Machine Learning Course

Linear Regression Assignment Solution

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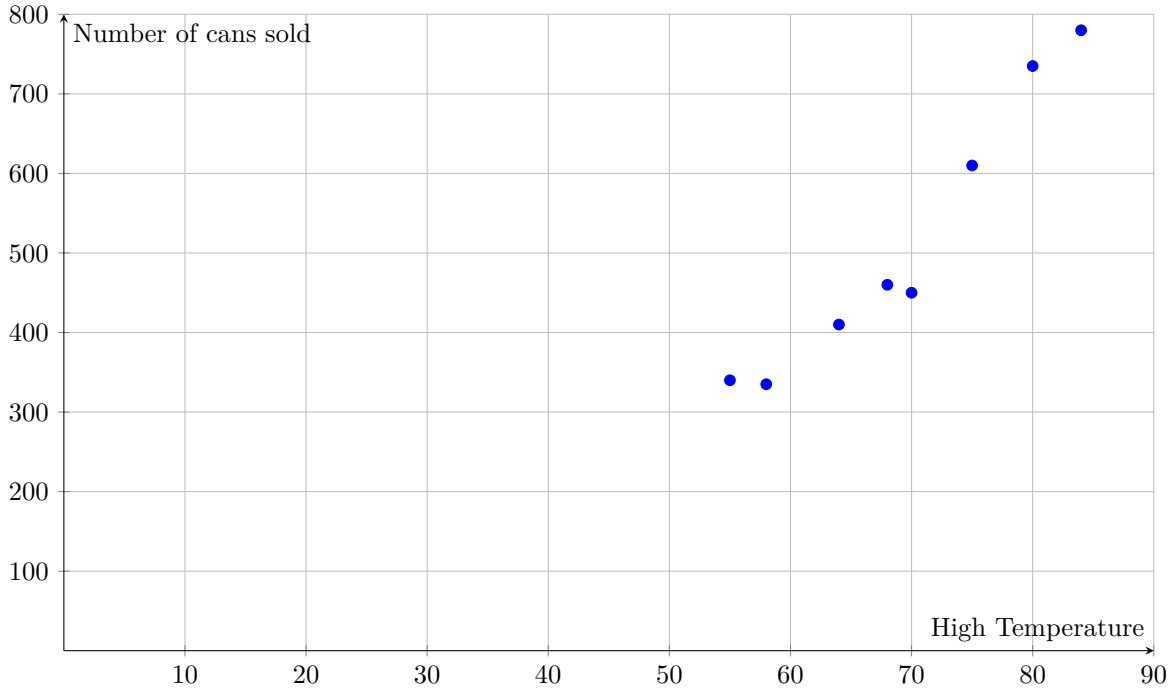
Question 1

0.1 Scatter Plot



0.2 Equation and Plot of Linear Regression for the Data

To obtain the regression parameters:

$$\theta = (X^T X)^{-1} X^T \vec{y}$$

$$= \left(\begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 55 & 58 & 64 & 68 & 70 & 75 & 80 & 84 \end{pmatrix} \begin{pmatrix} 1 & 55 \\ 1 & 58 \\ 1 & 64 \\ 1 & 68 \\ 1 & 70 \\ 1 & 75 \\ 1 & 80 \\ 1 & 84 \end{pmatrix} \right)^{-1} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 55 & 58 & 64 & 68 & 70 & 75 & 80 & 84 \end{pmatrix} \begin{pmatrix} 340 \\ 335 \\ 410 \\ 460 \\ 450 \\ 610 \\ 735 \\ 780 \end{pmatrix}$$

$$= \begin{pmatrix} 8 & 554 \\ 554 & 39090 \end{pmatrix}^{-1} \begin{pmatrix} 4120 \\ 297220 \end{pmatrix}$$

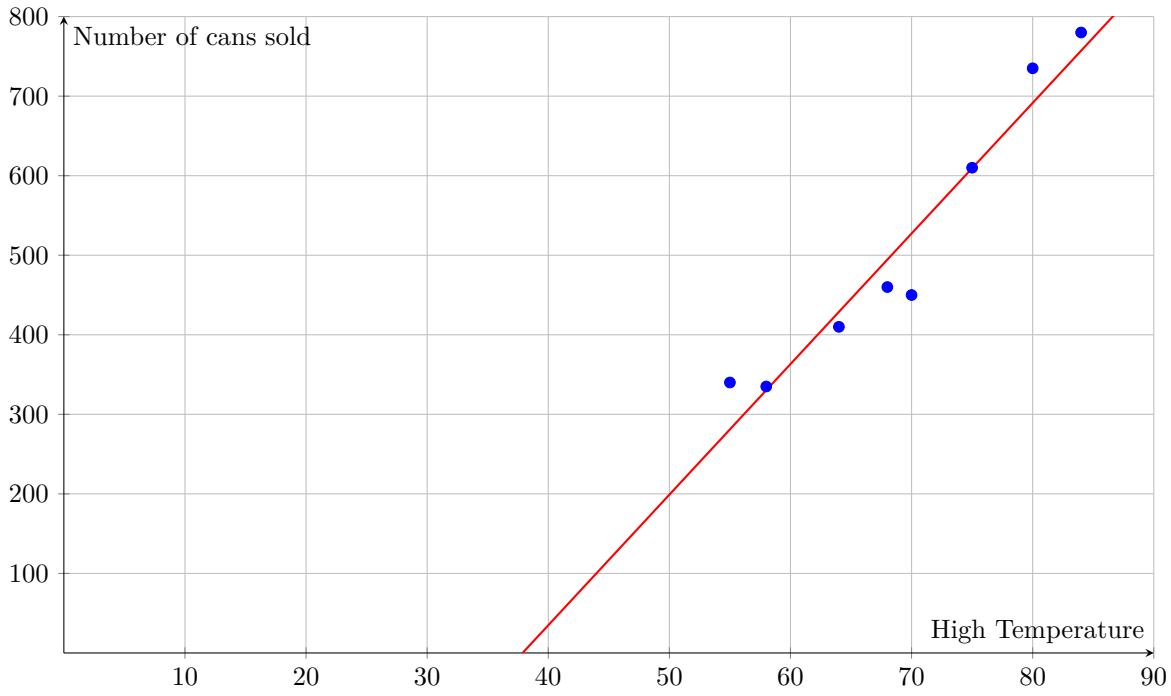
$$= \begin{pmatrix} 6.73501034 & -0.0954514128 \\ -0.0954514128 & 0.00137835975 \end{pmatrix} \begin{pmatrix} 4120 \\ 297220 \end{pmatrix}$$

$$= \begin{pmatrix} -621.82632667 \\ 16.41626465 \end{pmatrix}$$

Thus, the linear equation is:

$$y = -621.82632667 + 16.41626465x$$

And plotted as:



0.3 Prediction with the Obtained Model

For $x = 95°F$:

$$y = \begin{pmatrix} 1 & x \end{pmatrix} \begin{pmatrix} -621.82632667 \\ 16.41626465 \end{pmatrix}$$
$$= 937.71881508$$

0.4 Inverse Model Prediction

The equation is:

$$\theta_0 + \theta_1 x = 95 \rightarrow x = \frac{95 - \theta_0}{\theta_1}$$

From which the temperature is calculated:

$$x = \frac{95 - (-621.82632667)}{16.41626465} \rightarrow x = 43.6656$$

Question 2

0.5 Problem 1

The problem is Classification because the output is discrete.

0.6 Problem 2

The problem is Regression because we aim to examine the impact of features on a continuous target variable.

0.7 Problem 3

The problem is Classification because we predict class 0 or 1 for the specified disease.

Question 3

The second hyperparameter setting is chosen because it has the lowest training and test errors, with zero difference between them, indicating no overfitting.

Question 4

Here, the independent variable is y, and the dependent variable is x. Thus, we need to find the weights for the equation:

$$x = \theta_0 + \theta_1 y$$

We know:

$$\begin{aligned} \theta &= (\vec{y}^T \vec{y})^{-1} \vec{y}^T \vec{x} \\ &= \begin{pmatrix} 1.09549706 \\ 0.92451599 \end{pmatrix} \end{aligned}$$

The final model is:

$$x = 1.09549706 + 0.92451599y$$