

Simplex Solver

December 3, 2023

Problem

Given the following linear system and objective function, find the optimal solution.

$$\begin{aligned} & \max x_1 + 2x_2 \\ & \begin{cases} x_1 + x_2 \leq 5 \\ 2x_1 + x_2 \leq 8 \end{cases} \end{aligned}$$

Solution

Add slack variables to turn all inequalities to equalities.

$$\begin{cases} x_1 + x_2 + s_1 = 5 \\ 2x_1 + x_2 + s_2 = 8 \end{cases}$$

Create the initial tableau of the new linear system.

$$\left[\begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & b \\ 1 & 1 & 1 & 0 & 5 \\ 2 & 1 & 0 & 1 & 8 \\ -1 & -2 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} s_1 \\ s_2 \end{array}$$

There are negative elements in the bottom row, so the current solution is not optimal. Thus, pivot to improve the current solution. The entering variable is x_2 and the departing variable is s_1 .

Perform elementary row operations until the pivot element is 1 and all other elements in the entering column are 0.

$$\left[\begin{array}{cccc|c} x_1 & x_2 & s_1 & s_2 & b \\ 1 & 1 & 1 & 0 & 5 \\ 1 & 0 & -1 & 1 & 3 \\ 1 & 0 & 2 & 0 & 10 \end{array} \right] \begin{array}{l} x_2 \\ s_2 \end{array}$$

There are no negative elements in the bottom row, so we know the solution is optimal. Thus, the solution is:

$$s_1 = 0, s_2 = 3, x_1 = 0, x_2 = 5, z = 10$$