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ROBOTICS DESIGN CAPSTONE

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Off-center spinning mass controller for Quadcopters

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Abstract

This project is insanely hard...

1 Symbols

Here is a list of all symbols used in this paper:

$\mathbf{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	linear position vectors
$\mathbf{q} = \begin{bmatrix} q_r \\ q_i \\ q_j \\ q_k \end{bmatrix}$	angular orientation in quaternion
\mathbf{F}_T	thrust force
\mathbf{F}_G	gravitational force
\mathbf{F}_{AB}	reaction force acted from A on B
$\boldsymbol{\tau}_{AB}$	reaction torque acted from A on B
$\boldsymbol{\tau}_M$	torque generated by the motor
$\boldsymbol{\tau}_{RF}$	torque generated by the reaction force
m_A	mass of A
I_A	moment of inertial of A
S_x, C_x, T_x	$\sin(x), \cos(x), \tan(x)$ respectively

2 Mathematical Derivation

2.1 Appendix

The Quaternion-derived Rotation matrix is defined as follow,

$${}^O_B R = R(\mathbf{q}_B) = \begin{bmatrix} q_r^2 + q_i^2 - q_j^2 - q_k^2 & 2q_i q_j - 2q_r q_k & 2q_i q_k + 2q_r q_j \\ 2q_i q_j + 2q_r q_k & q_r^2 - q_i^2 + q_j^2 - q_k^2 & 2q_j q_k - 2q_r q_i \\ 2q_i q_k - 2q_r q_j & 2q_j q_k + 2q_r q_i & q_r^2 - q_i^2 - q_j^2 + q_k^2 \end{bmatrix}$$

2.2 Quadcopter Body Dynamics

Forces and Torques:

$$\begin{aligned}
 {}^B\mathbf{F}_T &= \begin{bmatrix} 0 \\ 0 \\ F_{TB} \end{bmatrix} \\
 {}^O\mathbf{F}_{GB} &= \begin{bmatrix} 0 \\ 0 \\ -m_bg \end{bmatrix} \\
 {}^O\mathbf{F}_{CB} &= \begin{bmatrix} F_{CBx} \\ F_{CB y} \\ F_{CBz} \end{bmatrix} \\
 {}^B\boldsymbol{\tau}_{CB} &= \begin{bmatrix} \tau_{CBx} \\ \tau_{CB y} \\ -\tau_M \end{bmatrix}
 \end{aligned}$$

Net Force and Torque

$${}^O\mathbf{F}_{B,net} = {}^O\mathbf{F}_{GB} + {}^O\mathbf{F}_T + {}^O\mathbf{F}_{CB} = m_B {}^O\mathbf{a}_B \quad (1)$$

$${}^O\boldsymbol{\tau}_{B,net} = R(\mathbf{q}_B) {}^B\boldsymbol{\tau}_{CB} = {}^OI_B {}^O\boldsymbol{\alpha}_B \quad (2)$$

2.3 Controller Dynamics

Forces and Torques:

$$\begin{aligned}
{}^O \mathbf{F}_{BC} &= \begin{bmatrix} F_{BCx} \\ F_{BCy} \\ F_{BCz} \end{bmatrix} \\
{}^O \mathbf{F}_{GC} &= \begin{bmatrix} 0 \\ 0 \\ -m_c g \end{bmatrix} \\
{}^C \boldsymbol{\tau}_{BC} &= \begin{bmatrix} \tau_{BCx} \\ \tau_{BCy} \\ \tau_M \end{bmatrix} \\
{}^O \mathbf{r}_{CB} &= R(\mathbf{q}_C) \begin{bmatrix} -L_{Mx} \\ 0 \\ -L_{Mz} \end{bmatrix} \\
{}^O \boldsymbol{\tau}_{RF} &= {}^O \mathbf{r}_{CB} \times {}^O \mathbf{F}_{BC}
\end{aligned}$$

Net Force and Net Torque:

$${}^O \mathbf{F}_{net,C} = {}^O \mathbf{F}_{BC} + {}^O \mathbf{F}_{GC} = m_C {}^O \mathbf{a}_C \quad (3)$$

$${}^O \boldsymbol{\tau}_{net,C} = R(\mathbf{q}_C) {}^C \boldsymbol{\tau}_{BC} + {}^O \boldsymbol{\tau}_{RF} = {}^O I_c {}^O \boldsymbol{\alpha}_C \quad (4)$$

2.4 Constraints and Manipulation

In the derivation below, assume everything is in the inertial frame unless explicitly stated.

The two bodies are constrained (attached together), there are some relationship between the states and the forces between the body and the controller,

Let $\mathbf{p}_{sys} = \mathbf{p}_B$ and $\mathbf{q}_{sys} = \mathbf{q}_B$,

$$\begin{bmatrix} \mathbf{p}_C \\ \mathbf{q}_C \end{bmatrix} = \begin{bmatrix} \mathbf{p}_B + \mathbf{r}_{BC} \\ \mathbf{q}_\theta \mathbf{q}_B \end{bmatrix} = \begin{bmatrix} \mathbf{p}_{sys} + \mathbf{r}_{BC} \\ \mathbf{q}_\theta \mathbf{q}_{sys} \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} \dot{\mathbf{p}}_C \\ \dot{\mathbf{q}}_C \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{p}}_{sys} + \dot{R}(\mathbf{q}_{sys})^B \mathbf{r}_{BC} \\ \mathbf{q}_\theta \dot{\mathbf{q}}_{sys} + \dot{\mathbf{q}}_\theta \mathbf{q}_{sys} \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} \ddot{\mathbf{p}}_C \\ \ddot{\mathbf{q}}_C \end{bmatrix} = \begin{bmatrix} \ddot{\mathbf{p}}_{sys} + \ddot{R}(\mathbf{q}_{sys})^B \mathbf{r}_{BC} \\ \mathbf{q}_\theta \ddot{\mathbf{q}}_{sys} + 2[\dot{\mathbf{q}}_\theta \dot{\mathbf{q}}_{sys}] + \ddot{\mathbf{q}}_\theta \mathbf{q}_{sys} \end{bmatrix} \quad (7)$$

Newton's Third Law

$${}^O\mathbf{F}_{BC} = -{}^O\mathbf{F}_{CB} \quad (8)$$

$${}^O\boldsymbol{\tau}_{BC} = -{}^O\boldsymbol{\tau}_{CB} \quad (9)$$

To limit our degree of freedom in the system, we have set a constraint for our quaternions, namely unit quaternion:

$$q_r^2 + q_i^2 + q_j^2 + q_k^2 = 1 \quad (10)$$

$$q_r\dot{q}_r + q_i\dot{q}_i + q_j\dot{q}_j + q_k\dot{q}_k = 0 \quad (11)$$

$$q_r\ddot{q}_r + q_i\ddot{q}_i + q_j\ddot{q}_j + q_k\ddot{q}_k + \dot{q}_r^2 + \dot{q}_i^2 + \dot{q}_j^2 + \dot{q}_k^2 = 0 \quad (12)$$

Last but not least, in the derivation below we use \mathbf{q}_θ directly for ease of typesetting, however, \mathbf{q}_θ is not our state variable but θ , their relationship is defined below

$$\mathbf{q}_\theta = \cos\left(\frac{\theta}{2}\right) + \sin\left(\frac{\theta}{2}\right)R(\mathbf{q}_{sys})^B\hat{\mathbf{z}}_B \quad (13)$$

$$\dot{\mathbf{q}}_\theta = -\frac{1}{2}\sin\left(\frac{\theta}{2}\right)\dot{\theta} + \frac{1}{2}\cos\left(\frac{\theta}{2}\right)\dot{\theta}R(\mathbf{q}_{sys})^B\hat{\mathbf{z}}_B + \sin\left(\frac{\theta}{2}\right)R(\dot{\mathbf{q}}_{sys})^B\hat{\mathbf{z}}_B \quad (14)$$

where ${}^B\hat{\mathbf{z}}_B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

2.4.1 Combining the Force equations

From (1),

$${}^O\mathbf{F}_{CB} = m_B {}^O\mathbf{a}_B - {}^O\mathbf{F}_{GB} - {}^O\mathbf{F}_T$$

From (3),

$${}^O\mathbf{F}_{BC} = m_C {}^O\mathbf{a}_C - {}^O\mathbf{F}_{GC}$$

Using (6),

$$m_B {}^O\mathbf{a}_B + m_C {}^O\mathbf{a}_C = {}^O\mathbf{F}_{GC} + {}^O\mathbf{F}_{GB} + {}^O\mathbf{F}_T$$

Simplifying the above expression, we get

$$(m_b + m_c)\ddot{\mathbf{p}}_{sys} + m_c\ddot{R}(\mathbf{q}_{sys})^B\mathbf{r}_{BC} = \mathbf{F}_{GC} + \mathbf{F}_{GB} + \mathbf{F}_T \quad (15)$$

2.4.2 Combining the Torque equations

From (2),

$${}^O\tau_{CB} = {}^OI_B {}^O\alpha_B$$

From (4),

$${}^O\tau_{BC} = {}^OI_c {}^O\alpha_C - {}^O\tau_{RF}$$

Using (7),

$${}^OI_B {}^O\alpha_B + {}^OI_c {}^O\alpha_C = {}^O\tau_{RF}$$

Assuming all the vectors are represented in the inertial O frame, using the quaternion representation for angular acceleration,

$$2I_B [\ddot{\mathbf{q}}_B \mathbf{q}_B^* - (\dot{\mathbf{q}}_B \mathbf{q}_B^*)^2] + 2I_c [\ddot{\mathbf{q}}_C \mathbf{q}_C^* - (\dot{\mathbf{q}}_C \mathbf{q}_C^*)^2] = {}^O\tau_{RF} \quad (16)$$

Substituting (5)-(7) in the above expression and isolating second derivative on the left, we have

$$2I_B [\ddot{\mathbf{q}}_{sys} \mathbf{q}_{sys}^*] + 2I_C [\mathbf{q}_\theta \ddot{\mathbf{q}}_{sys} (\mathbf{q}_\theta \mathbf{q}_{sys})^*] + 2I_C [\ddot{\mathbf{q}}_\theta \mathbf{q}_{sys}] (\mathbf{q}_\theta \mathbf{q}_{sys})^* - \mathbf{r}_{CB} \times \mathbf{F}_{BC} = \zeta \quad (17)$$

where

$$\zeta = 2I_B (\dot{\mathbf{q}}_{sys} \mathbf{q}_{sys}^*)^2 + 2I_C [(\mathbf{q}_\theta \dot{\mathbf{q}}_{sys} + \dot{\mathbf{q}}_\theta \mathbf{q}_{sys}) (\mathbf{q}_\theta \mathbf{q}_{sys})^*]^2 - 4I_C (\dot{\mathbf{q}}_\theta \dot{\mathbf{q}}_{sys}) (\mathbf{q}_\theta \mathbf{q}_{sys})^*$$

Note that we put τ_{RF} on the left hand side, this is because we can express \mathbf{F}_{BC} in terms of $\ddot{\mathbf{p}}_{sys}$ from (1), a second derivative of positional state

$$\mathbf{F}_{BC} = m_B \ddot{\mathbf{p}}_{sys} - \mathbf{F}_{GB} - \mathbf{F}_T$$