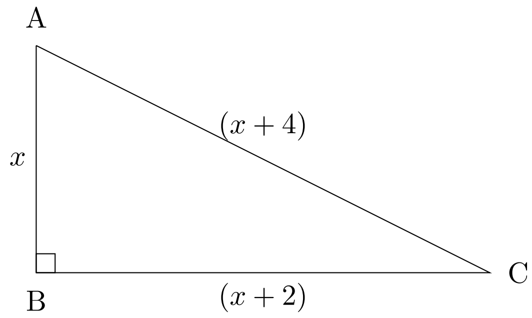


Diagram **NOT** accurately drawn

ABC is a right-angled triangle.

All the measurements are in centimetres.

$AB = x$

$BC = (x+2)$

$AC = (x+4)$

(a) Show that $x^2 - 4x - 12 = 0$. **(3)**

(b) **(4)**

i. Solve $x^2 - 4x - 12 = 0$

ii. Hence, write down the length of AC.

13. Prove that the difference between the squares of two consecutive odd numbers is a multiple of 8.

14. Prove that $n^2 + n + 1$ is always odd for all integers n.

15. Factorise $2t^2 + 5t + 2$. Hence explain why $2t^2 + 5t + 2$ can never be a prime number for any positive whole number value of t.

Chapter 2

GCSE Revision - Algebra - (Excluding Geometric problems and proofs)

1. (a) Simplify $x^7 \times x^3$

   (b) Simplify $(m^4)^3$

   (c) Simplify $\dfrac{36af^8}{12a^5f^2}$

Total for Question 1 is 7 marks

2. (a) Solve $\dfrac{4(8x-2)}{3x} = 10$ **(3)**

(b) Write as a single fraction in its simplest form **(3)**

$$\frac{2}{y+3} - \frac{1}{y-6}$$

3. Solve the simultaneous equations

$$3x + 4y = 5$$

$$2x - 3y = 9$$

$x =$ ....

$y =$ ....

Total for Question 3 is 4 marks

4.

$$A = 4bc$$

$$A = 100$$

$$b = 2$$

(a) Work out the value of c. **(2)**

(b) Make k the subject of the formula, $m = \sqrt{\dfrac{k+1}{4}}$. **(3)**

5. Solve $\dfrac{4x-1}{5} + \dfrac{x+4}{2} = 3$ **(3)**

$x =$ ....

6. (a) Simplify $a^4 \times a^5$ **(1)**

(b) Simplify $\dfrac{45e^6f^8}{5ef^2}$ **(2)**

(c) Write down the value of $9^{\frac{1}{2}}$. **(1)**

7. Solve the simultaneous equations **(6)**

$$x^2 + y^2 = 9$$
$$x + y = 2$$

Give your answers correct to 2 decimal places.

$x =$ .... $y =$ ....

or $x =$ .... $y =$ ....

8. Make p the subject of the formula $y = 3p^2 - 4$. **(3)**

9. (a) Factorise $6 + 9x$. **(1)**

(b) Factorise $y^2 - 16$. **(1)**

(c) Factorise $2p^2 - p - 10$. **(2)**

10. Solve $\dfrac{2-y}{5} = 1$. **(5)**

$y =$ ....

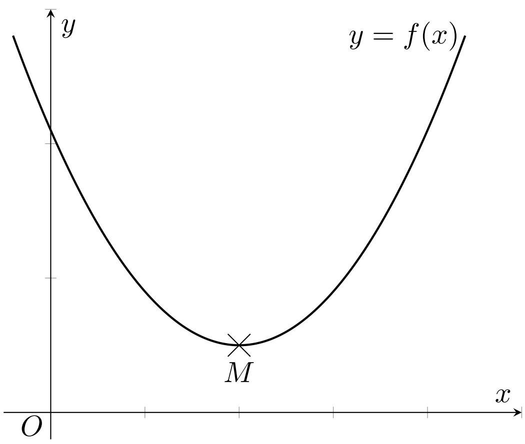
11. The expression $x^2 - 8x + 21$ can be written in the form $(x-a)^2 + b$ for all values of x.

(a) Find the value of a and the value of b. **(3)**

$a =$ ....

$b =$ ....

The equation of a curve is $y = f(x)$ where $f(x) = x^2 - 8x + 21$. The diagram shows part of a sketch of the graph of $y = f(x)$.



The minimum point of the curve is M.

(b) Write down the coordinates of M. **(1)**

(.........., ...)

12. Simplify $\dfrac{4(x+5)}{x^2+2x-15}$. **(2)**

13. Solve the simultaneous equations **(4)**

$$4x + 7y = 1$$
$$3x + 10y = 15$$

$x =$ ....

$y =$ ....

(a) Solve $2x^2 + 9x - 7 = 0$. Give your solutions correct to 3 significant figures. **(3)**

(b) Solve $\dfrac{2}{y^2} + \dfrac{9}{y} - 7 = 0$. Give your solutions correct to 3 significant figures. **(2)**

14. Simplify $\dfrac{x+1}{2} + \dfrac{x+3}{3}$. **(3)**

15. (a) i. Factorise $2t^2 + 5t + 2$.

ii. t is a positive whole number.

The expression $2t^2 + 5t + 2$ can never have a value that is a prime number. Explain why. **(3)**

16. Make t the subject of the formula **(4)**

$$p = \frac{3 - 2t}{4 + t}$$

17. Solve $\dfrac{5(2x+1)^2}{4x+5} = 5x - 1$. **(5)**

18. Solve the equations **(4)**

$$3x + 5y = 19$$

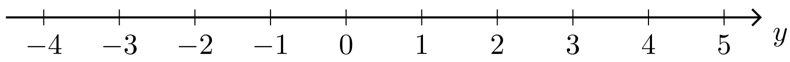
$$4x - 2y = -18$$

$x =$ ....

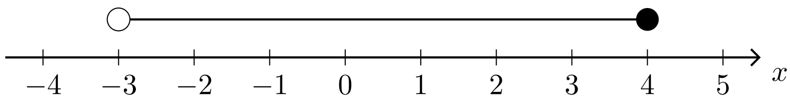
$y =$ ....

19. Solve the equation $5x^2 + 8x - 6 = 0$. Give each solution correct to 2 decimal places. **(3)**

20. (a) On the number line below, show the inequality $-2 < y < 3$. **(1)**



(b) Here is an inequality, in x, shown on a number line. **(2)**



Write down the inequality.

(c) Solve the inequality $4t - 5 > 9$. **(2)**

21. (a) Factorise fully $2x^2 - 4xy$. **(2)**

(b) Factorise $p^2 - 6p + 8$. **(2)**

(c) Simplify $\dfrac{(x+2)^2}{x+2}$. **(1)**

(d) Simplify $2a^2b \times 3a^3b$. **(2)**

22. Solve $3x^2 - 4x - 2 = 0$ Give your solutions correct to 3 significant figures. **(3)**

23. Make t the subject of the formula $2(d - t) = 4t + 7$. **(3)**

$t =$ ....

24. (a) Simplify fully $\dfrac{x^2 + 3x - 4}{2x^2 - 5x + 3}$. **(3)**

(b) Write $\dfrac{4}{x+2} + \dfrac{3}{x-2}$ as a single fraction in its simplest form. **(3)**

25. (a) Factorise $x^2 + px + qx + pq$. **(2)**

(b) Factorise $m^2 - 4$. **(1)**

(c) Write as a single fraction in its simplest form $\dfrac{2}{x-4} - \dfrac{1}{x+3}$. **(3)**

26. Find the exact solutions of $x + \frac{3}{x} = 7$. **(3)**

27. $-2 \leq n < 5$, n is an integer.

(a) Write down all the possible values of n. **(2)**

(b) Solve the inequality $4x + 1 > 11$. **(2)**

28. Simplify $(2n^3)^4$. **(2)**

29. (a) Factorise $2x^2 - 9x + 4$. **(2)**

Hence, or otherwise,

(b) Solve $2x^2 - 9x + 4 = (2x - 1)^2$ **(4)**

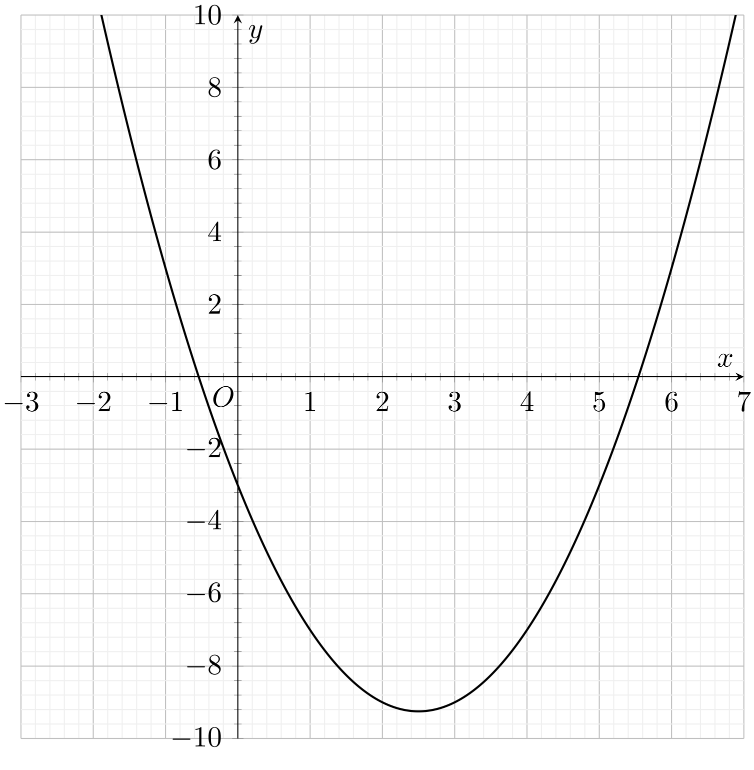
30. $y = p - 2qx^2$

$p = -10, \quad q = 3, \quad x = -5$

(a) Work out the value of y. **(2)**

(b) Rearrange $y = p - 2qx^2$ to make x the subject of the formula. **(3)**

31. The diagram shows the graph of $y = x^2 - 5x - 3$



(a) Use the graph to find estimates for the solutions of. **(3)**

i. $x^2 - 5x - 3 = 0$.

ii. $x^2 - 5x - 3 = 6$.

(b) Use the graph to find estimates for the solutions of the simultaneous equations **(3)**

$$y = x2 - 5x - 3$$

$$y = x - 4$$

32. The table shows six expressions. n is a positive integer.

$2n - 3$	$3n - 2$	$3(n + 4)$	$4n + 1$	$4(3n + 1)$	$2n + 1$

(a) From the table, write the expression whose value is **(2)**

i. always even.

ii. always a multiple of 3.

(b) From the table, write the expression which is a factor of $4n^2 - 1$. **(1)**

33. Solve the equation $\dfrac{x}{2} - \dfrac{2}{x + 1} = 1$. **(4)**

34. Make k the subject of the formula $t = \dfrac{k}{k-2}$. **(4)**

35. (a) Simplify completely $\dfrac{12xy^3}{3x^2y^3}$. **(2)**

36. (a) Expand and simplify $(2x+4y)(4x-5y)$. **(2)**

(b) Simplify fully $\dfrac{(x+10)^5}{(x+10)^4}$. **(1)**

(c) Simplify fully $\dfrac{x^2-25}{x^2+7x+10}$ **(3)**

(d) For all values of x, $x^2+6x-2=(x+p)^2+q$. Find the value of p and the value of q. **(3)**

$p =$ -------, $q =$ -------.

37. Make v the subject of the formula $t = \dfrac{v}{5} + 2$. **(2)**

$v =$ ....