UCLA

Robotics Design Capstone

183DB

Off center spinning mass controller for Quad Copters

Author:
Lin Li
Angel
Wilson
Amirali Omidfar

Professor:
Ankur Metha

April 17, 2018

Abstract

Your abstract.

1 Symbols

Here is a list of all symbols used in this paper:

$\boldsymbol{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	linear position vectors
$\boldsymbol{q} = \begin{bmatrix} q_r \\ q_i \\ q_j \\ q_k \end{bmatrix}$	angular orientation vectors in quaternion
$\boldsymbol{F_T}$	thrust force
$\boldsymbol{F_G}$	gravitational force
$\boldsymbol{F_S}$	reaction force acted on the surface
$\boldsymbol{\tau_S}$	reaction torque acted on the surface
$\boldsymbol{\tau_M}$	torque introduced by the off-center mass
m_b	mass of the body
m_c	mass of the controller
S_x, C_x, T_x	$\sin(x), \cos(x), \tan(x)$ respectively

2 Mathematical Derivation

2.1 Assumptions

- Assume unit quaternions: $||\boldsymbol{q}|| = 1$

2.2 Quadcopter Body Dynamics

The state of body is defined as follow:

$$\boldsymbol{s_B} = \begin{bmatrix} \boldsymbol{p_B} \\ \boldsymbol{q_B} \end{bmatrix}$$

Forces and Torques:

$$ {}^{B}\boldsymbol{F_{TB}} = \begin{bmatrix} 0 \\ 0 \\ F_{TB} \end{bmatrix} $$

$$ {}^{O}\boldsymbol{F_{GB}} = \begin{bmatrix} 0 \\ 0 \\ -m_b g \end{bmatrix} $$

$$ {}^{B}\boldsymbol{F_{SB}} = \begin{bmatrix} F_{SBx} \\ F_{SBy} \\ F_{SBz} \end{bmatrix} $$

$$ {}^{B}\boldsymbol{\tau_{SB}} = \begin{bmatrix} \tau_{SBx} \\ \tau_{SBy} \\ \tau_{SBz} \end{bmatrix} $$

The Quaternion-derived Rotation matrix is defined as follow,

$$ {}^{B}_{O}R = R(\boldsymbol{q_B}) = \begin{bmatrix} q_r^2 + q_i^2 - q_j^2 - q_k^2 & 2q_iq_j - 2q_rq_k & 2q_iq_k + 2q_rq_j \\ 2q_iq_j + 2q_rq_k & q_r^2 - q_i^2 + q_j^2 - q_k^2 & 2q_jq_k - 2q_rq_i \\ 2q_iq_k - 2q_rq_j & 2q_jq_k + 2q_rq_i & q_r^2 - q_i^2 - q_j^2 + q_k^2 \end{bmatrix} $$

Net Force

$$ {}^{O}\boldsymbol{F_{B,net}} = {}^{O}\boldsymbol{F_{GB}} + {}^{O}\boldsymbol{F_{TB}} + {}^{O}\boldsymbol{F_{SB}} = m_B\, {}^{O}\boldsymbol{a_B} $$
$$ {}^{O}\boldsymbol{F_{GB}} + ({}^{O}_{B}R)^T\, {}^{B}\boldsymbol{F_{TB}} + ({}^{O}_{B}R)^T\, {}^{B}\boldsymbol{F_{SB}} = m_B \boldsymbol{a_B} = m_B\, {}^{O}\ddot{\boldsymbol{p}}_{\boldsymbol{B}} $$

Net Torque

$$ {}^{O}\boldsymbol{\tau_{B,net}} = I_B\, {}^{O}\boldsymbol{\alpha_B} $$
$$ ({}^{O}_{B}R)^T\, {}^{B}\boldsymbol{\tau_{SB}} = 2I_B(\ddot{\boldsymbol{q}}_{\boldsymbol{B}}\boldsymbol{q_B} - (\dot{\boldsymbol{q}}_{\boldsymbol{B}}\boldsymbol{q}_B)^2) $$

By appling Physics law, we arrive at two equations below,

$$ {}^{O}\boldsymbol{F_{GB}} + ({}^{O}_{B}R)^T\, {}^{B}\boldsymbol{F_{TB}} + ({}^{O}_{B}R)^T\, {}^{B}\boldsymbol{F_{SB}} = m_B\, {}^{O}\ddot{\boldsymbol{p}}_{\boldsymbol{B}} \tag{1} $$
$$ ({}^{O}_{B}R)^T\, {}^{B}\boldsymbol{\tau_{SB}} = 2I_B(\ddot{\boldsymbol{q}}_{\boldsymbol{B}}\boldsymbol{q_B} - (\dot{\boldsymbol{q}}_{\boldsymbol{B}}\boldsymbol{q}_B)^2) \tag{2} $$

We then aim to find the state evolution equation for the body from the above equations (1) and (2), namely

$$ \dot{\boldsymbol{s}}_{\boldsymbol{B}} = f_B(\boldsymbol{s}_B, \boldsymbol{F_{B,net}}, \boldsymbol{\tau_{B,net}}) \tag{3} $$

2.3 Controller Dynamics

Similarly as above we aim to find the state evolution equation for the controller from equations we got from Physics law, namely

$$\dot{\boldsymbol{s}}_{\boldsymbol{C}} = f_C(\boldsymbol{s}_C, \boldsymbol{F}_{\boldsymbol{C,net}}, \boldsymbol{\tau}_{\boldsymbol{C,net}}) \tag{4}$$

2.4 Constraints and Manipulation

The two bodies are contrainted (attached together), there are some relationship between the states and the forces between the body and the controller,

$$\boldsymbol{s}_{\boldsymbol{C}} = f_{BC}(\boldsymbol{s}_{\boldsymbol{B}}) \tag{5}$$

$$\boldsymbol{F}_{\boldsymbol{C}} = g_{BC,F}(\boldsymbol{F}_{\boldsymbol{B}}) \tag{6}$$

$$\boldsymbol{\tau}_{\boldsymbol{C}} = g_{BC,\tau}(\boldsymbol{\tau}_{\boldsymbol{B}}) \tag{7}$$

Combining the above equations with (3) and (4), we would like to do some algebraic manipulation to get rid of the unwanted parameters in our state evolution equations, such that

$$\dot{\boldsymbol{s}}_{\boldsymbol{sys}} = f_{sys}(\boldsymbol{s}_{\boldsymbol{sys}}, \boldsymbol{u})$$

with u being our input, whatever we defined our input to be, either being the yaw torque in the body frame τ_{SBz} or some other representations.