$$ {}^{O}\boldsymbol{F_{B,net}} = {}^{O}\boldsymbol{F_{GB}} + {}^{O}\boldsymbol{F_{T}} + {}^{O}\boldsymbol{F_{CB}} = m_B\, {}^{O}\boldsymbol{a_B} \tag{1} $$

$$ {}^{O}\boldsymbol{\tau_{B,net}} = R(\boldsymbol{q_B})\, {}^{B}\boldsymbol{\tau_{CB}} = {}^{O}I_B\, {}^{O}\boldsymbol{\alpha_B} \tag{2} $$

$$ {}^{O}\boldsymbol{F_{net,C}} = {}^{O}\boldsymbol{F_{BC}} + {}^{O}\boldsymbol{F_{GC}} = m_C\, {}^{O}\boldsymbol{a_C} \tag{3} $$

$$ {}^{O}\boldsymbol{\tau_{net,C}} = R(\boldsymbol{q_C})\, {}^{C}\boldsymbol{\tau_{BC}} + {}^{O}\boldsymbol{\tau_{RF}} = {}^{O}I_c\, {}^{O}\boldsymbol{\alpha_C} \tag{4} $$

$$ \begin{bmatrix} \boldsymbol{p_C} \\ \boldsymbol{q_C} \end{bmatrix} = \begin{bmatrix} \boldsymbol{p_B} + \boldsymbol{r_{BC}} \\ \boldsymbol{q_\theta q_B} \end{bmatrix} = \begin{bmatrix} \boldsymbol{p_{sys}} + \boldsymbol{r_{BC}} \\ \boldsymbol{q_\theta q_{sys}} \end{bmatrix} \tag{5} $$

$$ \begin{bmatrix} \dot{\boldsymbol{p}}_{\boldsymbol{C}} \\ \dot{\boldsymbol{q}}_{\boldsymbol{C}} \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{p}}_{\boldsymbol{sys}} + \dot{R}(\boldsymbol{q_{sys}})\, {}^{B}\boldsymbol{r_{BC}} \\ \boldsymbol{q_\theta}\dot{\boldsymbol{q}}_{\boldsymbol{sys}} + \dot{\boldsymbol{q}}_{\boldsymbol{\theta}}\boldsymbol{q_{sys}} \end{bmatrix} \tag{6} $$

$$ \begin{bmatrix} \ddot{\boldsymbol{p}}_{\boldsymbol{C}} \\ \ddot{\boldsymbol{q}}_{\boldsymbol{C}} \end{bmatrix} = \begin{bmatrix} \ddot{\boldsymbol{p}}_{\boldsymbol{sys}} + \ddot{R}(\boldsymbol{q_{sys}})\, {}^{B}\boldsymbol{r_{BC}} \\ \boldsymbol{q_\theta}\ddot{\boldsymbol{q}}_{\boldsymbol{sys}} + 2[\dot{\boldsymbol{q}}_{\boldsymbol{\theta}}\dot{\boldsymbol{q}}_{\boldsymbol{sys}}] + \ddot{\boldsymbol{q}}_{\boldsymbol{\theta}}\boldsymbol{q_{sys}} \end{bmatrix} \tag{7} $$

$$ {}^{O}\boldsymbol{F_{BC}} = -\, {}^{O}\boldsymbol{F_{CB}} \tag{8} $$

$$ {}^{O}\boldsymbol{\tau_{BC}} = -\, {}^{O}\boldsymbol{\tau_{CB}} \tag{9} $$

$$ q_r^2 + q_i^2 + q_j^2 + q_k^2 = 1 \tag{10} $$

$$ q_r\dot{q}_r + q_i\dot{q}_i + q_j\dot{q}_j + q_k\dot{q}_k = 0 \tag{11} $$

$$ q_r\ddot{q}_r + q_i\ddot{q}_i + q_j\ddot{q}_j + q_k\ddot{q}_k + \dot{q}_r{}^2 + \dot{q}_i{}^2 + \dot{q}_j{}^2 + \dot{q}_k{}^2 = 0 \tag{12} $$

$$ \boldsymbol{q_\theta} = \cos(\frac{\theta}{2}) + \sin(\frac{\theta}{2})R(\boldsymbol{q_{sys}})\, {}^{B}\hat{\boldsymbol{z}}_{\boldsymbol{B}} \tag{13} $$

$$ \dot{\boldsymbol{q}}_{\boldsymbol{\theta}} = -\frac{1}{2}\sin(\frac{\theta}{2})\dot{\theta} + \frac{1}{2}\cos(\frac{\theta}{2})\dot{\theta}R(\boldsymbol{q_{sys}})\, {}^{B}\hat{\boldsymbol{z}}_{\boldsymbol{B}} + \sin(\frac{\theta}{2})R(\dot{\boldsymbol{q}}_{\boldsymbol{sys}})\, {}^{B}\hat{\boldsymbol{z}}_{\boldsymbol{B}} \tag{14} $$

$$ (m_b + m_c)\ddot{\boldsymbol{p}}_{\boldsymbol{sys}} + m_c\ddot{R}(\boldsymbol{q_{sys}})\, {}^{B}\boldsymbol{r_{BC}} = \boldsymbol{F_{GC}} + \boldsymbol{F_{GB}} + \boldsymbol{F_T} \tag{15} $$

$$ 2I_B\left[\ddot{\boldsymbol{q}}_{\boldsymbol{B}}\boldsymbol{q_B^*} - (\dot{\boldsymbol{q}}_{\boldsymbol{B}}\boldsymbol{q_B^*})^2\right] + 2I_c\left[\ddot{\boldsymbol{q}}_{\boldsymbol{C}}\boldsymbol{q_C^*} - (\dot{\boldsymbol{q}}_{\boldsymbol{C}}\boldsymbol{q_C^*})^2\right] = {}^{O}\boldsymbol{\tau_{RF}} \tag{16} $$

$$ 2I_B[\ddot{\boldsymbol{q}}_{\boldsymbol{sys}}\boldsymbol{q_{sys}^*}] + 2I_C[\boldsymbol{q_\theta}\ddot{\boldsymbol{q}}_{\boldsymbol{sys}}(\boldsymbol{q_\theta q_{sys}})^*] + 2I_C[\ddot{\boldsymbol{q}}_{\boldsymbol{\theta}}\boldsymbol{q_{sys}}](\boldsymbol{q_\theta q_{sys}})^* - \boldsymbol{r_{CB}} \times \boldsymbol{F_{BC}} = \zeta \tag{17} $$

where

$$ \zeta = 2I_B(\dot{\boldsymbol{q}}_{\boldsymbol{sys}}\boldsymbol{q_{sys}^*})^2 + 2I_C[(\boldsymbol{q_\theta}\dot{\boldsymbol{q}}_{\boldsymbol{sys}} + \dot{\boldsymbol{q}}_{\boldsymbol{\theta}}\boldsymbol{q_{sys}})(\boldsymbol{q_\theta q_{sys}})^*]^2 - 4I_C(\dot{\boldsymbol{q}}_{\boldsymbol{\theta}}\dot{\boldsymbol{q}}_{\boldsymbol{sys}})(\boldsymbol{q_\theta q_{sys}})^* $$

$$ \boldsymbol{F_{BC}} = m_B\ddot{\boldsymbol{p}}_{\boldsymbol{sys}} - \boldsymbol{F_{GB}} - \boldsymbol{F_T} $$