Kinematic Kalman Filter

Professor : Dr. Mohmmad Omidalizarandi

St : AmirAbbas Saberi

University of Tehran

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1 Introduction

The motion of the point is explained within the framework of a descriptive kinematic model using recursive filtering. Typically, the kinematic Kalman filter is employed to depict the typical trajectory of the point's spatial and temporal displacement.

Figure 1 illustrates the point field of Hornbergl in Reutte, Sudtirol. This area experiences persistent landslides on the mountain. To monitor the landslide-prone region, an online monitoring system has been set up on points 1 and 2. These points are equipped with low-cost GPS receivers. For the remaining points within the hazardous area, their monitoring is carried out through periodic measurements utilizing tachymeter and GPS.

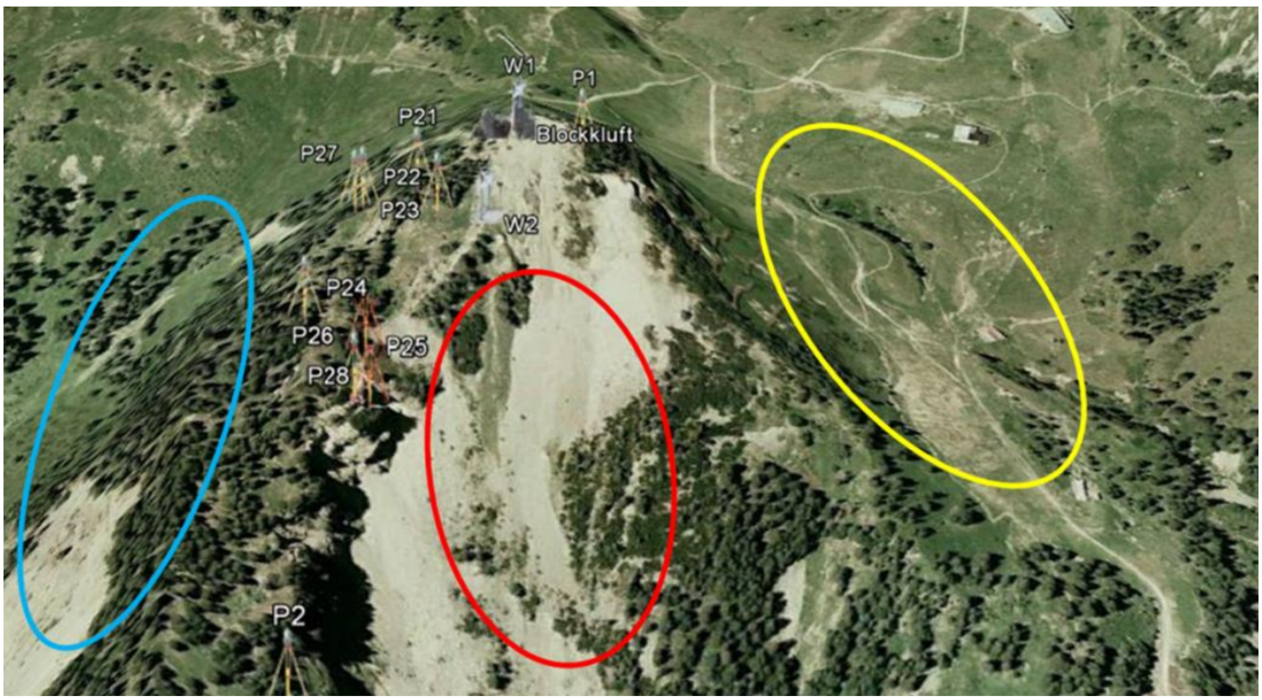

Figure 2: Overview from west of the Hornbergl, with the monitoring points and landslide zones

To perform the filtering process, the GPS coordinates of Points 1 and 6 are utilized. These coordinates are provided as Gauss-Krüger (GK) coordinates along with the corresponding ellipsoidal height. The data consists of measurements taken from 5 epochs, and each measurement is associated with its respective standard deviation.

The filtering is conducted using the "single point model," which corresponds to pointwise filtering. In this model, the a-posteriori standard deviation of the epochal adjustment is assumed to be $\hat{\sigma}_0 = 100$.

2 Task: Create the variance/covariance matrix, the matrix of the system noise and set the initail valuse

The initial value for the coordinates can be the coordinates of the first epoch, and for the speed, it can be obtained from the difference of the first two epochs, and this makes the value calculated by Freud to be the same as the real value, and finally, according to the physics of problem, which is a force It is not entered into the device. Acceleration was set to 0. Here, we took the initial value as follows:

$$y(t_0)_{P6,P1} = \begin{bmatrix} x_0 \\ y_0 \\ z_0 \\ v_{x0} \\ v_{y0} \\ v_{z0} \\ a_{x0} \\ a_{y0} \\ a_{z0} \end{bmatrix} = \begin{bmatrix} 23806.842 \\ 5260030.2 \\ 1791.863 \\ -0.016 \\ 0.0130000003 \\ -0.015 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 24127.55 \\ 5259334.014 \\ 1755.567 \\ 0.002 \\ 0.0299 \\ -0.03299 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

and :

$$y(t) = \Phi y(t_0) \rightarrow \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} = \begin{bmatrix} I_{3\times3} & \Delta t I_{3\times3} & 0.5 * \Delta t^2 I_{3\times3} \\ 0 & I_{3\times3} & \Delta I_{3\times3} \\ 0 & 0 & I_{3\times3} \end{bmatrix} \begin{bmatrix} x(t_0) \\ \dot{x}(t_0)\ddot{x}(t_0) \end{bmatrix}$$

The full variance covariance matrix of the initial state can be assembled as:

$$\Sigma_{yy} = \begin{bmatrix} \Sigma_{xx} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & \Sigma_{\dot{x}\dot{x}} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & \Sigma_{\ddot{x}\ddot{x}} \end{bmatrix} = \begin{cases} \Sigma_{xx} = \begin{bmatrix} \sigma_{xx}^2 & 0 & 0 \\ 0 & \sigma_{yy}^2 & 0 \\ 0 & 0 & \sigma_{zz}^2 \end{bmatrix} \\ \Sigma_{\dot{x}\dot{x}} = 10^{-4} * I_{3\times3} \\ \Sigma_{\ddot{x}\ddot{x}} = 10^{-4} * I_{3\times3} \end{cases}$$

The disturbance vector and its variance covariance matrix:

$$w_{\Delta t_{[9\times3]}} = \begin{bmatrix} 0.5\Delta^2 I_{[3\times3]} \\ \Delta t I_{[3\times3]} \\ I_{[3\times3]} \end{bmatrix}, \Sigma_{ww} = \begin{bmatrix} 10^{-4} & 0 & 0 \\ 0 & 10^{-4} & 0 \\ 0 & 0 & 10^{-4} \end{bmatrix}$$

3 Calculation and assessment of resualt

Calculations include three parts: prediction, innovation, and filtering, which repeat an iterative process as follows.

3.1 Prediction

$$y_-^{(i+1)} = \Phi^{(i)} y^(i)_+ \rightarrow \text{predicted state}$$
$$\Sigma^(i+1)_{yy,-} = \Phi^{(i)} \Sigma_{yy,+}^{(i)} \Phi^{(i)^T} + w_{\Delta t_{[9\times 3]}}^{(i)^T} \Sigma_{ww} w_{\Delta t_{[9\times 3]}}^{(i)^T}$$

3.2 Innovation

$$i^{(i+1)} = l^{(i+1)} - A y_-^{(i+1)}$$
$$A = [I_{3\times 3} 0_{3\times 3} 0_{3\times 3}]$$
$$\Sigma_{ii}^{(i+1)} = \Sigma_{ll}^{(i+1)} + A \Sigma_{yy,-}^{(i+1)} A^T$$

3.3 Filtering

$$K^{(i+1)} = \Sigma_{yy,-}^{(i+1)} A^T (\Sigma_{ii}^{(i+1)})^{-1} \rightarrow \text{Kalman gain}$$
$$y_+^{(i+1)} = y_-^{(i+1)} + K^{i+1)} i^{(i+1)} \rightarrow \text{Filtered state}$$
$$\Sigma_{yy,+}^{(i+1)} = (I - K^{(i+1)} A) \Sigma_{yy,-}^{(i+1)} \rightarrow \text{VCM of filtered state}$$

After these three steps, the result obtained is:

3.3.1 Point 1

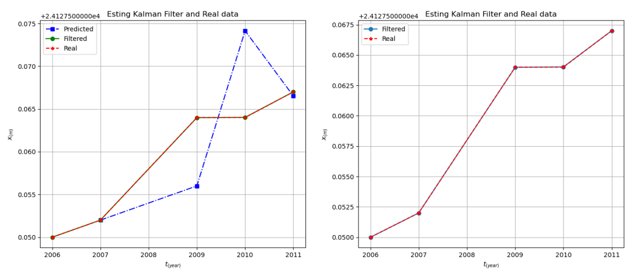

Figure 3: Esting Kalman Filter and Real data point1

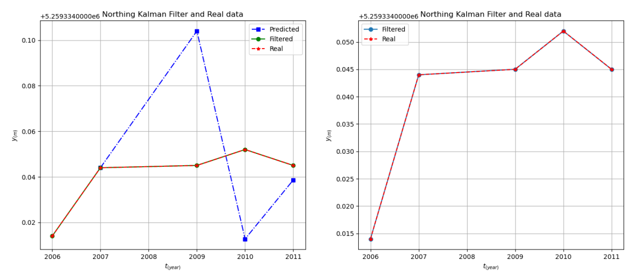

Figure 4: Northing Kalman Filter and Real data point1

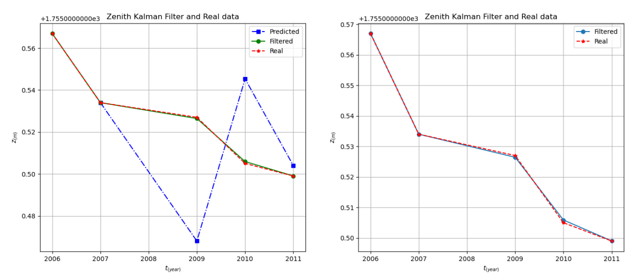

Figure 5: Zenith Kalman Filter and Real data point1

vlocity and acceleration:

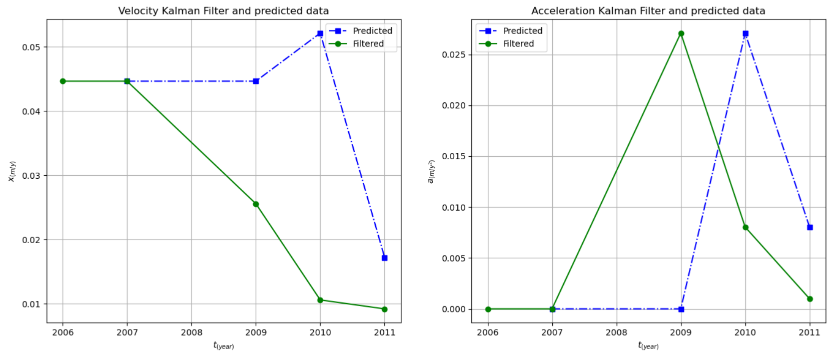

Figure 6: vlocity and acceleration Kalman Filter and Real data point1

3.3.2 Point 6

Figure 7: Esting Kalman Filter and Real data point6

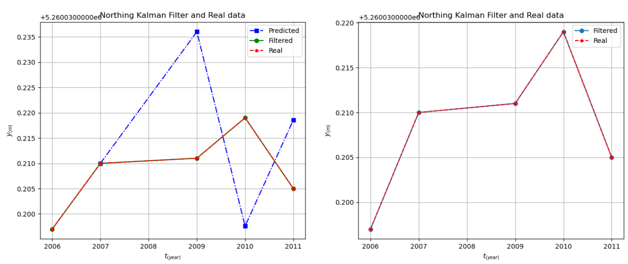

Figure 8: Northing Kalman Filter and Real data point6

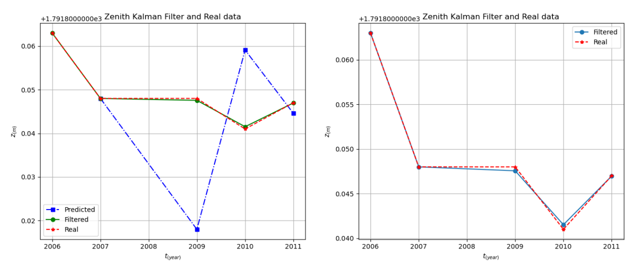

Figure 9: Zenith Kalman Filter and Real data point6

vlocity and acceleration:

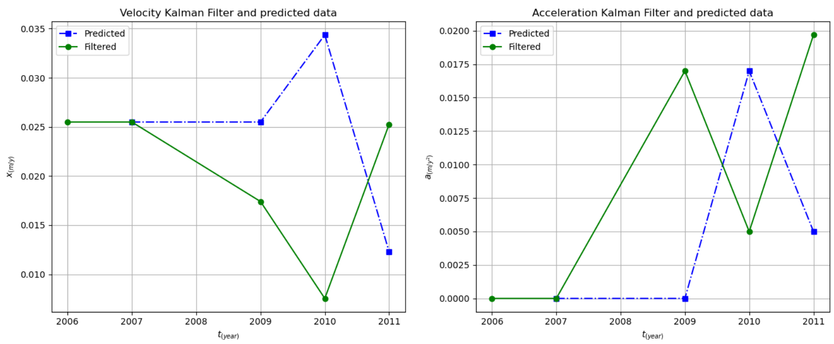

Figure 10: vlocity and acceleration Kalman Filter and Real data point6

3.4 Significant test

Test of significance of innovation and filtered state (velocity and acceleration) All test are accepted.