The University of California, Los Angeles

Robotics Design Capstone

EE 183DB

Off-center spinning mass controller for Quadcopters

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Abstract

Your abstract.

1 Symbols

Here is a list of all symbols used in this paper:

$\boldsymbol{p} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	linear position vectors
$\boldsymbol{q} = \begin{bmatrix} q_r \\ q_i \\ q_j \\ q_k \end{bmatrix}$	angular orientation vectors in quaternion
$\boldsymbol{F_T}$	thrust force
$\boldsymbol{F_G}$	gravitational force
$\boldsymbol{F_S}$	reaction force acted on the surface
$\boldsymbol{\tau_S}$	reaction torque acted on the surface
$\boldsymbol{\tau_M}$	torque introduced by the off-center mass
m_b	mass of the body
m_c	mass of the controller
S_x, C_x, T_x	$\sin(x), \cos(x), \tan(x)$ respectively

2 Mathematical Derivation

2.1 Assumptions

- Assume unit quaternions: $||\boldsymbol{q}|| = 1$

2.2 Quadcopter Body Dynamics

Forces and Torques:

$$^{B}\boldsymbol{F_{TB}} = \begin{bmatrix} 0 \\ 0 \\ F_{TB} \end{bmatrix}$$

$$^{O}\boldsymbol{F_{GB}} = \begin{bmatrix} 0 \\ 0 \\ -m_b g \end{bmatrix}$$

$$^{B}\boldsymbol{F_{SB}} = \begin{bmatrix} F_{SBx} \\ F_{SBy} \\ F_{SBz} \end{bmatrix}$$

$$^{B}\boldsymbol{\tau_{SB}} = \begin{bmatrix} \tau_{SBx} \\ \tau_{SBy} \\ \tau_{SBz} \end{bmatrix}$$

The Quaternion-derived Rotation matrix is defined as follow,

$$^{O}_{B}R = R(\boldsymbol{q_B}) = \begin{bmatrix} q_r^2 + q_i^2 - q_j^2 - q_k^2 & 2q_iq_j - 2q_rq_k & 2q_iq_k + 2q_rq_j \\ 2q_iq_j + 2q_rq_k & q_r^2 - q_i^2 + q_j^2 - q_k^2 & 2q_jq_k - 2q_rq_i \\ 2q_iq_k - 2q_rq_j & 2q_jq_k + 2q_rq_i & q_r^2 - q_i^2 - q_j^2 + q_k^2 \end{bmatrix}$$

Net Force

$$^{O}\boldsymbol{F_{B,net}} = {}^{O}\boldsymbol{F_{GB}} + {}^{O}\boldsymbol{F_{TB}} + {}^{O}\boldsymbol{F_{SB}} = m_B\, {}^{O}\boldsymbol{a_B}$$
$$^{O}\boldsymbol{F_{GB}} + {}^{O}_{B}R\, {}^{B}\boldsymbol{F_{TB}} + {}^{O}_{B}R\, {}^{B}\boldsymbol{F_{SB}} = m_B \boldsymbol{a_B} = m_B\, {}^{O}\boldsymbol{\ddot{p}_B}$$

Net Torque

$$^{O}\boldsymbol{\tau_{B,net}} = I_B\, {}^{O}\boldsymbol{\alpha_B}$$
$$^{O}_{B}R\, {}^{B}\boldsymbol{\tau_{SB}} = 2I_B(\boldsymbol{\ddot{q}_B q_B} - (\boldsymbol{\dot{q}_B q}_B)^2)$$

By appling Physics law, we arrive at two equations below,

$$^{O}\boldsymbol{F_{GB}} + {}^{O}_{B}R\, {}^{B}\boldsymbol{F_{TB}} + {}^{O}_{B}R\, {}^{B}\boldsymbol{F_{SB}} = m_B\, {}^{O}\boldsymbol{\ddot{p}_B} \tag{1}$$
$$^{O}_{B}R\, {}^{B}\boldsymbol{\tau_{SB}} = 2I_B(\boldsymbol{\ddot{q}_B q_B^*} - (\boldsymbol{\dot{q}_B q_B^*})^2) \tag{2}$$

We then aim to find the state evolution equation for the body from the above equations (1) and (2), namely

$$\boldsymbol{\dot{s}_B} = f_B(\boldsymbol{s}_B, \boldsymbol{F_{B,net}}, \boldsymbol{\tau_{B,net}}) \tag{3}$$

Let $\boldsymbol{v_B} = \dot{\boldsymbol{p}}_{\boldsymbol{B}}$ and $\boldsymbol{\omega_B} = \dot{\boldsymbol{q}}_{\boldsymbol{B}}$ and substitute new variables,

$$
\begin{aligned}
\dot{\boldsymbol{p}}_{\boldsymbol{B}} &= \boldsymbol{v_B} \\
\dot{\boldsymbol{v}}_{\boldsymbol{B}} &= \frac{1}{m_B}\left[R(\boldsymbol{q_B})\,{}^{B}\boldsymbol{F_{TB}} + R(\boldsymbol{q_B})\,{}^{B}\boldsymbol{F_{SB}} + {}^{O}\boldsymbol{F_{GB}}\right] \\
\dot{\boldsymbol{q}}_{\boldsymbol{B}} &= \boldsymbol{\omega_B} \\
\dot{\boldsymbol{\omega}}_{\boldsymbol{B}} &= \left[\frac{R(\boldsymbol{q_B})\boldsymbol{\tau_{SB}}}{2I_B} + (\boldsymbol{\omega_B}\boldsymbol{q_B^*})^2\right]\boldsymbol{q_B}
\end{aligned}
$$

Then, by letting our state variable $\boldsymbol{s_B} = \begin{bmatrix} \boldsymbol{p_B} \\ \boldsymbol{v_B} \\ \boldsymbol{q_B} \\ \boldsymbol{\omega_B} \end{bmatrix}$, we arrive at the desired state evolution equation.

2.3 Controller Dynamics

Similarly as above we aim to find the state evolution equation for the controller from equations we got from Physics law, namely

$$\dot{\boldsymbol{s}}_{\boldsymbol{C}} = f_C(\boldsymbol{s}_C, \boldsymbol{F_{C,net}}, \boldsymbol{\tau_{C,net}}) \tag{4}$$

Forces and Torques:

$$
\begin{aligned}
{}^{C}\boldsymbol{F_{SC}} &= \begin{bmatrix} F_{SCx} \\ F_{SCy} \\ F_{SCz} \end{bmatrix} \\
{}^{O}\boldsymbol{F_{GC}} &= \begin{bmatrix} 0 \\ 0 \\ -m_c g \end{bmatrix} \\
{}^{B}\boldsymbol{\tau_{SC}} &= \begin{bmatrix} \tau_{SCx} \\ \tau_{SCy} \\ \tau_{SCz} \end{bmatrix} \\
{}_{O}\boldsymbol{\tau_M} &= {}^{O}\boldsymbol{r_M} \times {}^{O}\boldsymbol{F_{GC}} = {}^{O}_{B}R\,{}^{B}\boldsymbol{r_M} \times {}^{O}\boldsymbol{F_{GC}} \\
&= R(q_C)\begin{bmatrix} L_{Mx} \\ 0 \\ L_{Mz} \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ -m_c g \end{bmatrix}
\end{aligned}
$$

Net Force and Net Torque:

$$
\begin{aligned}
m_c \ddot{\boldsymbol{p}}_{\boldsymbol{C}} &= {}^{O}F_{GC} + {}^{O}_{C}R\, {}^{B}\boldsymbol{F_{SC}} \\
I_c \boldsymbol{\alpha_C} &= {}^{O}_{C}R\, {}^{O}\tau_{SC} + {}^{O}\boldsymbol{\tau_M}
\end{aligned}
$$

Let $\boldsymbol{v_C} = \dot{\boldsymbol{p}}_{\boldsymbol{C}}$ and $\boldsymbol{\omega_C} = \dot{\boldsymbol{q}}_{\boldsymbol{C}}$ and substitute new variables,

$$
\begin{aligned}
\dot{\boldsymbol{p}}_{\boldsymbol{C}} &= \boldsymbol{v_C} \\
\dot{\boldsymbol{v}}_{\boldsymbol{C}} &= \frac{1}{m_C}\left[R(\boldsymbol{q_C})\, {}^{B}\boldsymbol{F_{SC}} + {}^{O}\boldsymbol{F_{GC}}\right] \\
\dot{\boldsymbol{q}}_{\boldsymbol{C}} &= \boldsymbol{\omega_C} \\
\dot{\boldsymbol{\omega}}_{\boldsymbol{C}} &= \left[\frac{R(\boldsymbol{q_C})\, {}^{B}\boldsymbol{\tau_{SC}} + {}^{O}\boldsymbol{\tau_M}(\boldsymbol{q_C})}{2I_C} + (\boldsymbol{\omega_C}\boldsymbol{q_C^*})^2\right]\boldsymbol{q_C}
\end{aligned}
$$

Then, by letting our state variable $\boldsymbol{s_C} = \begin{bmatrix} \boldsymbol{p_C} \\ \boldsymbol{v_C} \\ \boldsymbol{q_C} \\ \boldsymbol{\omega_C} \end{bmatrix}$, we arrive at the desired state evolution equation.

2.4 Constraints and Manipulation

The two bodies are contrainted (attached together), there are some relationship between the states and the forces between the body and the controller,

$$
\begin{aligned}
\boldsymbol{s_C} &= f_{BC}(\boldsymbol{s_B}) && (5) \\
\boldsymbol{F_C} &= g_{BC,F}(\boldsymbol{F_B}) && (6) \\
\boldsymbol{\tau_C} &= g_{BC,\tau}(\boldsymbol{\tau_B}) && (7)
\end{aligned}
$$

Combining the above equations with (3) and (4), we would like to do some algebraic manipulation to get rid of the unwanted parameters in our state evolution equations, such that

$$
\dot{\boldsymbol{s}}_{\boldsymbol{sys}} = f_{sys}(\boldsymbol{s_{sys}}, \boldsymbol{u})
$$

with u being our input, whatever we defined our input to be, either being the yaw torque in the body frame τ_{SBz} or some other representations.