we have 3 types of blocks each with a different volume

A) $2\times2\times4 \rightarrow V(A)=16$

$V(total) = 33\times8\times5 = 1320$

B) $2\times3\times4 \rightarrow V(B) = 24$

C) $3\times3\times3 \rightarrow V(C) = 27$

each time we put a type of block, the number of remaining empty spaces decrease by the volume of that block

properties: let's assume c_a is number of type A blocks that we're going to use, c_b blocks of type B, and c_c blocks of type C

Then if we want a complete tilling, we have this equation:

$$c_a \cdot V(A) + c_b \cdot V(B) + c_c \cdot V(C) = V(total)$$

Now we substitude the Known values: $c_a, c_b, c_c \geqslant 0$

$$16 \cdot c_a + 24 \cdot c_b + 27 \cdot c_c = 1320 \rightarrow \text{Eq. } 1$$

$\rightarrow$ remainder by 8

$16c_a + 24c_b + 27c_c \overset{8}{\equiv} 1320 \longrightarrow 27c_c \overset{8}{\equiv} 0 \longrightarrow c_c \overset{8}{\equiv} 0$

$\boxed{8 \mid c_c \longrightarrow \text{property \#1}}$

$\rightarrow$ remainder by 3

$16c_a + 24c_b + 27c_c \overset{3}{\equiv} 1320 \longrightarrow c_a \overset{3}{\equiv} 0 \longrightarrow 3 \mid c_a$

$\underline{3 \mid c_a \longrightarrow \text{property\#2}}$

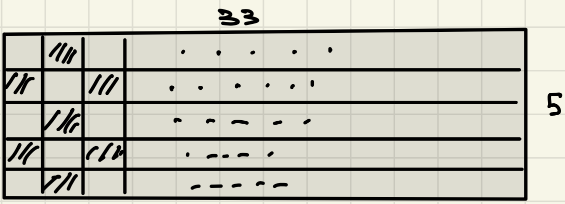
We can also find more limitations for blocks of type c:

we can have at most $\left\lfloor \frac{5}{3} \right\rfloor \times \left\lfloor \frac{8}{3} \right\rfloor \times \left\lfloor \frac{33}{3} \right\rfloor = 22$ blocks of type c. $\longrightarrow$ $c_c < 22$ property #3

Now using property #1 and #3 we can assume that:
$c_c \in \{0, 8, 16\}$

Now, we will prove that $c_c \geqslant 8$:

we can assume that our container is made of 8 slices of $33 \times 5 \times 1$. look at any of these slices as a 2D grid and color the cells like a checkerboard. Since 33×5 is odd, the number of black cells is not equal to the number of whites cells. Now, each block that we use to cover this grid uses two of its dimensions to cover a rectangle in this 2 dimensional grid.



The options for dimensions of the rectangles are:

$2 \times 2 \times 4$: $\{2\times2, 2\times4, 4\times2\}$

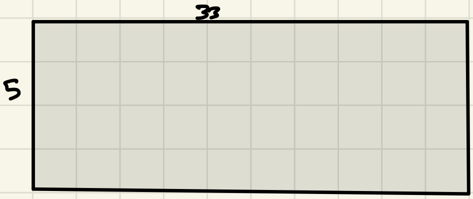
$2 \times 3 \times 4$: $\{2\times3, 2\times4, 3\times4, 3\times2, 4\times2, 4\times3\}$

$3 \times 3 \times 3$: $\{3\times3\}$

If you look at the sets, blocks of type A and B will produce even rectangles meaning the rectagles have equal number of white and black cells. So if we want to have different number of black and white cells, we have to use at least 1 block of type C $\longrightarrow$ $c_c \geqslant 1$ property #4

Now, using properties #3 and #4 we have:
$c_c \in \{8, 16\}$ property #5

we know that for all of the 33×5 slices we need odd number of 3×3 blocks. Now this means that the slices of 33×8 and 5×8s also contain these 3×3s and since these slices are even, we need even number of 3×3s for them.



at least 1

8 slices $\longrightarrow \left\lceil \frac{8}{3} \right\rceil$ $3\times3\times3 \longrightarrow 3$ different $3\times3\times3$ needed and since they also occupy a 3×3 in $33\times8\times1$ and $8\times5\times1$ slices, we need in each slice that uses a 3×3.

This means that for every 5 8×5 we need at least 2 3×3s. we can calculate a lower bound.

$$\left\lceil \frac{33}{5} \right\rceil \times 2 = 6\times2 = 12 \longrightarrow$$ the number of $3\times3\times3$s should be 16

Now we can calculate the possible combinations of (c_a, c_b, c_c) which only gives us 10 combinations. Now that we have limitations for the numbers we can use DLX to solve this problem. However, we didn't find an answer which shows there is no answer.