The University of California, Los Angeles

Robotics Design Capstone

EE 183DB

Off-center spinning mass controller for Quadcopters

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Abstract

Your abstract.

1 Symbols

Here is a list of all symbols used in this paper:

$\boldsymbol{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	linear position vectors
$\boldsymbol{q} = \begin{bmatrix} q_r \\ q_i \\ q_j \\ q_k \end{bmatrix}$	angular orientation vectors in quaternion
$\boldsymbol{F_T}$	thrust force
$\boldsymbol{F_G}$	gravitational force
$\boldsymbol{F_{AB}}$	reaction force acted from A on B
$\boldsymbol{\tau_{AB}}$	reaction torque acted from A on B
$\boldsymbol{\tau_M}$	torque generated by the motor
$\boldsymbol{\tau_{RF}}$	torque generated by the reaction force
m_b	mass of the body
m_c	mass of the controller
S_x, C_x, T_x	$\sin(x), \cos(x), \tan(x)$ respectively

2 Mathematical Derivation

2.1 Assumptions

- Assume unit quaternions: $||\boldsymbol{q}|| = 1$

2.2 Quadcopter Body Dynamics

Forces and Torques:

$$
{}^{B}\boldsymbol{F_T} = \begin{bmatrix} 0 \\ 0 \\ F_{TB} \end{bmatrix}
$$

$$
{}^{O}\boldsymbol{F_{GB}} = \begin{bmatrix} 0 \\ 0 \\ -m_b g \end{bmatrix}
$$

$$
{}^{O}\boldsymbol{F_{CB}} = \begin{bmatrix} F_{CBx} \\ F_{CBy} \\ F_{CBz} \end{bmatrix}
$$

$$
{}^{B}\boldsymbol{\tau_{CB}} = \begin{bmatrix} \tau_{CBx} \\ \tau_{CBy} \\ -\tau_M \end{bmatrix}
$$

The Quaternion-derived Rotation matrix is defined as follow,

$$
{}^{O}_{B}R = R(\boldsymbol{q_B}) = \begin{bmatrix} q_r^2 + q_i^2 - q_j^2 - q_k^2 & 2q_iq_j - 2q_rq_k & 2q_iq_k + 2q_rq_j \\ 2q_iq_j + 2q_rq_k & q_r^2 - q_i^2 + q_j^2 - q_k^2 & 2q_jq_k - 2q_rq_i \\ 2q_iq_k - 2q_rq_j & 2q_jq_k + 2q_rq_i & q_r^2 - q_i^2 - q_j^2 + q_k^2 \end{bmatrix}
$$

Net Force and Torque

$$
{}^{O}\boldsymbol{F_{B,net}} = {}^{O}\boldsymbol{F_{GB}} + {}^{O}\boldsymbol{F_T} + {}^{O}\boldsymbol{F_{CB}} = m_B \, {}^{O}\boldsymbol{a_B} \tag{1}
$$

$$
{}^{O}\boldsymbol{\tau_{B,net}} = R(\boldsymbol{q_B}) \, {}^{B}\boldsymbol{\tau_{CB}} = {}^{O}I_B \, {}^{O}\boldsymbol{\alpha_B} \tag{2}
$$

2.3 Controller Dynamics

Forces and Torques:

$$
\begin{aligned}
{}^{O}\boldsymbol{F_{BC}} &= \begin{bmatrix} F_{BCx} \\ F_{BCy} \\ F_{BCz} \end{bmatrix} \\
{}^{O}\boldsymbol{F_{GC}} &= \begin{bmatrix} 0 \\ 0 \\ -m_c g \end{bmatrix} \\
{}^{C}\boldsymbol{\tau_{BC}} &= \begin{bmatrix} \tau_{BCx} \\ \tau_{BCy} \\ \tau_{M} \end{bmatrix} \\
{}^{O}\boldsymbol{r_{RF}} &= R(\boldsymbol{q_C}) \begin{bmatrix} -L_{Mx} \\ 0 \\ -L_{Mz} \end{bmatrix} \\
{}^{O}\boldsymbol{\tau_{RF}} &= {}^{O}\boldsymbol{r_{RF}} \times {}^{O}\boldsymbol{F_{BC}}
\end{aligned}
$$

Net Force and Net Torque:

$$
{}^{O}\boldsymbol{F_{net,C}} = {}^{O}\boldsymbol{F_{BC}} + {}^{O}\boldsymbol{F_{GC}} = m_C\, {}^{O}\boldsymbol{a_C} \tag{3}
$$

$$
{}^{O}\boldsymbol{\tau_{net,C}} = R(\boldsymbol{q_C})\, {}^{C}\boldsymbol{\tau_{BC}} + {}^{O}\boldsymbol{\tau_{RF}} = {}^{O}I_c\, {}^{O}\boldsymbol{\alpha_C} \tag{4}
$$

2.4 Constraints and Manipulation

The two bodies are contrainted (attached together), there are some relationship between the states and the forces between the body and the controller,

Let $\boldsymbol{p_{sys}} = \boldsymbol{p_B}$ and $\boldsymbol{q_{sys}} = \boldsymbol{q_B}$,

$$
\begin{bmatrix} \boldsymbol{p_C} \\ \boldsymbol{q_C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{p_B} + \boldsymbol{p_\theta} \\ \boldsymbol{q_\theta q_B} \end{bmatrix} = \begin{bmatrix} \boldsymbol{p_{sys}} + \boldsymbol{p_\theta} \\ \boldsymbol{q_\theta q_{sys}} \end{bmatrix} \tag{5}
$$

Newton's Third Law

$$
{}^{O}\boldsymbol{F_{BC}} = -\,{}^{O}\boldsymbol{F_{CB}} \tag{6}
$$

$$
{}^{O}\boldsymbol{\tau_{BC}} = -\,{}^{O}\boldsymbol{\tau_{CB}} \tag{7}
$$

Combining the above equations (1) - (7), we would like to do some algebraic manipulation to get rid of the unwanted parameters in our state evolution equations.

2.4.1 Combining the Force equations

From (1),

$$^{O}\boldsymbol{F_{CB}} = m_B\,^{O}\boldsymbol{a_B} - {}^{O}\boldsymbol{F_{GB}} - {}^{O}\boldsymbol{F_T}$$

From (3),

$$^{O}\boldsymbol{F_{BC}} = m_C\,^{O}\boldsymbol{a_C} - {}^{O}\boldsymbol{F_{GC}}$$

Using (6),

$$m_B\,^{O}\boldsymbol{a_B} + m_C\,^{O}\boldsymbol{a_C} = {}^{O}\boldsymbol{F_{GC}} + {}^{O}\boldsymbol{F_{GB}} + {}^{O}\boldsymbol{F_T}$$

Since $\boldsymbol{p_\theta}$ is a constant vector, $\boldsymbol{a_C} = \boldsymbol{a_B} = \boldsymbol{\ddot{p}_{sys}}$, the above equation becomes

$$\boldsymbol{\ddot{p}_{sys}} = \frac{1}{m_B + m_C}({}^{O}\boldsymbol{F_{GC}} + {}^{O}\boldsymbol{F_{GB}} + {}^{O}\boldsymbol{F_T}) \tag{8}$$

2.4.2 Combining the Torqe equations

From (2),

$$^{O}\boldsymbol{\tau_{CB}} = {}^{O}I_B\,^{O}\boldsymbol{\alpha_B}$$

From (4),

$$^{O}\boldsymbol{\tau_{BC}} = {}^{O}I_c\,^{O}\boldsymbol{\alpha_C} - {}^{O}\boldsymbol{\tau_{RF}}$$

Using (7),

$$^{O}I_B\,^{O}\boldsymbol{\alpha_B} + {}^{O}I_c\,^{O}\boldsymbol{\alpha_C} = {}^{O}\boldsymbol{\tau_{RF}}$$

Assuming all the vectors are represented in the inertial O frame, using the quaternion representation for angular acceleration,

$$2I_B\left[\boldsymbol{\ddot{q}_B q_B^*} - (\boldsymbol{\dot{q}_B q_B^*})^2\right] + 2I_c\left[\boldsymbol{\ddot{q}_C q_C^*} - (\boldsymbol{\dot{q}_C q_C^*})^2\right] = {}^{O}\boldsymbol{\tau_{RF}} \tag{9}$$

Since $\boldsymbol{q_B} = \boldsymbol{q_{sys}}$ and $\boldsymbol{q_C} = \boldsymbol{q_\theta q_{sys}}$, by Chain Rule,

$$\begin{aligned} \boldsymbol{\dot{q}_C} &= \boldsymbol{q_\theta \omega_{sys}} + \boldsymbol{\omega_\theta q_{sys}} \\ \boldsymbol{\ddot{q}_C} &= \boldsymbol{q_\theta \dot{\omega}_{sys}} + 2(\boldsymbol{\omega_\theta \omega_{sys}}) + \boldsymbol{\alpha_\theta q_{sys}} \end{aligned}$$

where $\boldsymbol{\omega_{sys}} = \dot{\boldsymbol{q}}_{\boldsymbol{sys}}, \quad \boldsymbol{\omega_\theta} = \dot{\boldsymbol{q}}_{\boldsymbol{\theta}}, \quad \boldsymbol{\alpha_\theta} = \ddot{\boldsymbol{q}}_{\boldsymbol{\theta}}$. Combining the above, (9) becomes,

$$2I_B[\dot{\boldsymbol{\omega}}_{\boldsymbol{sys}}\boldsymbol{q}^*_{\boldsymbol{sys}}] + 2I_C[\boldsymbol{q_\theta}\dot{\boldsymbol{\omega}}_{\boldsymbol{sys}}(\boldsymbol{q_\theta q_{sys}})^*]$$
$$= {}^O\boldsymbol{\tau_{RF}} + 2I_B(\boldsymbol{\omega_{sys}}\boldsymbol{q}^*_{\boldsymbol{sys}})^2 + 2I_C[(\boldsymbol{q_\theta \omega_{sys}} + \boldsymbol{\omega_\theta q_{sys}})(\boldsymbol{q_\theta q_{sys}})^*]^2 - 2I_C[2(\boldsymbol{\omega_\theta \omega_C}) + \boldsymbol{\alpha_\theta q_C}](\boldsymbol{q_\theta q_{sys}})^*$$

Let the above R.H.S. sum be ζ, we have,

$$\dot{\boldsymbol{\omega}}_{\boldsymbol{sys}} = 2(I_B + I_C R(\boldsymbol{q_\theta}))^{-1}\zeta\boldsymbol{q_{sys}} \tag{10}$$

2.4.3 State Evolution Equation

From (8) and (10), we have the follow state evolution equation,

$$\boldsymbol{s_{sys}} = \begin{bmatrix} \dot{\boldsymbol{p}}_{\boldsymbol{sys}} \\ \dot{\boldsymbol{q}}_{\boldsymbol{sys}} \\ \dot{\boldsymbol{q}}_{\boldsymbol{theta}} \\ \dot{\boldsymbol{\omega}}_{\boldsymbol{theta}} \\ \dot{\boldsymbol{v}}_{\boldsymbol{sys}} \\ \dot{\boldsymbol{\omega}}_{\boldsymbol{sys}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{v_{sys}} \\ \boldsymbol{\omega_{sys}} \\ \boldsymbol{\omega_{theta}} \\ \boldsymbol{\alpha_{thete}} \\ \frac{1}{m_B + m_C}({}^O\boldsymbol{F_{GC}} + {}^O\boldsymbol{F_{GB}} + {}^O\boldsymbol{F_T}) \\ 2(I_B + I_C R(\boldsymbol{q_\theta}))^{-1}\zeta\boldsymbol{q_{sys}} \end{bmatrix}$$