Strain Analysis

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1 Introduction

It is necessary to perform a strain analysis in order to identify alterations in the internal shape of the structure. The anticipated deformations are relatively minor compared to the size of the object, allowing us to assume infinitesimal strain. This assumption holds true for deformations occurring in the vicinity of differential points. As a result, the deformations can be broadly described using the linear relationship X = Fx + t.

2 Task 1: Determining the strain tensor and the elements of the RBM

Here we assume infinitesimal strain.We can see the deformation as an Affine transformation $X_{epoch1} = FX_{epoch2} + t$. Then we can see the deformation as an Affine transformation. Deformation matrix F describes both distortion and rotation

$$F = \begin{bmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{bmatrix} \rightarrow F = RV$$

If we assume infinitesimal strain we have :

$$R \approx \begin{bmatrix} 1 & \omega \\ -\omega & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0\omega \\ -\omega 0 \end{bmatrix}$$

$$V = \begin{bmatrix} v_{xx} & v_{xy} \\ v_{yx} & v_{yy} \end{bmatrix} = I + E \approx \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} e_{xx} & e_{xy} \\ e_{yx} & e_{yy} \end{bmatrix}$$

We are looking for ω, E and t ? We can have the following relation between displacement vector and transformation parameters.

$$p = H^{-1}u \text{ or } p = (H^T H)^{-1} H^T u$$

$$u_i = \begin{bmatrix} u_y \\ u_x \end{bmatrix} = \begin{bmatrix} y_i^{(2)} - y_i^{(1)} \\ x_i^{(2)} - x_i^{(1)} \end{bmatrix},\ H_i = \begin{bmatrix} 0 & \Delta x_i & \Delta y_i & -\Delta x_i & 0 & 1 \\ \Delta x_i & \Delta y_i & 0 & \Delta_i & 1 & 0 \end{bmatrix},\ p_i = \begin{bmatrix} e_{xx} \\ e_{xy} \\ e_{yy} \\ \omega \\ t_x \\ t_y \end{bmatrix}$$

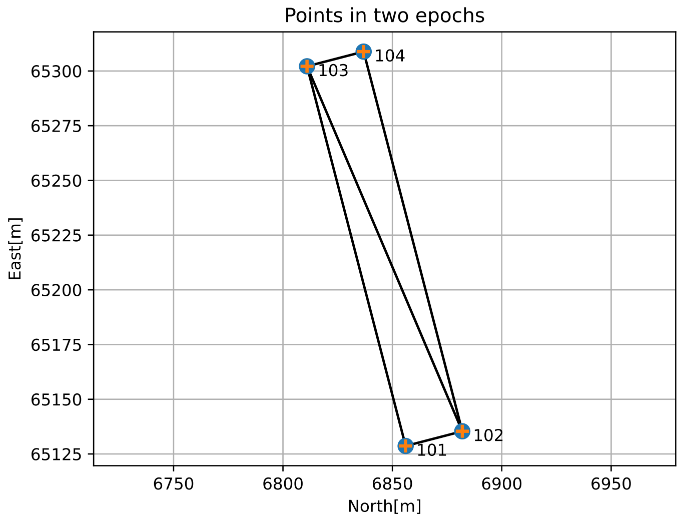

Figure 2: Points in two epochs

$$\Sigma_{pp} = H^{-1}(\Sigma_{ll}^{(1)} + \Sigma_{ll}^{(2)})(H^{-1})^T$$

$$p_{101,102,103} = \begin{bmatrix} 2.4362 \times 10^{-4} \\ 7.6965 \times 10^{-5} \\ 5.3282 \times 10^{-5} \\ -4.8258 \times 10^{-5} \\ 2.14891 \times 10^{-3} \\ 7.3994 \times 10^{-3} \end{bmatrix} \text{ and } p_{102,103,104} = \begin{bmatrix} 1.9504 \times 10^{-4} \\ -1.5125 \times 10^{-5} \\ -1.6291 \times 10^{-5} \\ 2.3162 \times 10^{-5} \\ 2.1491 \times 10^{-3} \\ 7.4001 \times 10^{-3} \end{bmatrix}$$

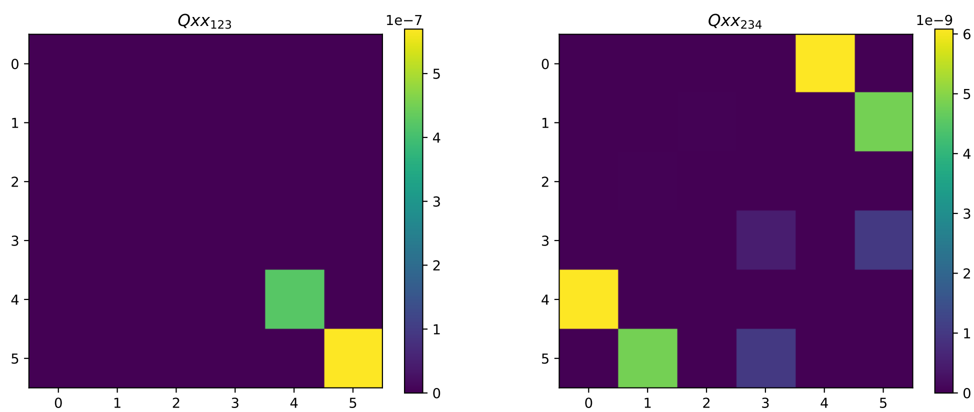

Figure 3: Points in two epochs

3 Task 2 : Determination of further distortions

More distortion measures for physical interpretation is given as follows:

3.1 Distance distortion

scale change of any arbitrary distance from epoch 1 to epoch 2

$$e = \frac{S^{(2)} - S^{(1)}}{S^{(1)}} \text{ and } e_{t_i^j} = e_{xx}cos^2 t_i^j + exysin2t_i^j + e_{yy}sin^2 t_i^j \text{ and } t_i^j = arctan^{-1}\frac{\Delta y}{\Delta x}$$

$$\begin{bmatrix} e_{101}^{102} \\ e_{102}^{103} \\ e_{101}^{103} \end{bmatrix} = \begin{bmatrix} 0.000269 \\ 2.7855 \times 10^{-5} \\ 2.7003 \times 10^{-5} \end{bmatrix}$$

4 Angle distortion

shear strain

$\gamma = tan(\alpha + \beta)$

if infinitesimal strain holds :

$$\gamma = 2e_{xy} = 0.000153$$

5 Surface distortion

change of area scale from epoch 1 to epoch 2 acn be compute from this formula $A = \frac{A^{(2)} - A^{(1)}}{A^{(1)}}$ if infinitesimal strain holds : $dA \approx e_1 + e_2$ In which e_1 and e_2 are the parameters of the distortion ellipse

$$e_1 = 0.5(e_{xx} + e_{yy} + ee) \text{ and } e_2 = 0.5(e_{xx} + e_{yy} - ee) \text{ and } ee = \sqrt{(e_{xx} - e_{yy})^2 + 4e_{xy}^2}$$

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}_{101,102,103} = \begin{bmatrix} 0.0002708 \\ 2.60556 \times 10^{-5} \end{bmatrix} \text{ and } \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}_{101,102,103} = \begin{bmatrix} 0.000196 \\ -1.73683 \times 10^{-5} \end{bmatrix}$$

6 Task 3 : Determination and presentation of the distortion ellipse

Semi-major and semi-minor axes of the ellipse are already determined.Orientation of the distortion ellipse can be compute from this formula $\theta 0.5atan(\frac{2e_{xy}}{e_{xx}-e_{yy}})$ and meaningful scale for presentation of the error ellipses is 100000.

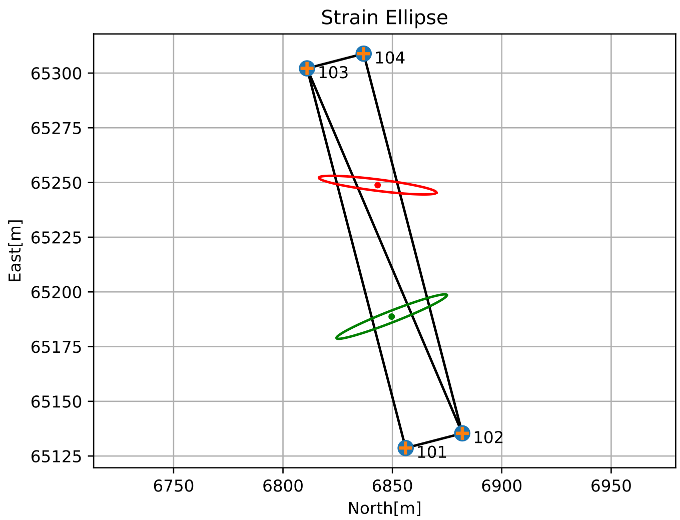

Figure 4: Main strain element and strain ellipse