Aperiodic deformation of structures

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1 Introduction

This exercise focuses on the non-periodic deformation of structures such as dams or barrages using the congruence model. The analysis is performed using two methods: pointwise analysis and global analysis. These structures are subject to geodetic monitoring at regular intervals due to their significant potential for hazard. This monitoring aims to detect any structural failures that may occur.

In the upcoming analysis, we will examine the deformation characteristics of a particular structure, specifically focusing on points 901-905 (depicted in Figure 1). These points are situated on the uppermost part of the structure's wall, and their positions are determined through precise tachymetric measurements using fixed reference points from the monitoring network.

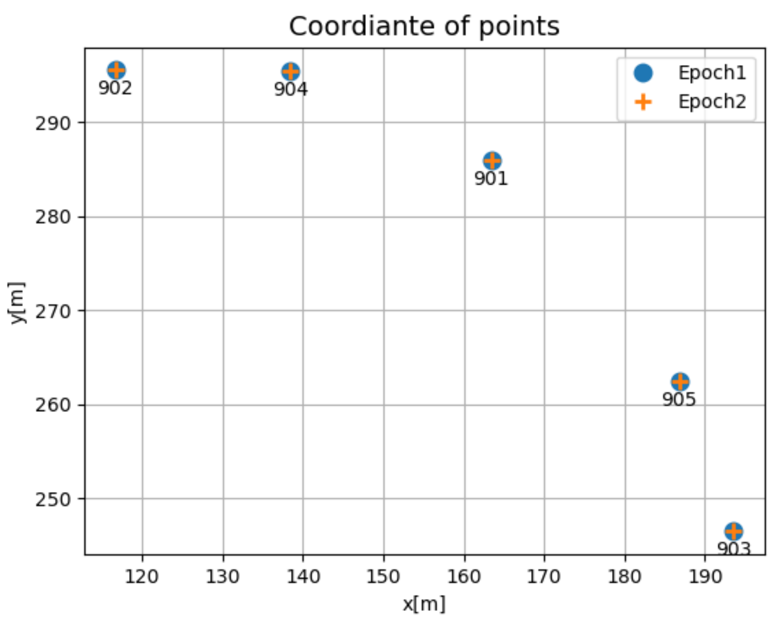

Figure 2: This plot illustrates the positions of points on a wall crown at two different epochs

To assess the deformation behavior, we have data from two measuring epochs, each separated by a time interval of 2 years. For each epoch, we have sets of coordinates along with their respective cofactor matrices. Additionally, we have the standard deviation of the unit weight, which is 1.08 mm for the first epoch and 1.01 mm for the second epoch. Both adjustments were performed with a number of degrees of freedom of 146.

2 Pointwise Analysis

2.1 Preparation

$$\hat{x}_1 = \begin{bmatrix} \hat{y}_{0901} \\ \hat{x}_{0901} \\ . \\ . \\ . \\ \hat{x}_{0905} \end{bmatrix} \rightarrow Epoch1 \quad \hat{x}_2 = \begin{bmatrix} \hat{y}_{0901} \\ \hat{x}_{0901} \\ . \\ . \\ . \\ \hat{x}_{0905} \end{bmatrix} \rightarrow Epoch2$$

$$\Sigma_{xx,1} = \hat{\sigma}_{0,1} Q_{xx,1} \quad \Sigma_{xx,2} = \hat{\sigma}_{0,2} Q_{xx,2}$$

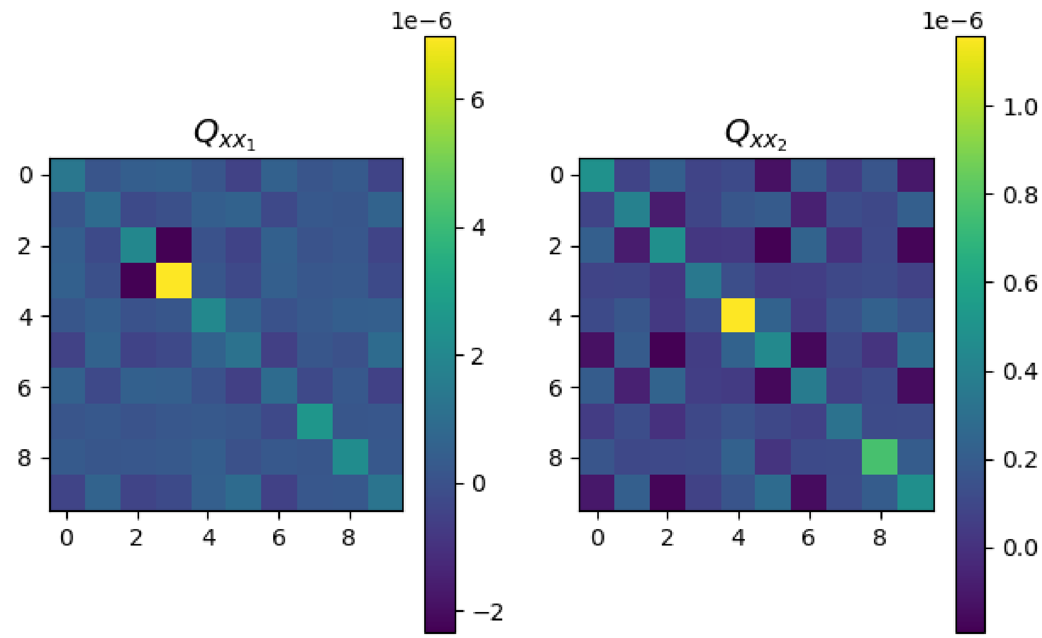

Figure 3: This plot illustrates the covarince-variance matrix on a wall crown at two different epochs

2.2 Calculations

Calculate the coordinate shift between the two epochs for all points and the corresponding VCM:

$$\hat{d}_i = \begin{bmatrix} \hat{y}_i \\ \hat{x}_i \end{bmatrix}_2 - \begin{bmatrix} \hat{y}_i \\ \hat{x}_i \end{bmatrix}_1 \quad with\ i = 901, 902, 903, 904, 905$$

$$\Sigma_{dd} = \Sigma_{xx,1} + \Sigma_{xx,2}$$

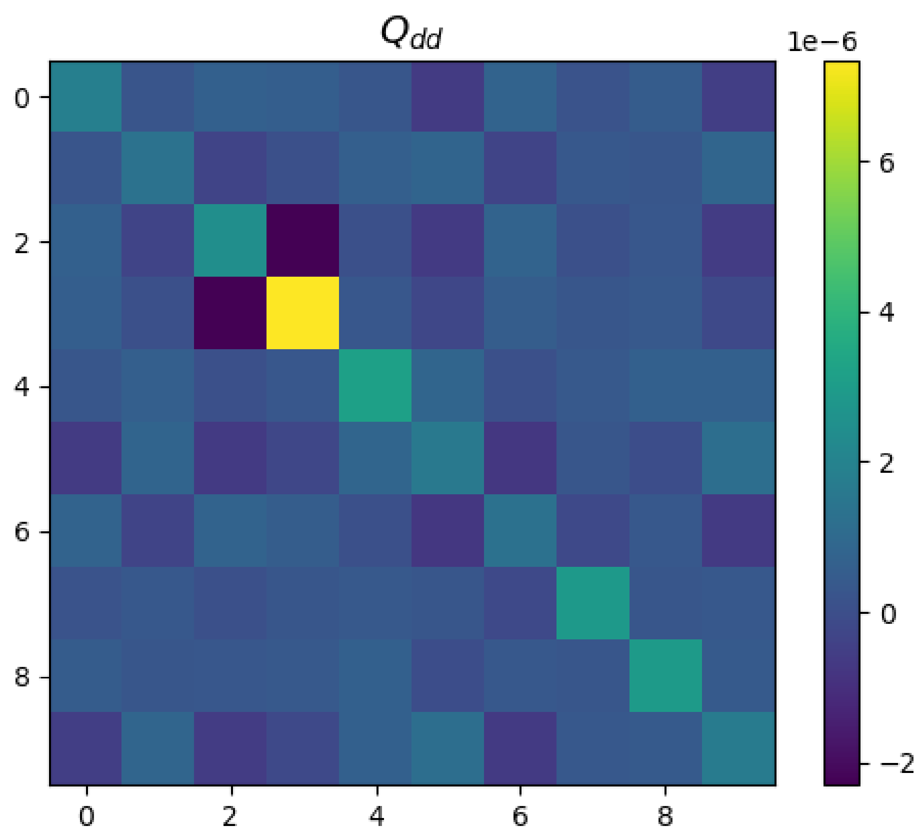

Figure 4: This plot illustrates difference the covarince-variance matrix

2.3 Significance test

The zero hypothesis is that the transition between two datum is 0.

$$H_0 = E[\hat{d}_i] = 0 \quad H_A = E[\hat{d}_i] \neq 0$$

Test value

$$\hat{F} = d_i^T.\Sigma_{d_i d_i}^{-1} d_i \sim 2F_{f_1+f_2,1-\alpha}$$

Pointwise Analysis hypothesis

T_i	$2F_{f_1+f_2,1-\alpha}$	
1.194		
0.733		
0.158	6.053	accepted
0.614		
0.881		

Based on the significance test value of 6.0533, and considering that the value of Ti is less than the statistic test value, we can accept the first hypothesis. This implies that there is no significant shift or difference between the two epochs. In other words, there is no observable change or deformation between the two measurement epochs.

We will perform a transformation to a local coordinate system where the y-axis passes through points 902 and 903. Additionally, we will reduce the coordinates by shifting them to the center of mass of points 902 and 903, making the origin the midpoint of these two points.

2.4 Set a local coordinate system in two steps

2.4.1 Rotate $\hat{x}_{rot}$

$$t_{902}^{903} = tan^{-1}(\frac{y_{903}-y_{902}}{x_{903}-x_{902}}) \rightarrow \epsilon = t_{902}^{903} - \frac{\pi}{2}$$

$$R_z = \begin{bmatrix} cos(\epsilon) & sin(\epsilon) \\ -sin(\epsilon) & cos(\epsilon) \end{bmatrix} \rightarrow \bar{R} = \begin{bmatrix} R_z & 0 & 0 \\ 0 & ... & 0 \\ 0 & 0 & R_z \end{bmatrix}$$

$$\text{Rotated coordinates from the 1st epoch and their VCM} = \begin{cases} \hat{x}_{rot,1} = \bar{R}\hat{x}_1 \\ \Sigma_{xx,rot,1} = \bar{R}\Sigma_{xx,1}\bar{R}^T \end{cases}$$

$$\text{Rotated coordinates from the 2nd epoch and their VCM} = \begin{cases} \hat{x}_{rot,2} = \bar{R}\hat{x}_2 \\ \Sigma_{xx,rot,2} = \bar{R}\Sigma_{xx,2}\bar{R}^T \end{cases}$$

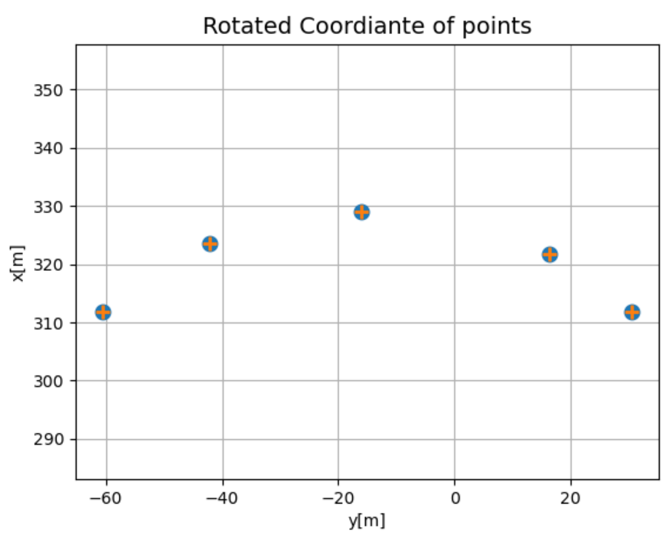

Figure 5: This plot illustrates the positions of Rotated points on a wall crown at two different epochs

3 Global Analysis

3.0.1 Reduce coordinates by the midpoint x_μ

Rotated and Reduced coordinates (ζ) for the two epochs

$$\hat{\xi}_1 = G\hat{x}_{rot,1} = \begin{bmatrix} \hat{\eta}_{0901,1} \\ \hat{\xi}_{0901,1} \\ . \\ . \\ . \\ \hat{\xi}_{0905,1} \end{bmatrix} \rightarrow Epoch1 \quad \hat{\xi}_2 = \hat{x}_{rot,2} - G_1\hat{x}_{rot,1} = \begin{bmatrix} \hat{\eta}_{0901,2} \\ \hat{\xi}_{0901,2} \\ . \\ . \\ . \\ \hat{\xi}_{0905,2} \end{bmatrix} \rightarrow Epoch2$$

$$\Sigma_{\zeta\zeta,1} = G\Sigma_{xx,rot,1}G^T \quad \Sigma_{\zeta\zeta} = \Sigma_{xx,rot,2} + (G_1\Sigma_{xx,rot,1}G_1^T)$$

$$\text{with: } = \begin{cases} G = I - G_1 \\ G_1 = \frac{1}{2}F^T F_1 \\ F = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \end{cases}$$

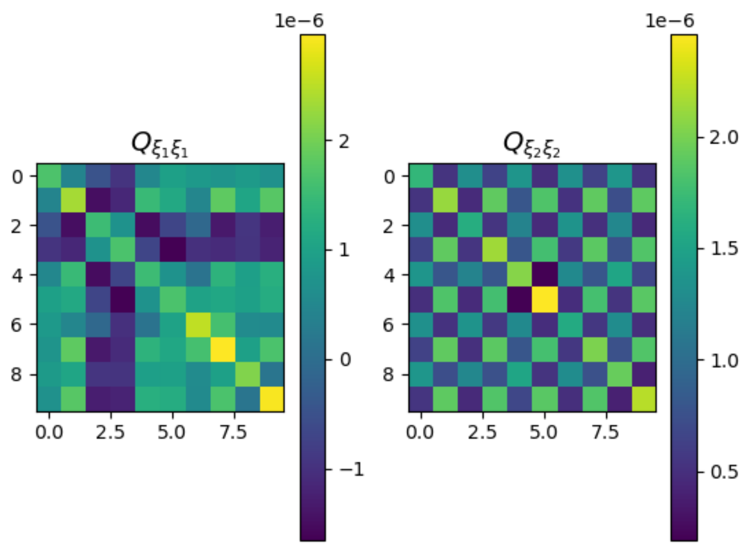

Figure 6: This plot illustrates the covarince-variance matrix $Q_{\xi\xi}$ epoch 1 and 2

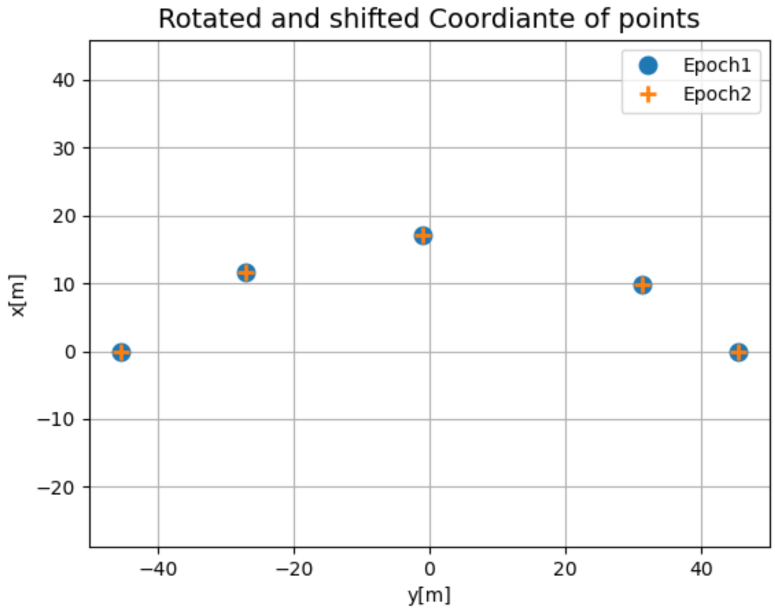

Figure 7: This plot illustrates the positions of rotated and shifted points on a wall crown at two different epochs

3.0.2 Calculate the difference of the rotated and reduced coordinates

$$d\xi = \xi_2 - \xi_1 = \begin{bmatrix} d\xi_{901} \\ d\eta_{901} \\ . \\ . \\ . \\ d\eta_{905} \end{bmatrix}$$

To evaluate the deformation, fit a geometric model to the differences (here a parabola)

$$\xi_{i,1}^2 = 2p(\eta_0 - d\eta_i)$$
$$d\eta_i = [1 - \xi_{i,1}^2] \begin{bmatrix} \eta_0 \\ q \end{bmatrix} \rightarrow q = \frac{1}{2p}$$

formulating as a Gauss-Markov-model :

$$p = (A^T \Sigma_{d\eta d\eta}^{-1} A)^{-1} A^T \Sigma_{d\eta d\eta}^{-1} d\eta$$
$$\Sigma_{pp} = (A^T \Sigma_{d\eta d\eta}^{-1} A)^{-1}$$

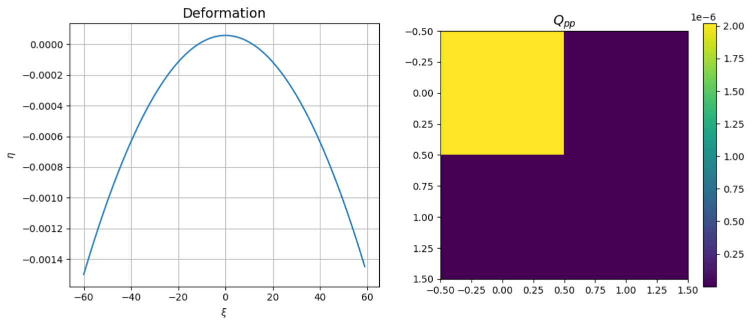

Figure 8: This plot illustrates the deformations and deformations parameters

Deformations parameters

η_0	q
5.76617e-05	-4.32774e-07

3.1 Significance test

The zero hypothesis is that the transition between two datum is 0.

$$H_0 = E[p] = 0 \quad H_A = E[p] \neq 0$$

Test value

$$\hat{F} = p^T \Sigma_{pp}^{-1} p \sim 2F_{2,3,95}$$

Global Analysis hypothesis

T	$2F_{2,3,95}$	
0.0104	19.104	accepted

Based on the calculated value of T (0.0104) and the significance test value (19.1041), since the value of T is less than the significance test value, we can accept the first hypothesis.

4 Conclusion

This suggests that there is no significant global deformation observed between the two epochs. In other words, there is no overall shift or change in the deformation behavior between the two measurement epochs.