LEAKY INTEGRATE AND FIRE MODELS

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1 Introduction

The LIF model captures the facts that a neuron:

- performs spatial and temporal integration of synaptic inputs
- generates a spike when the voltage reaches a certain threshold
- goes refractory during the action potential
- has a leaky membrane

The LIF model assumes that the spatial and temporal integration of inputs is linear. Also, membrane potential dynamics close to the spike threshold are much slower in LIF neurons than in real neurons. We have implemented the LIF model and its improved versions, ELIF and AELIF in the notebook. In this document, we are going to analyze the responses of the three models to different input currents.

2 Simulation

In this section, we inject a direct current into the LIF model and see its response (see figure 1).

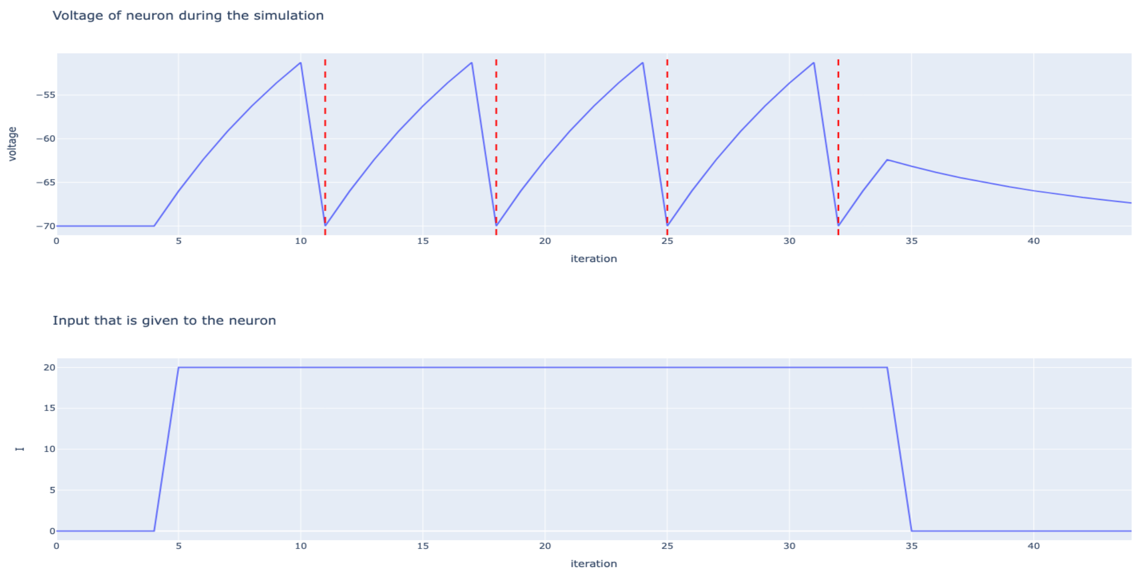
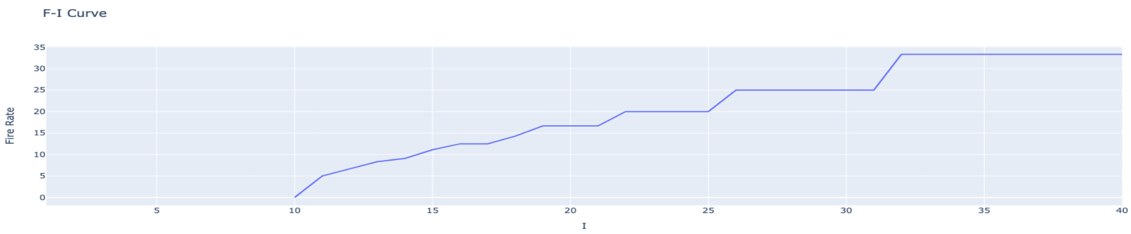


Figure 1: The LIF model and its response to the input. Red vertical lines demonstrate the spike times of the model.

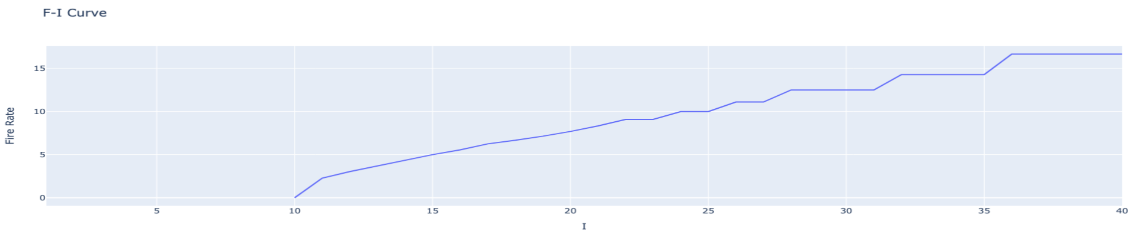
3 Analyse

3.1 Membrane time constant

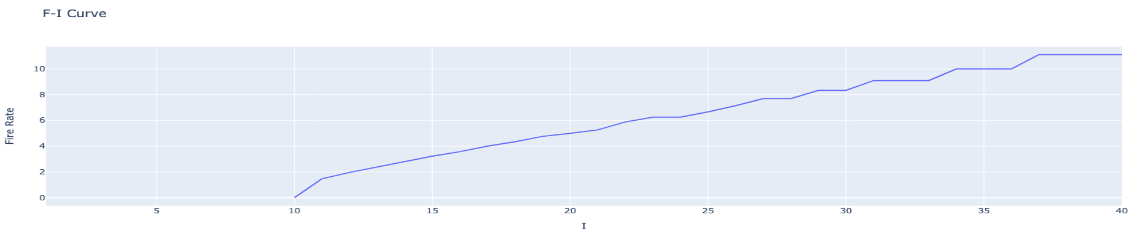
The membrane time constant, τ is the characteristic time of the decay. For a typical neuron, it is in the range of 10ms.



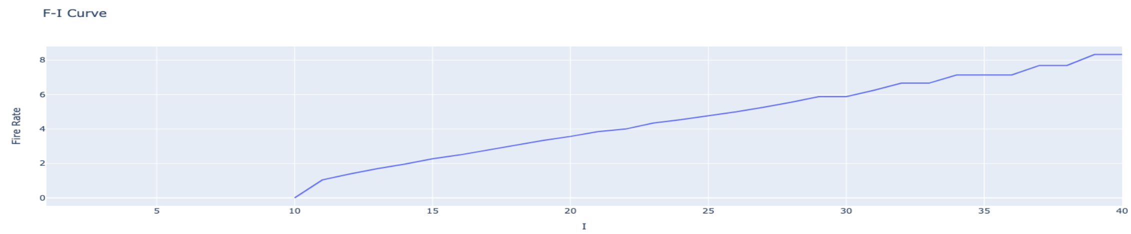
(a) $\tau = 8$



(b) $\tau = 18$



(c) $\tau = 28$



(d) $\tau = 40$

Figure 2: F-I Curve of the LIF model for increasing values of τ.

In this sub-section, we want to investigate the effect of τ in the responses of the model. To do that, first, a model with the following parameters is made:

$$v_{rest} = -70mV, v_{rest} = -70mV, R = 2.0, threshold = -50mV$$

After that, we plot the F-I Curves of the model for different values of τ (see figure 2).

It is clear that with the rise of τ, the fire rate of the model decreased significantly, and in addition, the F-I curve in the larger values of the time constant is more like a straight line than it was in lower values.

3.2 Gaussian White Noise

As you know, neurons usually receive complex, time-varying inputs. To mimic this, we will now consider the neuronal response when the model receives Gaussian white noise with mean μ and some standard deviation σ. In the previous section, because of the constant input, the ISI was constant and it depended on both input and τ. However, In this scenario, we don't inject a constant input to the model and particularly, we want to see the effect of σ in the inter-spike-intervals (ISI) of the model. (see figure 3)

We can see that, as the standard deviation of the input increases, the ISI of the model varies a lot. To measure the variation of ISI of the model we use CV_{ISI} coefficient of variation(CV) of interspike interval:

$$CV_{ISI} = \frac{std(ISI)}{mean(ISI)}$$

Now let's calculate the CV_{ISI} of the model for the three values of σ. You can find both CV_{ISI} and the histogram of the ISI values in figure 4.

We have seen the effect of σ in spike regularity. But how about μ? if we increase the mean of the GWN, CV_{ISI} increases? we keep the σ constant and increase the μ. You can check out the results in figure 5. As the input is so high all the neuron does is charge up to the spike threshold and reset. This essentially gives almost regular spiking. So the CV_{ISI} reduces.

Let's investigate the relationship between the fire rate of the model with its CV_{ISI}. To do that, we will calculate both the fire rate and CV_{ISI} of the model for different amounts of μ and σ in GWN. According to figure 6, as the fire rate increases, the CV_{ISI} falls. In addition, the fire rates are closer together when the CV_{ISI} is low.

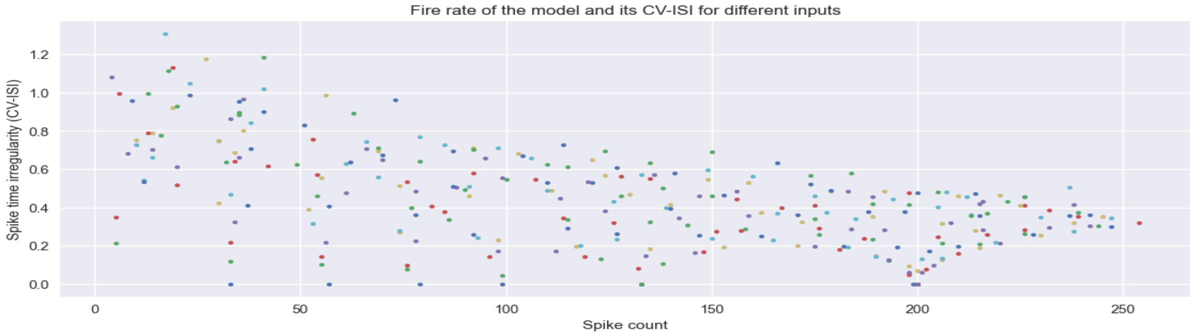


Figure 6: Fire rate of the LIF model and its CV_{ISI} for different inputs which are generated by GWN

4 Generalized Integrate and Fire Models

4.1 Exponential Integrate and Fire

The exponential integrate-and-fire model (EIF) is a biological neuron model, a simple modification of the classical leaky integrate-and-fire model describing how neurons produce action potentials. In the EIF, the threshold for spike initiation is replaced by a depolarizing non-linearity. The model was first introduced by Nicolas Fourcaud-Trocmé, David Hansel, Carl van Vreeswijk, and Nicolas Brunel.

4.1.1 Simulation

In the following figure, you can see the constant input and the response of the model.

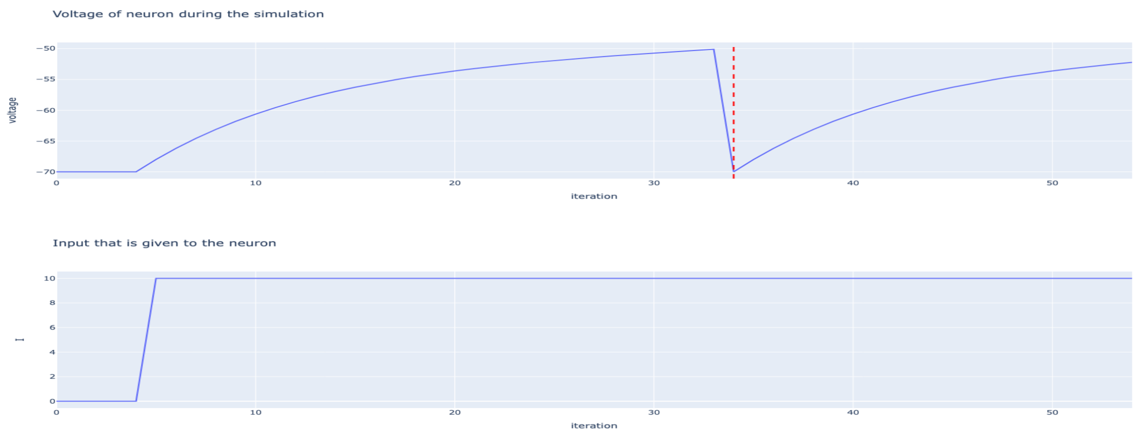
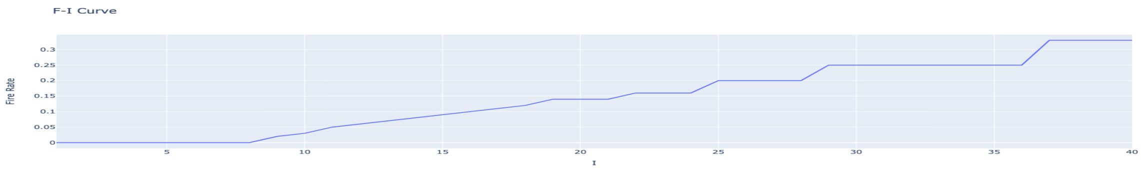


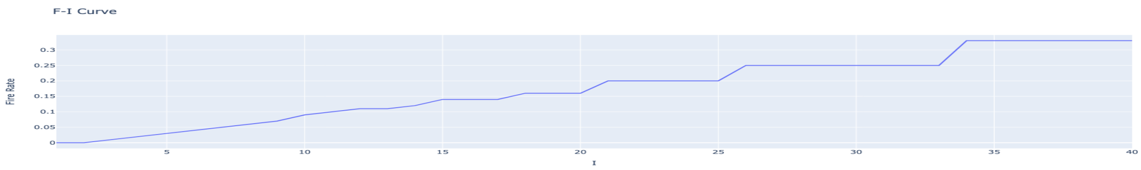
Figure 7: The ELIF model and its response to the input. Red vertical lines demonstrate the spike times of the model.

4.1.2 Analyse

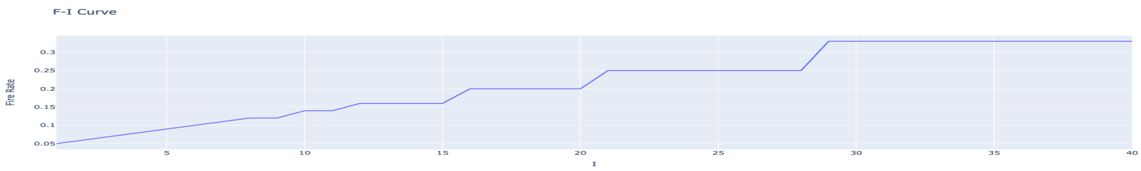
In this model, we have a new parameter Δ_T which is the sharpness parameter. We want to see the effect of this parameter in the model responses by plotting F-I curves.



(a) $\Delta_T = 3$



(b) $\Delta_T = 15$



(c) $\Delta_T = 27$

Figure 8: F-I Curve of ELIF model for different values of Δ_T

It is clear that when we increase the sharpness parameter, the fire rate of the model rises too and it reaches its peak with lower amounts of input. In the figure below we can see that the relationship between CV_{ISI} and fire rate in the ELIF model is very similar to the LIF model.

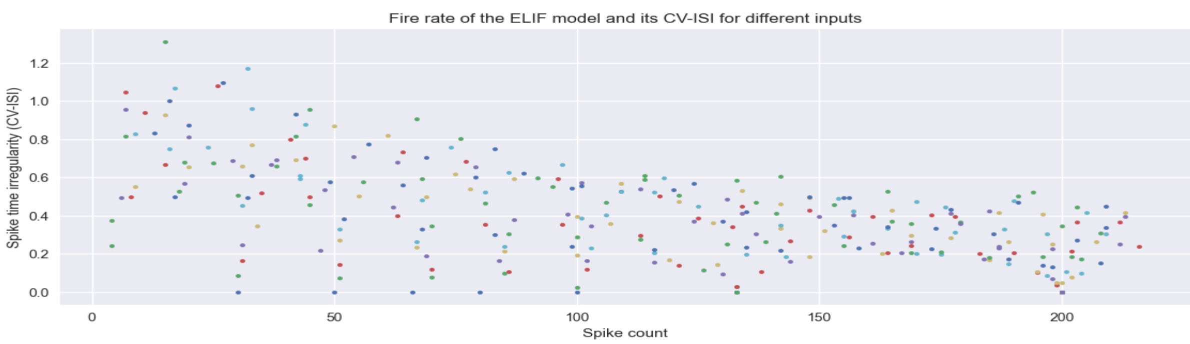


Figure 9: Fire rate of the ELIF model and its CV_{ISI} for different inputs which are generated by GWN

4.2 Adaptive Exponential Integrate and Fire

The Adaptive exponential integrate-and-fire model, also called AELIF, is a spiking neuron model with two variables (see the notebook). The first equation describes the dynamics of the membrane potential and includes an activation term with an exponential voltage dependence. Voltage is coupled to a second equation that describes adaptation. The adaptive exponential integrate-and-fire model is capable of describing known neuronal firing patterns, e.g., adapting, bursting, delayed spike initiation, initial bursting, fast-spiking, and regular spiking. Introduced by Brette and Gerstner in 2005.

4.2.1 Simulation

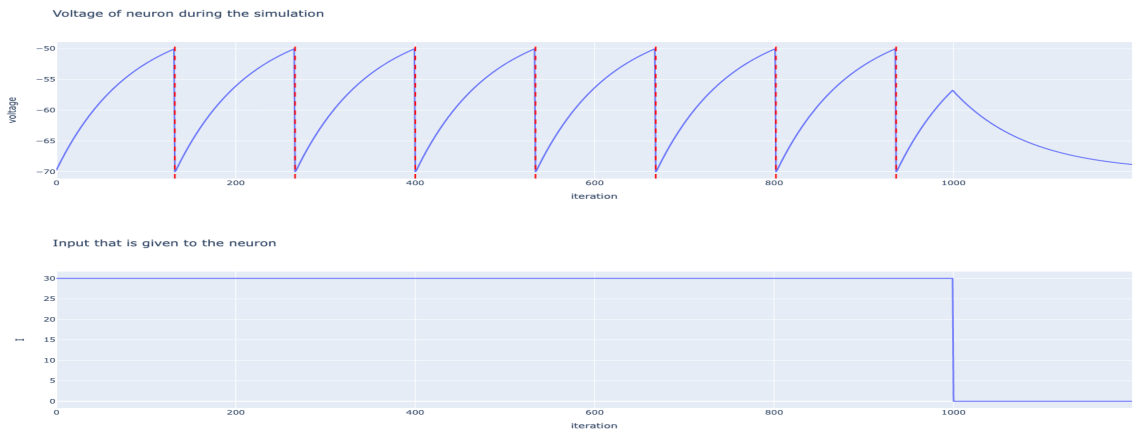


Figure 10: The AELIF model and its response to the input. Red vertical lines demonstrate the spike times of the model.

Now we will consider the effects of α on the fire rate of the model through the F-I curve (see figure 12) According to the figure, as the α increases, the minimum amount of input to activate the neuron rises as well. We can see that it increased from 35 in $\alpha = 1$ to 50 in $\alpha = 2$. In addition, in $\alpha = 5$ the neuron didn't fire at all.

Let's take a look at the relationship between the fire rate of the model and its CV_{ISI}. We plot the spike regularity and the fire rate of the model for GWN in figure 11. Unlike LIF and ELIF, there isn't any visible pattern in the figure.

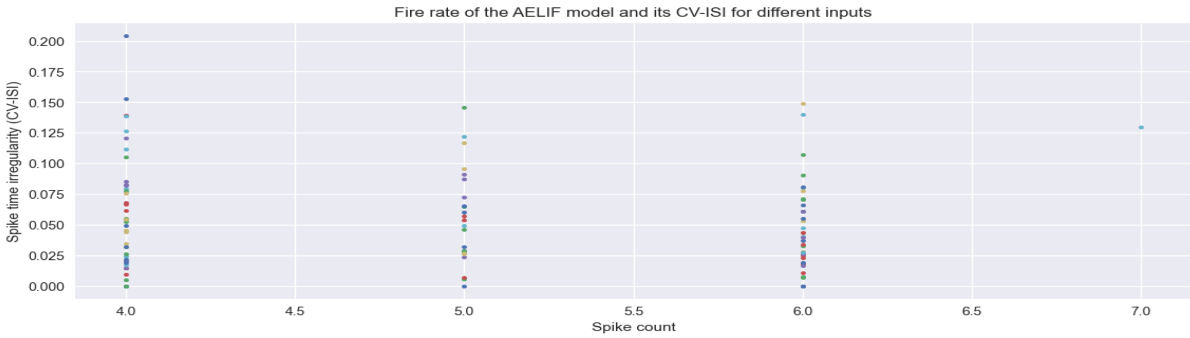
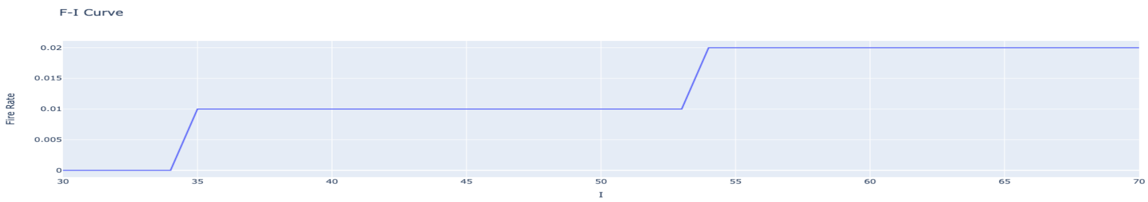
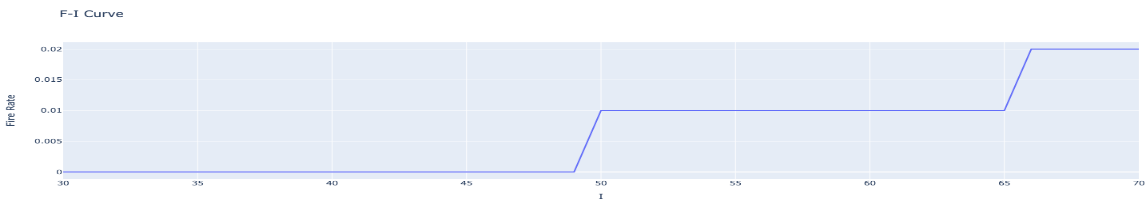


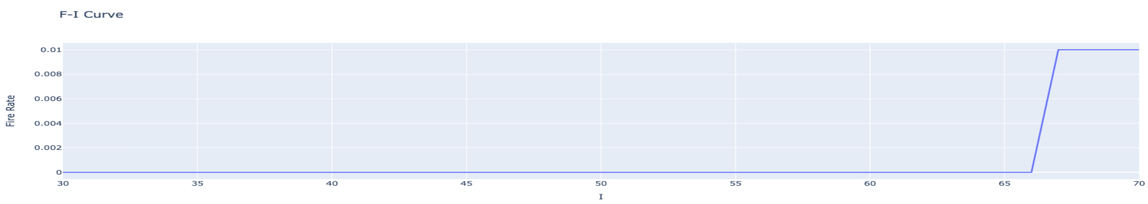
Figure 11: Fire rate of the AELIF model and its CV_{ISI} for different inputs which are generated by GWN.



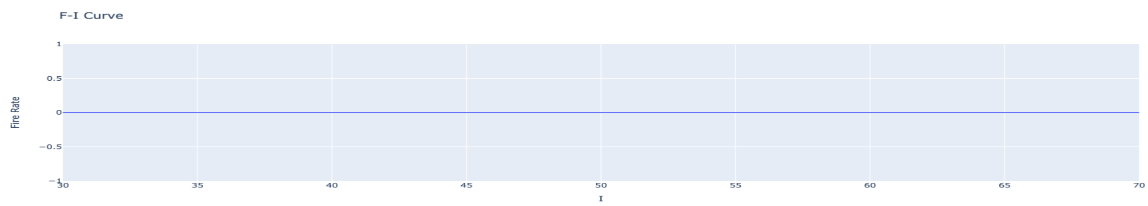
(a) $\alpha = 1$



(b) $\alpha = 2$

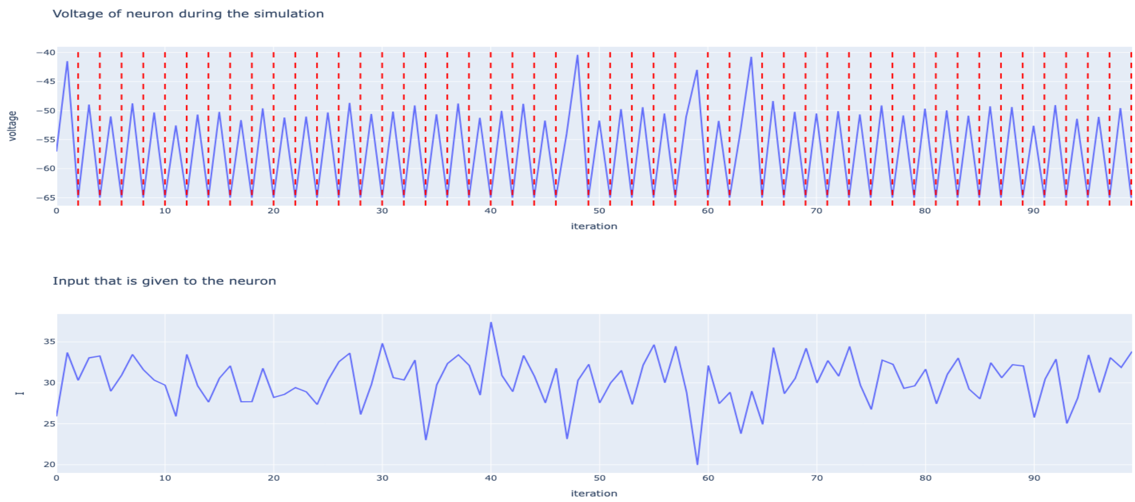


(c) $\alpha = 3$

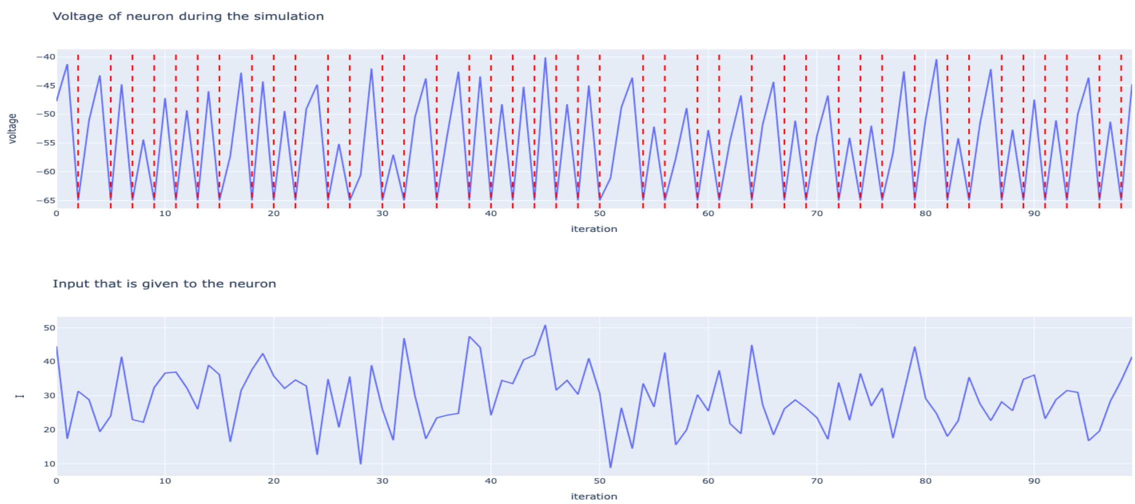


(d) $\alpha = 5$

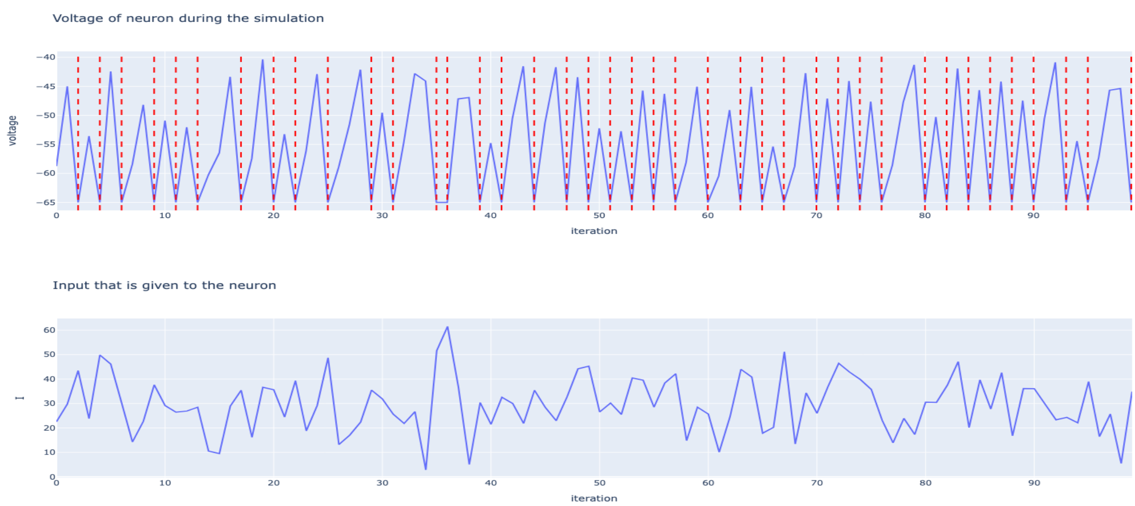
Figure 12: F-I Curve of AELIF model for different values of α



(a) $\sigma = 3$

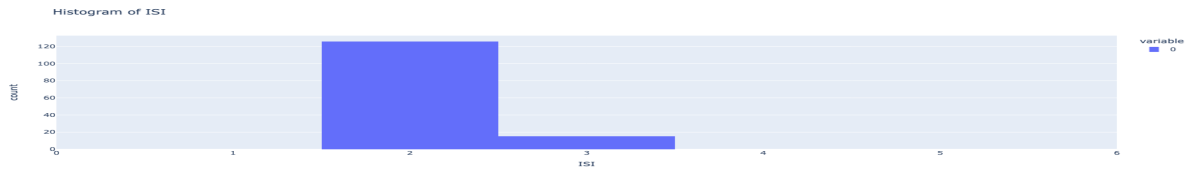


(b) $\sigma = 8$

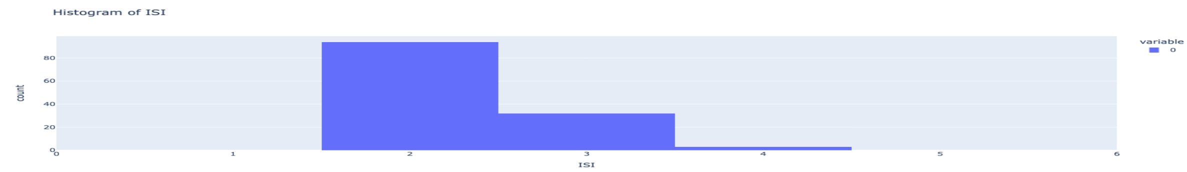


(c) $\sigma = 10$

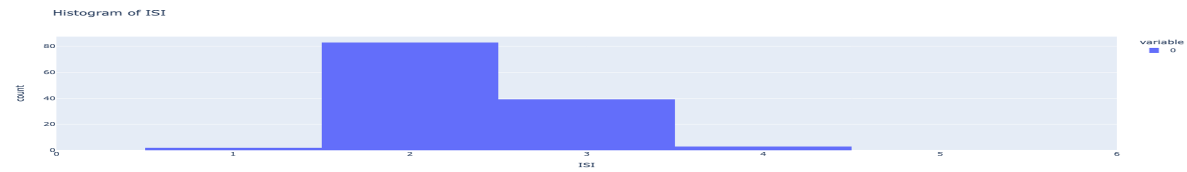
Figure 3: The responses of the model to the inputs that are generated from Gaussian distribution.



(a) $\sigma = 3, CV_{ISI} = 0.14$

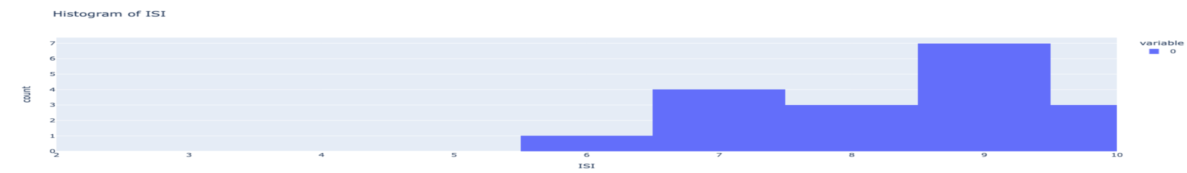


(b) $\sigma = 8, CV_{ISI} = 0.21$

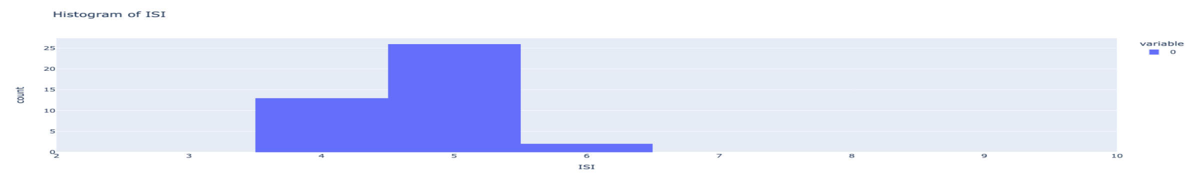


(c) $\sigma = 10, CV_{ISI} = 0.24$

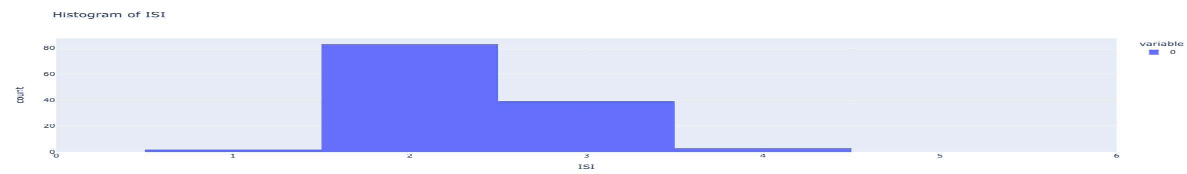
Figure 4: Histogram of ISI values for different amounts of σ with their CV_{ISI}.



(a) $\mu = 10, CV_{ISI} = 0.18$



(b) $\mu = 15, CV_{ISI} = 0.11$



(c) $\mu = 20, CV_{ISI} = 0.14$

Figure 5: Histogram of ISI values for different amounts of μ with their CV_{ISI}.