The University of California, Los Angeles

Robotics Design Capstone

EE 183DB

Off-center spinning mass controller for Quadcopters

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Abstract

Your abstract.

1 Symbols

Here is a list of all symbols used in this paper:

$\boldsymbol{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	linear position vectors
$\boldsymbol{q} = \begin{bmatrix} q_r \\ q_i \\ q_j \\ q_k \end{bmatrix}$	angular orientation in quaternion
$\boldsymbol{F_T}$	thrust force
$\boldsymbol{F_G}$	gravitational force
$\boldsymbol{F_{AB}}$	reaction force acted from A on B
$\boldsymbol{\tau_{AB}}$	reaction torque acted from A on B
$\boldsymbol{\tau_M}$	torque generated by the motor
$\boldsymbol{\tau_{RF}}$	torque generated by the reaction force
m_A	mass of the A
I_A	moment of inertial of A
S_x, C_x, T_x	$\sin(x), \cos(x), \tan(x)$ respectively

2 Mathematical Derivation

2.1 Assumptions

- Assume unit quaternions: $||\boldsymbol{q}|| = 1$

2.2 Quadcopter Body Dynamics

Assuming all the vectors are represented in the inertial O frame, using the quaternion representation for angular acceleration,

$$2I_B\left[\ddot{\boldsymbol{q}}_{\boldsymbol{B}}\boldsymbol{q}_{\boldsymbol{B}}^* - (\dot{\boldsymbol{q}}_{\boldsymbol{B}}\boldsymbol{q}_{\boldsymbol{B}}^*)^2\right] + 2I_c\left[\ddot{\boldsymbol{q}}_{\boldsymbol{C}}\boldsymbol{q}_{\boldsymbol{C}}^* - (\dot{\boldsymbol{q}}_{\boldsymbol{C}}\boldsymbol{q}_{\boldsymbol{C}}^*)^2\right] = {}^O\boldsymbol{\tau}_{\boldsymbol{RF}} \tag{1}$$

Since $\boldsymbol{q}_{\boldsymbol{B}} = \boldsymbol{q}_{\boldsymbol{sys}}$ and $\boldsymbol{q}_{\boldsymbol{C}} = \boldsymbol{q}_{\boldsymbol{\theta}}\boldsymbol{q}_{\boldsymbol{sys}}$, by Chain Rule,

$$\dot{\boldsymbol{q}}_{\boldsymbol{C}} = \boldsymbol{q}_{\boldsymbol{\theta}}\boldsymbol{\omega}_{\boldsymbol{sys}} + \boldsymbol{\omega}_{\boldsymbol{\theta}}\boldsymbol{q}_{\boldsymbol{sys}}$$
$$\ddot{\boldsymbol{q}}_{\boldsymbol{C}} = \boldsymbol{q}_{\boldsymbol{\theta}}\dot{\boldsymbol{\omega}}_{\boldsymbol{sys}} + 2(\boldsymbol{\omega}_{\boldsymbol{\theta}}\boldsymbol{\omega}_{\boldsymbol{sys}}) + \boldsymbol{\alpha}_{\boldsymbol{\theta}}\boldsymbol{q}_{\boldsymbol{sys}}$$

where $\boldsymbol{\omega}_{\boldsymbol{sys}} = \dot{\boldsymbol{q}}_{\boldsymbol{sys}}$, $\boldsymbol{\omega}_{\boldsymbol{\theta}} = \dot{\boldsymbol{q}}_{\boldsymbol{\theta}}$, $\boldsymbol{\alpha}_{\boldsymbol{\theta}} = \ddot{\boldsymbol{q}}_{\boldsymbol{\theta}}$. Combining the above, (9) becomes,

$$2I_B[\dot{\boldsymbol{\omega}}_{\boldsymbol{sys}}\boldsymbol{q}_{\boldsymbol{sys}}^*] + 2I_C[\boldsymbol{q}_{\boldsymbol{\theta}}\dot{\boldsymbol{\omega}}_{\boldsymbol{sys}}(\boldsymbol{q}_{\boldsymbol{\theta}}\boldsymbol{q}_{\boldsymbol{sys}})^*]$$
$$= {}^O\boldsymbol{\tau}_{\boldsymbol{RF}} + 2I_B(\boldsymbol{\omega}_{\boldsymbol{sys}}\boldsymbol{q}_{\boldsymbol{sys}}^*)^2 + 2I_C[(\boldsymbol{q}_{\boldsymbol{\theta}}\boldsymbol{\omega}_{\boldsymbol{sys}} + \boldsymbol{\omega}_{\boldsymbol{\theta}}\boldsymbol{q}_{\boldsymbol{sys}})(\boldsymbol{q}_{\boldsymbol{\theta}}\boldsymbol{q}_{\boldsymbol{sys}})^*]^2 - 2I_C[2(\boldsymbol{\omega}_{\boldsymbol{\theta}}\boldsymbol{\omega}_{\boldsymbol{C}}) + \boldsymbol{\alpha}_{\boldsymbol{\theta}}\boldsymbol{q}_{\boldsymbol{C}}](\boldsymbol{q}_{\boldsymbol{\theta}}\boldsymbol{q}_{\boldsymbol{sys}})^*$$