٤٠٢١٣٣٤٠١١٩٠٣٩ $x[n-77]$

«بنام خدا»

مهندس صالح

«درس سیگنال ها و سیستم ها»

تمرینات سری دوم فصل ۲

۱- الف- $y_1[n] = h[-1]x[n+1] + h[n]x[n-1] = 2x[n-1]$

$y_1[n] = 2\delta[n+1] + 4\delta[n] + 2\delta[n-1] + 2\delta[n-2] - 2\delta[2-4]$

ب- $y_2[n] = x[n+2] * h[n] = \sum_{k=-n}^{\infty} h[k]x[n+2-k]$

$y_2[n] = y_1[n+2]$

ج- $y_1[n] = x[n] * h[n] = \sum_{k=\infty}^{\infty} x[k]h[n-k]$ $\qquad y_3[n] = y[n+2]$

۲- $h[k] = (\frac{1}{4})^{k-1}\{u[k+3] - u[k-10]\}$ $\qquad A = n-9 \quad B = n+3$

۳- $x_1[n] = (\frac{1}{2})^n u[n] \qquad h[n] = u[n] \qquad y[n] = x[n] * h[n]$

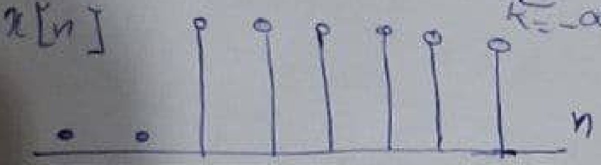
$h[n] = h_1[n+2]$ و $x[n] = x_1[n-2]$ $\qquad = x_1[n-2] * h[n+2]$

$y[n] = \sum_{-\infty}^{\infty} x_1[m]h_1[n-m] = x_1[n]x_1[n * h_1[n] \sum_{k=-\infty}^{\infty} x_1[k-2]h_1[n-k+2]$

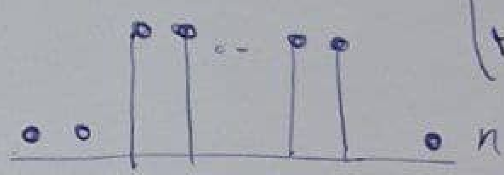
$y[n] = 2[1-(\frac{1}{2})^{n+1}]u[n]$

٤- $y[n] = \cancel{x[n] * h[n]} = \sum_{k=-\infty}^{\infty} [k]h[n-k]$

$x[n]$



$$y[n] = \begin{cases} n-6 & 7 \le n \le 11 \\ 6 & 12 \le n \le 18 \\ 24-n & 19 \le n \le 24 \\ 0 & \text{سایر نقاط} \end{cases}$$



ه- $y[n] = \sum_{k=0}^{9} x[k]h[n-k] = \sum_{k=0}^{9} h[n-k]$

۶-

۷- الف)

$$x[n]=\delta[n-2]$$

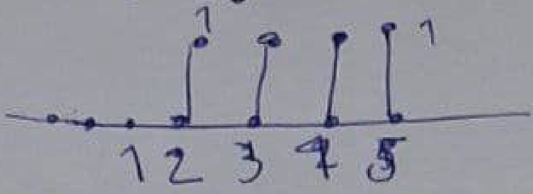
$$y[n]=\sum_{-\infty}^{+\infty} x[k]g[n-2k]=g[n-4]=u[n-4]-u[n-8]$$

ب)

$$y[n]=\sum_{-\infty}^{+\infty} x[k]g=\sum_{k=0}^{\infty} g[n-2k]$$

$$y[n]=\begin{cases}1 & n=0,1\\ 2 & n>=2u[n]-\delta[n]-\delta[n-1]\\ 0\end{cases}$$

$g[n-1]$



$g[n-4]$

$g[n]$

۸-

سوال ←

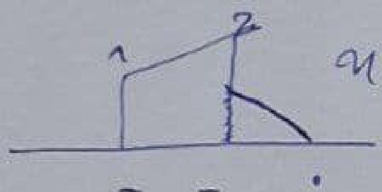
$$x(t)=\begin{cases}t+1, & 0\leq t\leq 1\\ 2-1, & 1\leq t\leq 2\\ 0 & \text{در غیر این صورت}\end{cases}$$

$$h(t)=\delta(t+2)+2\delta(t+1)$$

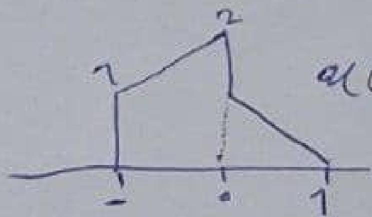
حل :

$$x(t)*h(t)=\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau=\int_{-\infty}^{+\infty} h(\tau)x(t-\tau)d\tau$$

$$x(t)*y(t)=x(t+2)+2x(t+1)$$



$x(t+2)$



$x(t+1)$

$$y(t)=\begin{cases}t+3 & -2<t\leq -1\\ t+4 & -1t<0\\ 2-2t & 0<t\leq 1\\ 0\end{cases}$$

b

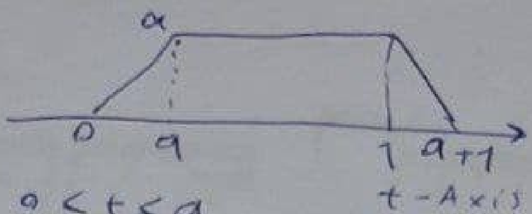
$$h(\tau)=e^{2\tau}u(-\tau+4)+e^{-2\tau}u(\tau-5)=\begin{cases}e^{-2\tau} & \tau>5\\ e^{2\tau} & \tau>4\\ 0 & 4<\tau<5\end{cases}$$ ٩

$$h(-\tau)=\begin{cases}e^{2\tau} & \tau>-5\\ e^{-2\tau} & \tau>-4\\ 0 & -5<\tau-4\end{cases}$$

$A=t-5$,

$B=t-4$

١٠



$$y(t)=\begin{cases}t & 0<t<a\\ a & a<t<1\\ 1+a-t & 1<t<1+a\\ 0 & \text{سایر نقاط}\end{cases}$$

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١١ الف)

$$y(t)=x(t)*h(t)=\int_{-\infty}^{+\infty}h(\tau)x(t-\tau)\,d\tau$$
$$=\sum_{0}^{\infty}e^{-3\tau}\left(u(t-\tau-3)\right)-u(t-\tau-5)\,d\tau$$

$$y(t)=\int_0^{t-3}e^{-3\tau}\,d\tau=\frac{1-e^{-3(t-3)}}{3}$$

$t>5$

$$y(t)=\int_{t-5}^{t-3}e^{-3\tau}\,d\tau=\frac{(1-e^{-5})e^{-3(t-5)}}{3}$$

$$y(t)\begin{cases}0 & -\infty<t<3\\ \frac{1-e^{-3(t-3)}}{3} & 3>t\le 5\\ \frac{(1-e^{-5})e^{-3(t-5)}}{3} & 5<t\le\infty\end{cases}$$

ب)

$$\frac{dx(t)}{dt}=\delta(t-3)-\delta(t-5)\longrightarrow g(t)=\frac{dx(t)}{dt}*h(t)$$

$$=e^{-3(t-3)}u(t-3)-^{-}(t-5)\,u(t-5)$$

$$\frac{dy(t)}{dt} = \begin{cases} 0 & -\infty < t < 3 \\ e^{-3(t-3)} & 3 < t \leq 5 \\ (e^{-6}-1)e^{-3(t-5)} & 5 < t \leq \infty \end{cases}$$ (ج

بنابراین = $g(t) = \frac{dy(t)}{dt}$

-12 سوال ← $y(t) = e^{-t}u(t) * \sum_{k=-\infty}^{\infty} \delta(t-3k)$

$y(t) = \cdots + e^{-(t-6)} u(t+6) + e^{-(t+3)} u(t+3) + e^{-t} u(t) + e^{-(t-3)} u(t-3) + e^{-(t-6)} u(t-6) + \cdots$ اصل

$y(t) = \cdots + e^{-(t+6)} u(t+6) + e^{-(t+3)} u(t+3) + e^{-t}u(t) = e^{-t} + e^{-(t+3)} + e^{-(t+6)} + \cdots = e^{-t}(1 + e^{-3} + e^{-6} + \cdots) = e^{-t}\frac{1}{1-e^{-3}}$

بنابراین: $A = \frac{1}{1-e^{-3}}$

-13 الف) $\left(\frac{1}{5}\right)^n u[n] - A\left(\frac{1}{5}\right)^{n-1} u[n-1] = \delta[n]$

$A = \frac{1}{3} \leftarrow n = 1$

ب) $h[n] - \frac{1}{5}h[n-1] = \delta[n] \Rightarrow h[n] * \left(\delta[n] - \frac{1}{5}\delta[n-1]\right) = \delta[n]$

$g[n] = \delta[n] - \delta[n] - \frac{1}{2}\delta[n-1]$

-14 الف) $\int_{-\infty}^{+\infty} |h_1(\tau)| d\tau = \int_0^{\infty} e^{-\tau} d\tau = 1$

ب) $\int_{-\infty}^{+\infty} |h_2(\tau)| d\tau = \int_0^{\infty} e^{-t} |\cos 2t| d\tau$

-15 الف) $\sum_{k=-\infty}^{\infty} |h_1[k]| = \sum_{k=0}^{\infty} k \left|\cos\left(\frac{\pi}{4}k\right)\right|$

ب) $\sum_{k=-\infty}^{+\infty} |h_2[k]| = \sum_{k=-\infty}^{10} 3^k \cong \frac{3^{11}}{2}$