Alternative Models for ΛCDM

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1 Introduction

Dark energy is an unknown form of energy that affects the universe on the largest scales, and it constitutes about 70% of the universe.
The remaining 30% is matter, including both baryonic (i.e., normal matter) and cold dark matter.

We began the story of dark energy with A Einstein and General theory of Relativity in 1915

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} \tag{1}$$

where $G_{\mu\nu}$ is the so-called Einstein tensor, $g_{\mu\nu}$ and $R_{\mu\nu}$ are the metric and Ricci tensors, respectively, $R = g_{\mu\nu} R_{\mu\nu}$ is the Ricci scalar, and $T_{\mu\nu}$ is the energy-momentum tensor. After his fascinating discovery, Einstein quickly realized the potential power of his theory. He began studying the implications of his new framework in cosmology, starting with a hypothetical static Universe. The second assumption was reasonable at the time, based on empirical evidence supporting the low velocities of observed stars. The question was whether GR can explain this type of Universe. Einstein knew, of course, that the Newtonian Poisson's equation

$$\nabla^2 \phi = 4\pi G \rho_m \tag{2}$$

is immediately recovered from (1) in the low-energy limit. Einstein saw the necessity of incorporating GR into his theory, as he recognized that the Newtonian model of gravity would not fit his envisioned Universe. Whether finite or infinite, a Euclidean Universe collapses due to gravitational attraction. In a static Universe, a uniformly distributed matter energy density cannot satisfy Poisson's equation. Thus, he needed to modify his original field equations to obtain a modified form of Poisson's equation in the low-energy regime, since Einstein's research on the general theory of relativity resulted in a paper called "Cosmological considerations on the general theory of relativity", which he submitted to the Royal Prussian Academy of Sciences in Berlin on February 8th, 1917. Einstein pointed out that the following modified form of Poisson's equation can eliminate the existing drawback

$$\nabla^2\phi - \lambda\phi = 4\pi G\rho_m \tag{3}$$

where λ is a universal constant According to this equation, a static and homogeneous Universe becomes possible if the gravitational potential takes the constant value

$$\phi = -4\pi G\rho m/\lambda$$

Analogously to the addition of this new term containing the constant in the original Poisson's equation, Einstein modified his original field equations (1) by also adding a constant so as to be able to explain in a consistent way his static Universe,

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{4}$$

Einstein proposed Λ as a new constant of Nature, which preserves general covariance and, if small, passes Solar System tests. He demonstrated that $\Lambda = 4\pi G\rho m$ can counteract the attractive gravitational force of the uniformly distributed mass in the Universe, enabling a homogeneous, static, and closed spherical Universe with curvature radius $R_E = \Lambda^{-1/27}$. This is a pivotal moment in the development of modern Cosmology, closely tied to the emergence of the Cosmological Constant.

The first observational evidence for the existence of dark energy came from measurements of supernovas, which showed that the universe does not expand at a constant rate; rather, the universe's expansion is accelerating.

Understanding the universe's evolution requires knowledge of its starting conditions and composition.

Dark energy's status as a hypothetical force with unknown properties makes

it a very active target of research.
The problem is attacked from a great variety of angles, such as modifying the prevailing theory of gravity (general relativity), attempting to pin down the properties of dark energy, and finding alternative ways to explain the observational data.
There are several forms the dark energy could take, but even now, more than two decades after it was first detected, scientists are still not certain which of the possible forms is the right one.

Some of the best candidates for dark energy are:

1. **Vacuum Energy:** Quantum Field Theory reveals that empty space is not actually empty. Vacuum fluctuations of matter fields exist even without interactions and are purely quantum. They are Feynman closed loop diagrams without external legs (vacuum blob diagrams), representing the contribution of infinite oscillators vibrating in all frequencies. This is due to the Heisenberg uncertainty principle. These diagrams contribute infinitely to the total energy density of the quantum system, equal to the predicted energy of the system in its ground state.this infinite contribution is equal to the theoretically predicted energy of the system when it is in the ground state.

2. **Quintessence:** Quintessence is a new kind of dynamical energy fluid or field, something that fills all of space but something whose effect on the expansion of the universe is the opposite of that of matter and normal energy.

3. **Modified Gravity:** Some scientists suggest that dark energy is not a new form of energy but rather a modification of the laws of gravity on cosmological scales.

Vacuum energy is the energy that exists in a vacuum or empty space. According to quantum mechanics, the vacuum of empty space should contain a type of energy known as vacuum energy, which is thought to be spread throughout the universe and exert a force opposing gravity.

The vacuum energy is an underlying background energy that exists in space throughout the entire universe.

It is related to the universe's accelerating expansion and is thought to have a negative pressure, which is believed to be responsible for the accelerating expansion of the universe.

Vacuum energy can also be thought of in terms of virtual particles (also known as vacuum fluctuations) which are created and destroyed out of the vacuum.

These particles are always created out of the vacuum in particle-antiparticle pairs, which in most cases shortly annihilate each other and disappear. However, these particles and antiparticles may interact with others before disappearing, a process which can be mapped using Feynman diagrams.

The vacuum energy is a potential source of dark energy, which is different from Einstein's cosmological constant.

The effects of vacuum energy can be experimentally observed in various phenomena such as **spontaneous emission, the Casimir effect, and the Lamb shift**, and are thought to influence the behavior of the Universe on cosmological scales.

Vacuum energy is a good candidate for dark energy for several reasons, as suggested by the search results:

1. **Quantum Mechanics:** According to quantum mechanics, the vacuum of empty space should contain a type of energy known as vacuum energy, which is thought to be spread throughout the universe and exert a force opposing gravity.

2. **Negative Pressure:** The vacuum energy is thought to have a negative pressure, which is believed to be responsible for the accelerating expansion of the universe.

3. **Dynamic Quantum Vacuum Energy:** Some scientists suggest that dark energy is a type of dynamic quantum vacuum energy, which is different from Einstein's cosmological constant.

4. **Vacuum Energy and Dark Energy:** Vacuum energy and dark energy are not the same thing, but vacuum energy is a potential source of dark energy.

In summary, vacuum energy is a good candidate for dark energy because it is a form of energy that is thought to be spread throughout the universe and exert a force opposing gravity, which is believed to be responsible for the accelerating expansion of the universe.

1.1 Differences between CC and the Dynamic Vacuum Energy:

The cosmological constant and dynamic quantum vacuum energy are two different concepts related to the vacuum energy. Here are the differences between them:

- **Cosmological Constant:**
  The cosmological constant is a mathematical term in the equations of general relativity that represents an intrinsic energy density of the vacuum, ρ_{vac}, and an associated pressure.
  It was introduced by Einstein to counteract the gravitational attraction of matter and achieve a static universe.
  The cosmological constant has the same effect as vacuum energy, but it is a constant value that does not change with time or space.
  The cosmological constant is a source of dark energy, which is responsible for the accelerating expansion of the universe.
  The cosmological constant problem arises because the predicted amount of vacuum energy is many orders of magnitude bigger than the observed cosmological constant.

- **Dynamic Quantum Vacuum Energy:**
  Dynamic quantum vacuum energy is a type of vacuum energy that is not constant but changes with time and space.
  It is a type of dark energy that is different from the cosmological constant.
  Dynamic quantum vacuum energy is a potential source of dark energy, and it is thought to be spread throughout the universe and exert a force opposing gravity.
  The vacuum energy in quantum field theory can be set to any value by re-normalization, which treats the cosmological constant as simply

another fundamental physical constant not predicted or explained by theory.

The cosmological constant and dynamic quantum vacuum energy are two different concepts related to vacuum energy. The cosmological constant is a constant value that does not change with time or space, while dynamic quantum vacuum energy changes with time and space. The cosmological constant is a source of dark energy, while dynamic quantum vacuum energy is a potential source of dark energy and is different from the cosmological constant.

1.2 Why do we prefer Dynamic vacuum energy?

Dynamic vacuum energy models are preferred over the ΛCDM model for several reasons, as suggested by the search results:

1. **Time Evolution:** Dynamic vacuum energy models incorporate a time-evolving vacuum energy density, which is different from the constant vacuum energy density in the ΛCDM model

2. **Observational Status:** Theoretical background and current observational status of ΛCDM and Running Vacuum Models suggest that dynamic vacuum energy models are consistent with current observations.

3. **Challenges to ΛCDM:** Some studies challenge the ΛCDM model, which assumes a constant vacuum energy density, and suggest that dynamic vacuum energy models may provide a better explanation for the observed accelerating expansion of the universe.

4. **Incorporation of Dark Matter:** Some dynamic vacuum energy models incorporate dark matter, which is not included in the ΛCDM.

2 Alternatives to ΛCDM model:

In this article, we have provided some models that can replace ΛCDM model according to observational data sets. these models use "Dark Energy" as a **Dynamic** parameter. We can categorize these models into 5 types. these types are actually the major differences the models have with ΛCDM.

2.1 The Scalar Field Model ϕCDM:

We can consider that the accelerating expansion of the universe is caused by an isotropic, homogeneous field $\phi(t)$. Note that the field density ρ and pressure p are derived from the Lagrangian:

$$\rho_\phi = \frac{M_P^2}{16\pi}\left[\frac{\dot{\phi}^2}{2} + V(\phi)\right], P_\phi = \frac{M_P^2}{16\pi}\left[\frac{\dot{\phi}^2}{2} - V(\phi)\right] \tag{5}$$

And the Klein-Gordon equation is:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0 \tag{6}$$

Where the potential term V(ϕ) can obtain various forms as below:

Potential form	Equation
Linear	$V(\phi) = \lambda\phi$
Inverse power-law	$V(\phi) = \frac{1}{2}\kappa M_p^2 \phi^{-\alpha}$
Exponential	$V(\phi) = V_0 e^{-\kappa\lambda\phi}$
Double-Exponential	$V(\phi) = V_0(e^{-\kappa\lambda_1\phi} + e^{-\kappa\lambda_2\phi})$
Pseudo-Nambu-Goldstone	$V(\phi) = V_0[C + cos(\frac{\phi}{f})]$
SUGRA motivated	$V_0^{4+\alpha}\phi^{-\alpha}e^{\frac{\kappa^2 n^2 \phi}{2}}$

in all of the equations above, we defined $\kappa^2 = \frac{8\pi}{M_p^2}$.
Now we can solve the Klein-Gordon equation with different forms of potential. for example, for the **Inverse power-law form of potential (also known as Peebles & Ratra potential)**, we can assume the field's time relation is a power-law:

$$\phi(t) = At^p$$

and then, place the $\phi(t)$ in the Klein-Gordon equation. Of course we cannot solve the H(t) analytically, but we can calculate it for the Matter-dominant and the Radiation-dominant era separately. From the continuity equation and the Friedmann's first equation, we know:

$$\rho(z) = \rho_0(1+z)^{3(1+\omega)} = \rho_0(1+z)^n \tag{7}$$

$$H^2 = \frac{8\pi G}{3}\rho \tag{8}$$

$$\Longrightarrow H(t) = \frac{2}{nt} \tag{9}$$

then for $\phi(t) = At^p$, we have:

$$p = \frac{2}{\alpha+2}, \quad A^{\alpha+2} = \frac{\alpha(\alpha+2)^2 M_{pl}^2 \kappa n}{4(6\alpha + 12 - n\alpha)} \tag{10}$$

Hence, the field equation in time t becomes as follows:

$$\phi(t) = \left[\frac{\alpha(\alpha+2)^2 M_{pl}^2 \kappa n}{4(6\alpha + 12 - n\alpha)}\right]^{1/(\alpha+2)} t^{2/(\alpha+2)} \tag{11}$$

If we assume that the field obeys the equation $P_\phi = \omega_\phi \rho_\phi$, from the equation (5), the Equation of State is derived as below:

$$\omega_\phi = \frac{P_\phi}{\rho_\phi} = -1 + \frac{\alpha n}{3(2+\alpha)} \tag{12}$$

Although the ω_ϕ seems to be constant from the equation above, but note that we calculated it for different era's of universe. in reality, the H(t)'s evolution is not that simple. As we can see, at the late times, the matter and radiation dominance fades away and as we expected, $n \longrightarrow 0$ or $\omega_\phi \longrightarrow -1$.

We also can re-write the field equation in terms of the scale-factor. For example, in the Matter-dominant era we have:

$$\phi(z) = \left[\frac{\alpha(\alpha+2)^2 \bar{\kappa}}{9 \times 10^4 \Omega_m h^2 (\alpha+4)}\right]^{1/(\alpha+2)} (1+z)^{-3/(\alpha+2)} \tag{13}$$

Where $\bar{\kappa} \equiv \kappa M_p^2/\zeta^2$ and $\zeta = 1Km/s/Mpc$ is a constant to make $\bar{\kappa}$ dimensionless.

Compared, say to the exponential potential, $V(\phi) = V_0 e^{-\kappa\lambda\phi}$, the latter is inconsistent with BBN (if λ is too small) or cannot be important enough to cause accelerated expansion at the current time (if λ is too large). This can be cured with a sum of two exponentials with different values of λ, but of course it is less motivated since involves more parameters. Thus, the PR-potential seems to have the minimal number of ingredients to successfully accomplish the job. In fact, it is what we have now verified at a rather significant confidence level in the light of the modern cosmological data.

2.2 The Equation of State Parameterized Models:

These models obey the equation $P = \omega\rho$ as the ΛCDM does; But unlike the ΛCDM, the ω parameter is not exactly equal to -1; Hence the DE density is not completely constant.

2.2.1 The XCDM Model:

The simplest example of these models is the XCDM, where the Dark Energy is a fluid with a constant Equation of State ω_0. In general, this fluid can have a quintessence behaviour ($\omega_0 \geq -1$) or a phantom behaviour ($\omega_0 \leq -1$); but the ω_0 has to be around -1, Because the XCDM, like any suggested model, has to behave nearly like the ΛCDM.

In a flat universe, we will have $\Omega_X = 1 - \Omega_m - \Omega_r$. From the equation (7), the density in terms of redshift reads:

$$\rho_X = \rho_X^0 (1+z)^{3(1+\omega_0)}$$

$$\rho_X^0 = \rho_c^0 \Omega_X^0 = \rho_c^0 (1 - \Omega_m^0)$$

$$\Longrightarrow \rho_X = \rho_c^0 (1 - \Omega_m^0)(1+z)^{3(1+\omega_0)}$$

So, the First Friedmann's equation becomes as follows:

$$E^2(z) = \Omega_m^0 (1+z)^3 + \Omega_r^0 (1+z)^4 + \Omega_X^0 (1+z)^{3(1+\omega_0)} \tag{14}$$

Where $H(z) = H_0 E(z)$.

2.2.2 The CPL Model:

A more generalized form of XCDM is the CPL Model. in this model, we assume that Equation of State has two free parameters ω_0 and ω_a:

$$\omega = \omega_0 + (1-a)\omega_a = \omega_0 + \frac{z}{1+z}\omega_a$$

By putting this ω in the equation (7), the Hubble's parameter in a Flat universe in terms of z reads:

$$E^2(z) = \Omega_m^0(1+z)^3 + \Omega_r^0(1-\Omega_m^0-\Omega_r^0)(1+z)^{3(1+w_0+w_1)}e^{-3w_1\frac{z}{1+z}} \tag{15}$$

2.3 Chaplygin Gas:

This model is quite interesting because it unifies Dark Energy and Cold Dark Matter in a single form of fluid which has a strange Equation of State. This model can be viewed as interacting dark energy models with interaction parameter $Q \propto \frac{\rho_{cdm}\cdot\rho_{de}}{\rho_{cdm}+\rho_{de}}$.

2.3.1 The Original Chaplygin Gas:

The Original Chaplygin gas has an Equation of State in a simple form:

$$P_g = -A\rho_g^{-1}$$

Where A is a positive constant. Although, this model has been excluded by observations.

2.3.2 The Generalized Chaplygin Gas (GCG):

The generalized form however, could result in a good fit for our data sets. In this form, the Equation of State becomes as below:

$$P_g = -A\rho_g^{-\alpha}$$

Where β is a free parameter. By using the Continuity Equation, this gas' density equation can be derived:

$$\dot{\rho}_g + 3H(\rho_g + P_g) = 0$$

$$\Longrightarrow \rho_g(a) = [A_s + \frac{1 - A_s}{a^{3(1+\beta)}}]^{\frac{1}{1+\beta}}$$

Where $A_s \equiv \frac{A}{(\rho_g^0)^{1+\beta}}$ and (a) is the scale factor. If we limit our equation to the early times ($a \longrightarrow 0$), we can see that the fluid behaves like a pressure-less matter; And we also see that at the late times, the density tends to be constant and the fluid behaves like the Cosmological Constant. That's exactly why this model can unify the DE and CDM into a single fluid.

By this density equation, we have:

$$E^2(z) = \Omega_b^0(1+z)^3 + \Omega_r^0(1 - \Omega_b^0 - \Omega_r^0)(A_s + (1 - A_s)(1+z)^{3(1+\beta)})^{\frac{1}{1+\beta}}$$

2.3.3 New Generalized Chaplygin Gas (NGCG):

in the GCG model, we had the constant vacuum energy interacting with cold dark matter. If we want to extend this model one step further, we can replace the constant vacuum energy with a dynamic dark energy. In this model, dark energy has a constant ω but interacts with CDM with a different rate:

$$Q = 3\beta\omega H \frac{\rho_{cdm} \cdot \rho_{de}}{\rho_{cdm} + \rho_{de}}$$

The equation of State of the NGCG is as reads:

$$P = -\bar{A}(a)\rho_{ng}^{-\beta}$$

If we solve the Continuity Equation for this fluid, we get:

$$\rho_{ng}(a) = [Aa^{-3(1+\omega)(1+\beta)} + Ba^{-3(1+\beta)}]^{\frac{1}{1+\beta}}$$

And the form of $\bar{A}(a)$ function will be $\bar{A}(a) = -\omega a^{-3(1+\omega)(1+\beta)}$; Which results in a same form of CGC if $\omega = -1$.

Now if we re-write E(z) from the CGC model, we get:

$$E^2(z) = \Omega_b^0(1+z)^3+\Omega_r^0(1+z)^4+(1-\Omega_b^0-\Omega_r^0)(1+z)^3\left[1-\frac{\Omega_{de}^0}{1-\Omega_b^0-\Omega_r^0}\left(1-(1+z)^{3\omega(1+\beta)}\right)\right]^{\frac{1}{1+\beta}}$$

2.4 DGP Model and its extension:

Dvali-Gabadadze-Porrati braneworld model (DGP for short) is an example of Modified Gravity. In this model, the braneworld yields self-acceleration without dark energy, and it's described by an additional term in our Friedmann's Equation.

2.4.1 The DGP Model:

This model is the most basic form of the Modified Gravity correction of the Friedmann's first equation:

$$H^2 - \frac{H}{r_c} = \frac{8\pi G}{3}\left(\rho_m^0(1+z)^3 + \rho_r^0(1+z)^4\right)$$

Where r_c is called "the crossover scale" and is defined by:

$$r_c \equiv [H_0(1-\Omega_m^0-\Omega_r^0)]^{-1}$$

If we define $\Omega_{rc} = (1-\Omega_m-\Omega_r)^2/4$, then our E(z) function is given by:

$$E(z) = \sqrt{\Omega_m^0(1+z)^3 + \Omega_r^0(1+z)^4 + \Omega_{rc}^0} + \sqrt{\Omega_{rc}^0}$$

2.4.2 The α dark energy Model:

Just like taking an extra step from CG model to GCG (Section 2.3), We can somehow "generalize" our DGP model with an extra degree of freedom. This idea led to a new model: αDE. The Friedmann's Equation now has this form:

$$H^2 - \frac{H^\alpha}{{r_c}^{2-\alpha}} = \frac{8\pi G}{3}\left(\rho_m^0(1+z)^3 + \rho_r^0(1+z)^4\right)$$

Where r_c in this model, is defined as below:

$$r_c \equiv H_0^{-1}(1 - \Omega_m^0 - \Omega_r^0)^{\frac{1}{\alpha-2}}$$

Notice in a situation where $\alpha = 1$, we get the same equation as DGP, and for $\alpha = 0$, our equation reduces to the ΛCDM model.

If we calculate E(z) for this model, the function will be as follows:

$$E^2(z) = \Omega_b^0(1+z)^3 + \Omega_r^0(1+z)^4 + E^\alpha(z)(1 - \Omega_b^0 - \Omega_r^0)$$

Which again, results the same as ΛCDM in case of $\alpha = 0$.

3 Observational Data:

In this chapter, the Planck 2015 data is categorized in 4 major observation types:

1. **Ia-Supernovae**
2. **CMB Spectrum**
3. **BAO data**
4. H_0 **Measurements** ($h_0 = 0.706 \pm 0.033$)

3.1 Methodology:

To fit the models to observational data, the χ^2 method is used. First we define χ^2_ξ as below:

$$\chi^2_\xi = \frac{(\xi_{th} - \xi_{obs})^2}{\sigma^2_\xi}$$

Where ξ_{th} is the theory expected value and ξ_{obs} is the observed value. ξ indicates the observation which we are comparing our models to. Now the total χ value is defined by:

$$\chi^2 = \sum_\xi \chi^2_\xi = \chi^2_{SN} + \chi^2_{CMB} + \chi^2_{BAO} + \chi^2_{H_0}$$

it's obvious that the lesser the χ^2 value is for a model, the better it fits our data.

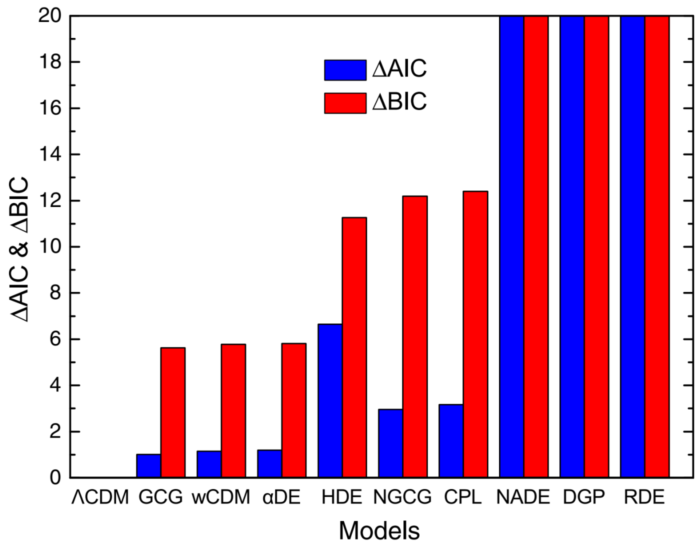
Now we can compare each of our models to the ΛCDM, and to the observational data. We have ignored the "0" indices (for present time) to make the writing simpler:

Model	Parameter			
ΛCDM	$h = 0.667^{+0.006}_{-0005}$	$\Omega_m = 0.324^{+0.007}_{-0.008}$		
ϕCDM	$h = 0.671 \pm 0.006$	$\Omega_m = 0.312 \pm 0.002$	$\omega_\phi = -0.931 \pm 0.017$	$\alpha = 0.219 \pm 0.057$
$XCDM$	$h = 0.662^{+0.008}_{-0.007}$	$\Omega_m = 0.326^{+0.009}_{-0.008}$	$\omega_0 = -0.964^{+0.030}_{-0.036}$	
CPL	$h = 0.663^{+0.007}_{-0.008}$	$\Omega_m = 0.326^{+0.009}_{-0.007}$	$\omega_0 = -0.969^{+0.098}_{-0.098}$	$\omega_a = 0.007^{+0.366}_{-0.431}$
GCG	$h = 0.663^{+0.008}_{-0.007}$	$A_s = 0.695^{+0.024}_{-0.023}$	$\beta = -0.03^{+0.067}_{-0.057}$	
NGCG	$h = 0.662^{+0.015}_{-0.014}$	$\Omega_{de} = 0.673^{+0.008}_{-0.007}$	$\omega = -0.969^{+0.031}_{-0.041}$	$\eta = 1.004^{+0.013}_{-0.010}$
DGP	$h = 0.601^{+0.004}_{-0.006}$	$\Omega_m = 0.367^{+0.004}_{-0.006}$		
αDE	$h = 0.663^{+0.007}_{-0.008}$	$\Omega_m = 0.326^{+0.008}_{-0.008}$	$\alpha = 0.106^{+0.140}_{-0.111}$	

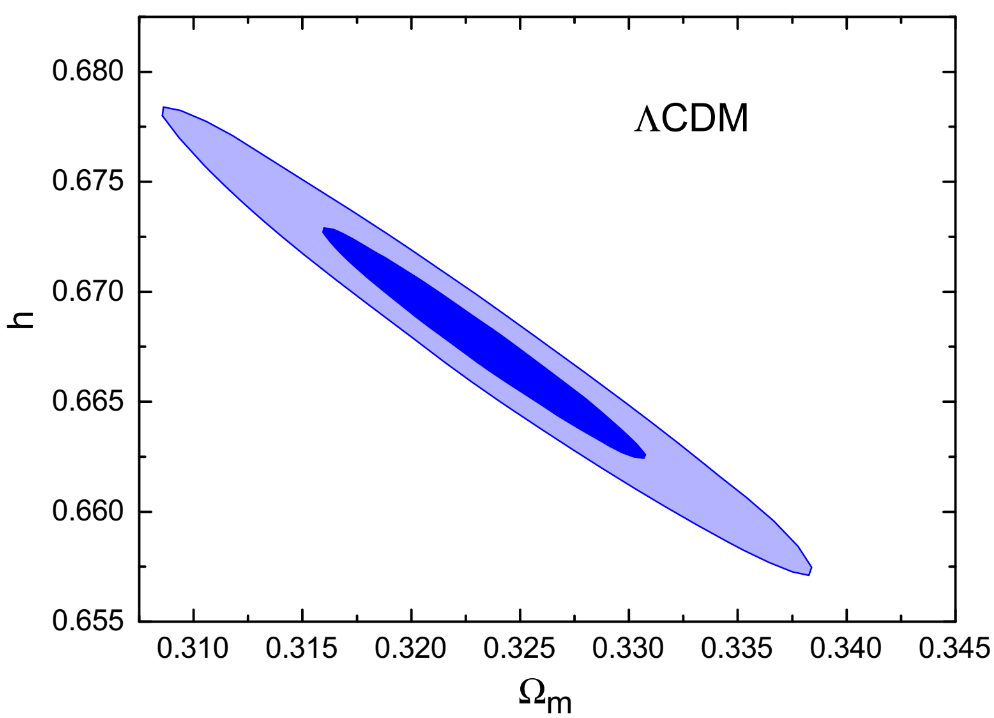
And for the minimum χ^2 values, we have the table below. notice that AIC and BIC values are some other methods to calculate deviation from our data sets. We put the ΛCDM model as a reference and calculate ΔAIC and ΔBIC values of each model compared to it. The rows are sorted from the smallest χ^2 value to the largest:

Model	χ^2_{min}	ΔAIC	ΔBIC
$NGCG$	698.331	2.956	12.191
GCG	698.381	1.006	5.623
$XCDM$	698.524	1.149	5.766
CPL	698.543	3.199	12.401
αDE	698.574	1.199	5.816
ΛCDM	699.375	0	0
DGP	786.326	86.951	86.951

(16)



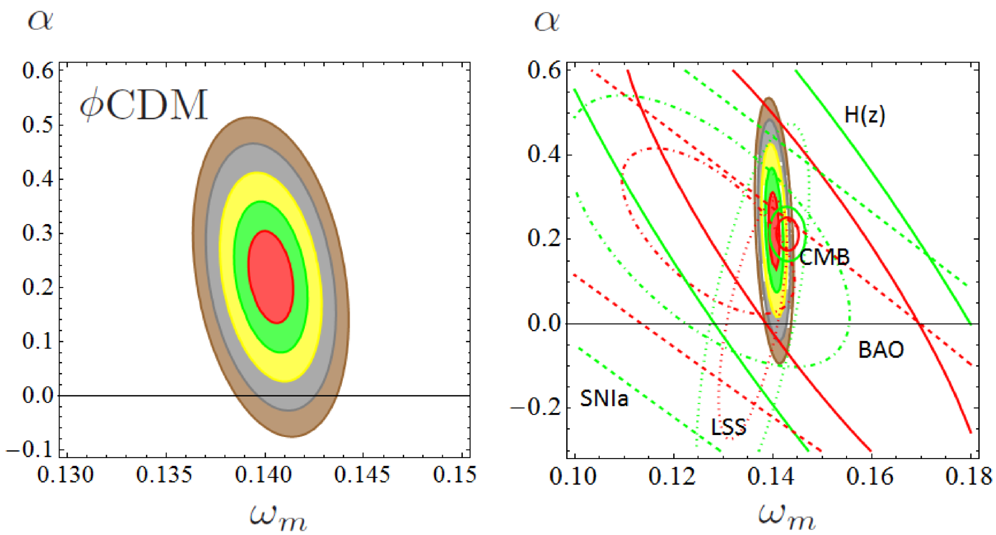
Here we have the h-Ω_m graph for the ΛCDM model to use as a reference:



The center of 1σ region gives $h = 0.667, \Omega_m = 0.324$ with $\chi^2_{min} = 699.375$.

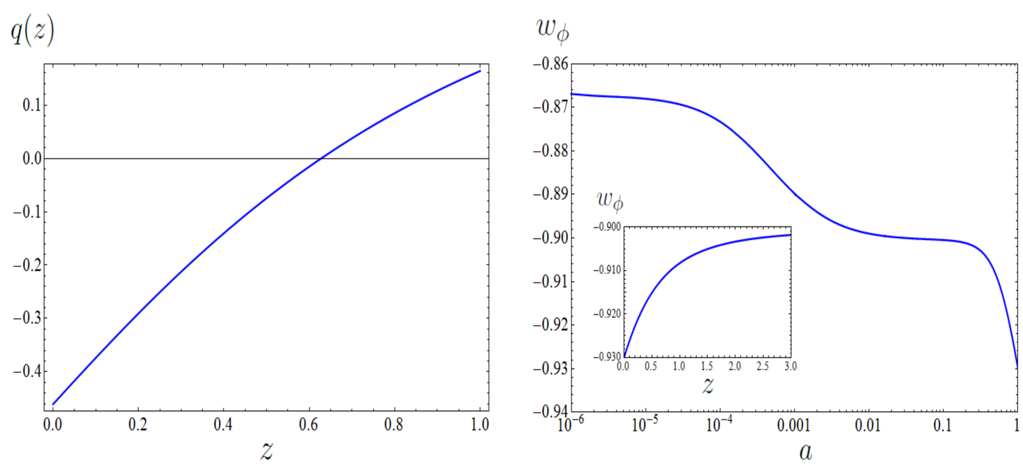
3.2 The Scalar Field Model ϕCDM:

If we fit our equations on observational data sets, we have:



Model	$\Omega_m h^2$	α	$\bar{\kappa}$
ϕCDM	0.1403 ± 0.0008	0.219 ± 0.057	$(325 \pm 1.1) \times 10^3$

Now we draw the deceleration parameter q(z) around $z_{tr} \sim 0.628$ where $q(z_{tr}) = 0$, the graph with participation of ϕ looks like below:



As expected, the ω_ϕ decreases as scale factor grows, therefore our field density tends to remain constant at late times and become the dominant factor of the expansion of the universe. We can calculate that at present $z = 0$,

$$\omega_\phi = -0.931 \pm 0.017$$

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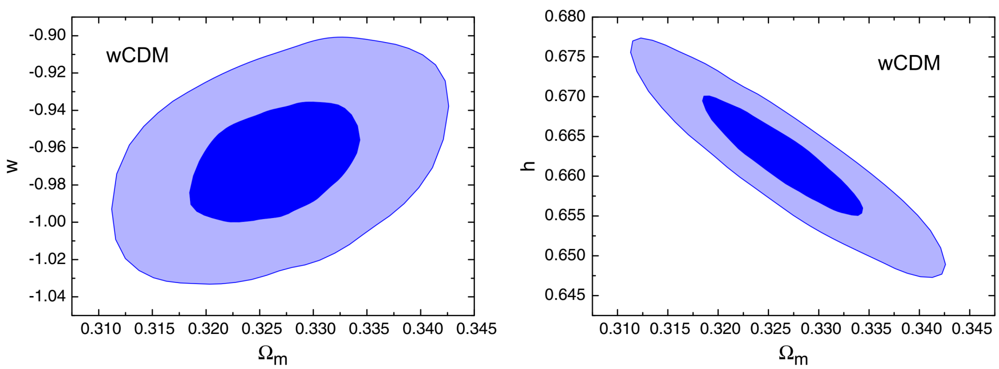
3.3 The Equation of State Parameterized Models:

3.3.1 The XCDM model:

for XCDM model (or as some people might call it, ωCDM model, because of the constant ω), the $\omega - \Omega_m$ and the h-Ω_m graphs look as below:



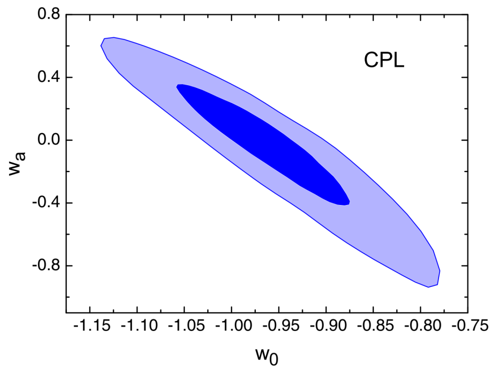
And the center of graphs give us the following values:

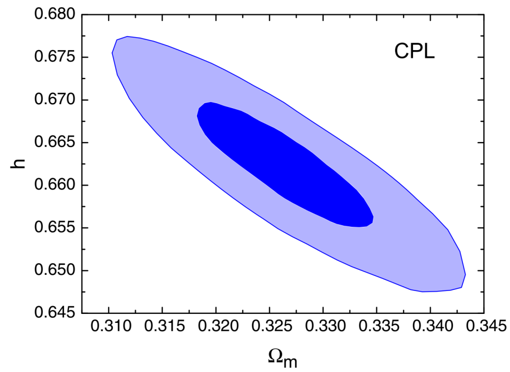
$$h = 0.662, \quad \Omega_m = 0.326, \quad \omega_0 = -0.964,$$

$$\chi^2_{min} = 698.524, \quad \Delta AIC = 1.149, \quad \Delta BIC = 5.766$$

3.3.2 The CPL model:

The CPL model has an extra degree of freedom ω_1, so the graphs look different than the XCDM model. Here we have $\omega_a - \omega_0$ and $h - \Omega_m$ graphs:





From center points of the 1σ regions of these graphs, we get:

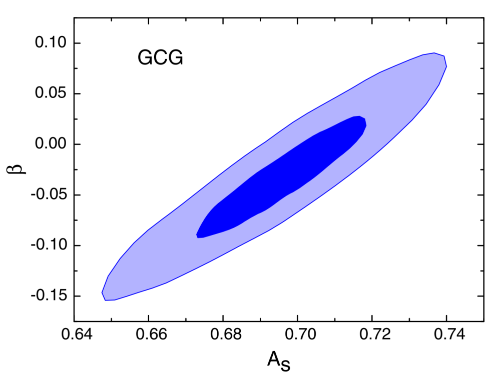
$$h = 0.663, \quad \Omega_m = 0.326, \quad \omega_0 = -0.969, \quad \omega_a = 0.007,$$

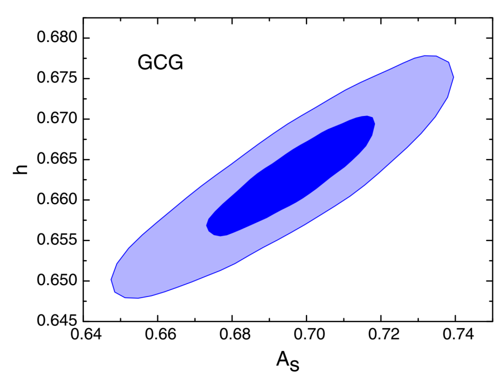
$$\chi^2_{min} = 698.543, \quad \Delta AIC = 3.199, \quad \Delta BIC = 12.401$$

3.4 The Chaplygin Gas models:

3.4.1 The CGC model:

This model, unlike the prior ones, doesn't provide an Ω_m value, because it doesn't include Dark Matter in the Matter-like part of the universe:





Which give us the following values for the model's free parameters:

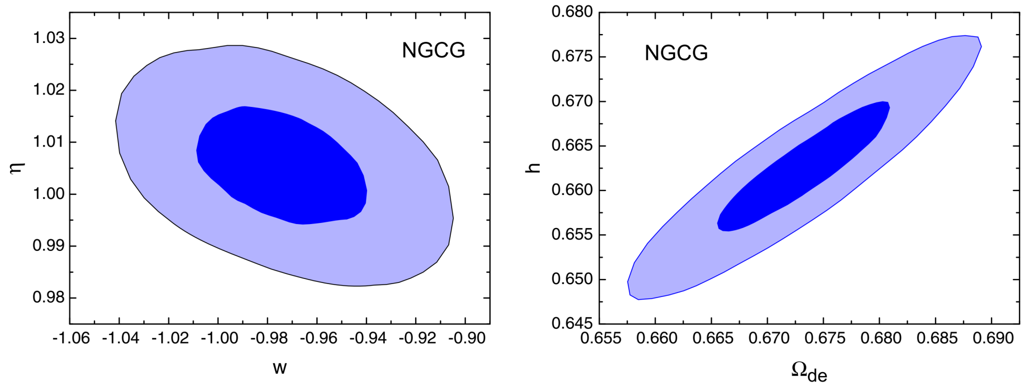
$$h = 0.663, \quad A_s = 0.695, \quad \beta = -0.03,$$

$$\chi^2_{min} = 698.381, \quad \Delta AIC = 1.006, \quad \Delta BIC = 5.623$$

Which fits our data slightly better than the prior models.

3.4.2 The NGCG model:

For the new GCG model, we have:



Where we defined $\eta \equiv 1 + \beta$. From these graphs, we get:

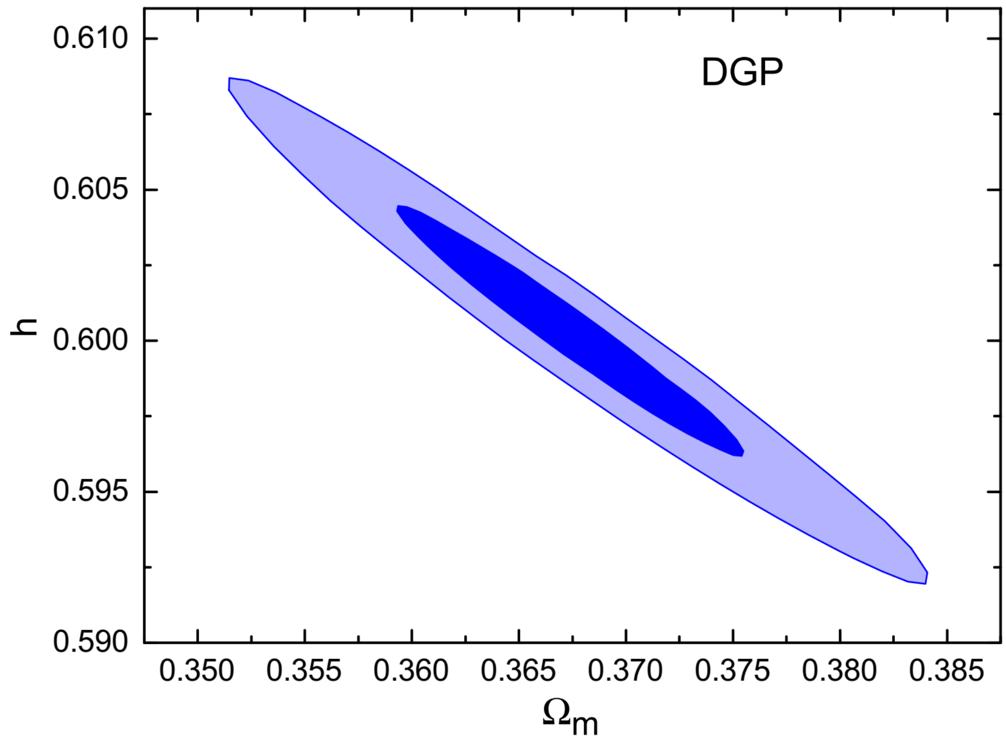
$$h = 0.662, \quad \Omega_{de} = 0.673, \quad \omega = -0.969, \quad \eta = 1.004,$$

$$\chi^2_{min} = 698.331, \quad \Delta AIC = 2.956, \quad \Delta BIC = 12.191$$

3.5 The DGP model and its extension:

3.5.1 The DGP model:

The DGP model only has two free parameters, which in the first sight, is a good thing; but it's lack of free parameters causes a seemingly large deviation from data, and from the ΛCDM model:



From this graph, we get:

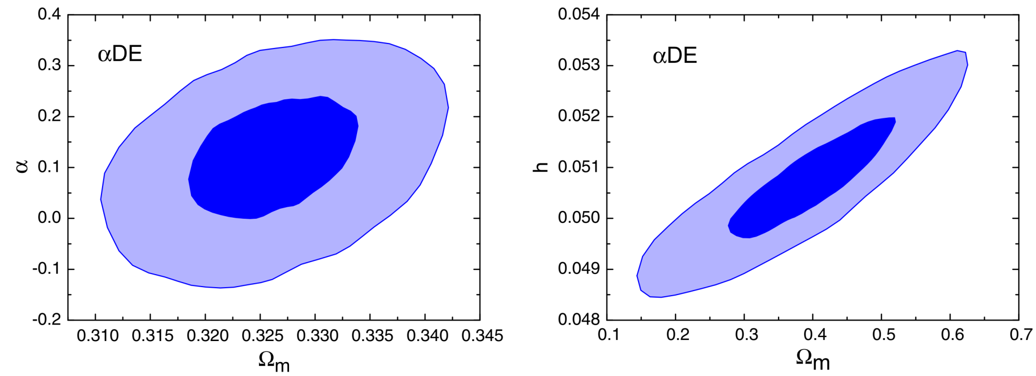
$$h = 0.601, \quad \Omega_m = 0.367$$

$$\chi^2_{min} = 786.326, \quad \Delta AIC = 86.951, \quad \Delta BIC = 86.951$$

The second row shows that because $\Delta AIC, \Delta BIC \gg 10$ and the χ^2_{min} value is seemingly large compared to the other models, this model is not a good candidate for replacing ΛCDM.

3.5.2 The αDE model:

This model although, by adding an extra free parameter α, has corrected the DGP model's results by far:



The center of 1σ region of these graphs, give us:

$$h = 0.663 \Omega_m = 0.326 \alpha = 0.106$$

$$\chi^2_{min} = 698.574, \quad \Delta AIC = 1.199, \quad \Delta BIC = 5.816$$

Which unlike the DGP, is comparable to the prior suggested models.

4 Discussion and conclusions:

In summary, all of the models above (except the DGP) have accomplished a slightly better result than the current model. Although the difference from ΛCDM is not much, but it can be considered as a promising fact; because large deviations from ΛCDM would result in considerable drift from original data, and therefore they lose their validity.

Among all the suggested models, The CGC models (2.3) have shown the best fits for our data sets (smallest χ^2_{min} values).
The GCG (2.3.2), The XCDM (2.2.1) and the αDE (2.4.2) have result in smallest ΔAIC and ΔBIC values; Which roughly means that they have the least drifts from ΛCDM compared to the other models.

The CGC model has more parameters than The ΛCDM, and although it makes it more suitable and flexible to fit our data, it also makes it more complex to use or calculate more precisely and also harder to find implications and calculate other cosmological quantities from it; Hence the ΛCDM is yet not officially replaced by any other model.

5 References:

Comparison of dark energy models after Planck 2015

Vacuum Energy in Quantum Field Theory and Cosmology

And some useful websites:

https://www.science.org/content/article/dark-energy-supermassive-black-holes-phys

https://www.space.com/25238-dark-energy-quantum-vacuum-theory.html

https://physics.stackexchange.com/questions/678536/vacuum-energy-and-dark-matter

https://www.dictionary.com/browse/vacuum-energy

https://en.wikipedia.org/wiki/Vacuum_energy

https://revolutionized.com/what-is-vacuum-energy-really/

https://www.scientificamerican.com/article/follow-up-what-is-the-zer/

https://en.wikipedia.org/wiki/Dark_energy

https://science.nasa.gov/astrophysics/focus-areas/what-is-dark-energy

https://societyofmodernastronomy.wordpress.com/2014/07/14/vacuum-energy-as-a-cand

https://www.space.com/25238-dark-energy-quantum-vacuum-theory.html

https://www.science.org/content/article/dark-energy-supermassive-black-holes-phys

https://en.wikipedia.org/wiki/Cosmological_constant

https://www.scientificamerican.com/article/the-cosmological-constant-is-physics-m

https://en.wikipedia.org/wiki/Cosmological_constant_problem

https://www.sciencedirect.com/science/article/abs/pii/S1355219802000333

https://link.springer.com/article/10.1140/epjc/s10052-018-5621-7

https://philsci-archive.pitt.edu/398/1/cosconstant.pdf

https://www.researchgate.net/publication/355219447_BD-LambdaCDM_and_Running_Vacuum_Models_Theoretical_background_and_current_observational_status

https://www.researchgate.net/publication/271140451_The_barLambdarm_CDM_cosmology_From_inflation_to_dark_energy_through_running_L

https://arxiv.org/abs/1612.02449

https://link.springer.com/article/10.1140/epjc/s10052-019-7600-z