

**EEE 204: Numerical Techniques in Engineering**

Section (1):

**Term Project**

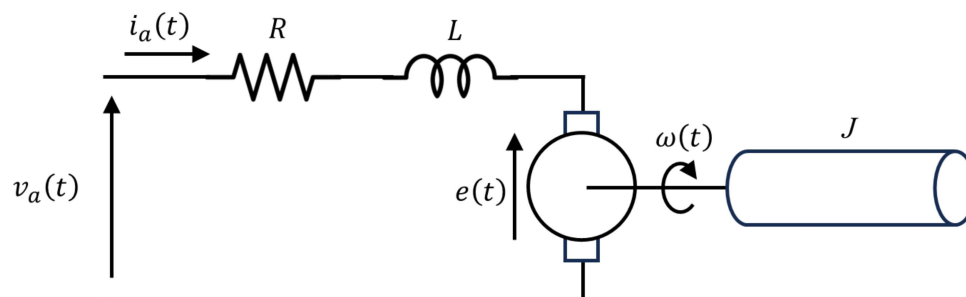
[Deadline: see checkpoint breakdown]

Instructions:

1. Prepare a report with problem description, MATLAB code and relevant results and plots.
2. There will be a viva after full report submission.

**Circuit parameter extraction to predict the circuit transient behavior.**

A DC motor converts electrical energy to mechanical energy (motion). The equivalent circuit of a motor includes the motor coil resistance ( $R$ ) and inductance ( $L$ ), and an equivalent back EMF voltage generated due to the mechanical part.



With an applied motor voltage of  $v_a(t)$ , the motor angular speed  $\omega(t)$  (in rad/s). The electrical and mechanical parts of the system are represented by the following differential equations:

$$L \frac{di_a(t)}{dt} + Ri_a(t) + k_b \omega(t) = V_a(t) \quad (1)$$

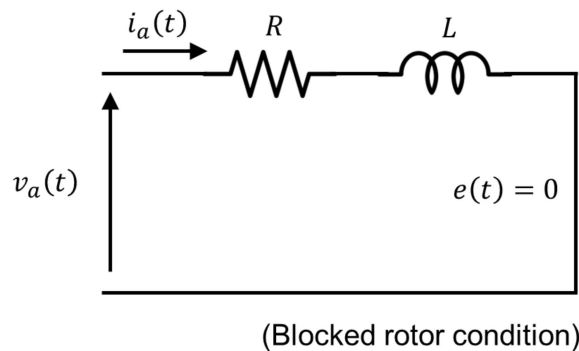
$$J \frac{d\omega(t)}{dt} + b_{mL} \omega(t) - k_t i_a(t) = 0 \quad (2)$$

Here,  $J$  and  $b$  are inertia and friction constants;  $k_t$  and  $k_b$  are torque constant and motor constant. The parameter  $b_{mL}$  includes motor friction ( $b_m$ ) and the constant related to driving the load ( $b_L$ ), i.e.,  $b_{mL} = b_m + b_L$ .

We will first extract electrical parameters of the motor from a dataset of an experiment. Once we have the parameters, we can solve the differential equations to find the motor speed response to the applied voltage.

### [Part-A]

Say, the rotor (mechanical part of the motor) is locked in place while a voltage is being applied to the motor input. In this case, the equivalent circuit does not have any mechanical part and is reduced to the following RL circuit. This experiment is called the blocked rotor test.



The frequency response of the conductance (  $G = |I_a|/|V_a|$  ) of the system was tested. The measured data of  $G$  [unit:  $1/\Omega$ ] at various frequencies  $f$  [unit: Hz] are given in the spreadsheet. The goal is to find the circuit parameters:  $R, L$  using the given data.

- (1) Rewrite the equation to convert the problem into a polynomial Least Square regression (LS curve fitting) problem. [Hint: how many unknowns are there? What could be the order of your polynomial?]
- (2) Find the best fit curve and plot with the data. You should provide the relevant code and figures.
- (3) Find the values of  $R, L$  after the best fit.
- (4) Find all the relevant error-statistics of the regression and comment on how good the fit is.

**Part-A deadline: [Q-1 to 4]: Saturday 07-Dec-2024**

## [Part-B]

The motor was initially turned off ( $v_a(t < 0) = 0V$ ). So, at time  $t = 0$ , we have  $\omega = 0$  and  $d\omega/dt = 0$ . You have already found  $R$  and  $L$  from curve fitting. Additional experiments were done to find:  $J = 0.15 \text{ kgm}^2$ ,  $k_t = k_b = 0.1$ ,  $b_m = 0.05 \text{ Ns/rad}$ , and  $b_L = 0.15 \text{ Ns/rad}$ .

The input voltage  $v_a(t)$  was varied to control the speed of the motor: the  $v_a$  vs  $t$  data is given in the spreadsheet. The goal of part-B of the project is to analyze the transient behavior of the system.

- (5) Setting up the equations to numerically solve the problem:
  - a. Find the differential equation for angular speed  $\omega$ . You can find the expression for  $i_a(t)$  from equ (2) and replace it in equ (1). What are the independent and dependent variables? Also, write down the boundary conditions.
  - b. Then derive the difference equation along with appropriate coefficient values.
  - c. Write the system equations in a matrix form including the boundary conditions.
- (6) Write a code to solve the differential equation (attach all codes at the end of the reports as appendix). Visualizations and plots:
  - a. Plot the given data  $v_a$  vs  $t$ . You will observe that the voltage is slowly increased to a high value, and then suddenly lowered at  $t = 10s$ . We can use the voltage to control the speed of the motor.
  - b. Plot the numerically solved  $\omega$  vs  $t$ .
  - c. Write down the equation for torque ( $\tau$ ) and its corresponding finite difference formula. Find and plot torque ( $\tau$ ) vs  $t$ . Interpret the values of the torque, e.g., what does the +ve and -ve values mean? (Hint: the moment of inertia  $J$  is given)
  - d. Now use your answer from 'a' and 'b', and equ (2) to find and plot  $i$  vs  $t$ . You should explain the indices used.
  - e. In the same figure: plot input power ( $P_{in} = v_a i$ ) vs  $t$ , output power ( $P_{out} = \tau \omega$ ) vs  $t$
  - f. Plot efficiency ( $\eta = P_{out} / P_{in}$ ) vs  $t$ .

**\*\* The final report should include part-A and B.**

**\*\* viva will be on the following Lab class: date-TBD**