Analysis of rigid body motions

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1 Introduction

One of the main tasks of surveyors is to measure geometrical changes due to the action of forces affecting the object. In general, geometric changes are divided into two categories. The first category is **rigid body movement**, in which the components of rotation and transfer are examined, it is practically an isometric transformation. The second category is called **distortion**, which can be said to include twisting, bending and strain, in other words, it causes deformation in the body.

In this project, our focus is on investigating a geodetic network that has been established. The network includes points 904 to 911 and 9400 to 9402, along with points 101 to 104. The primary objective is to ensure the stability and functional reliability of a structure. To achieve this, points 904 to 911 and 9400 to 9402 have been marked with deeply founded pillars, indicating their stability for the specific task at hand. The structure itself is represented by points 101 to 104 and is comprised of a monolithic block.

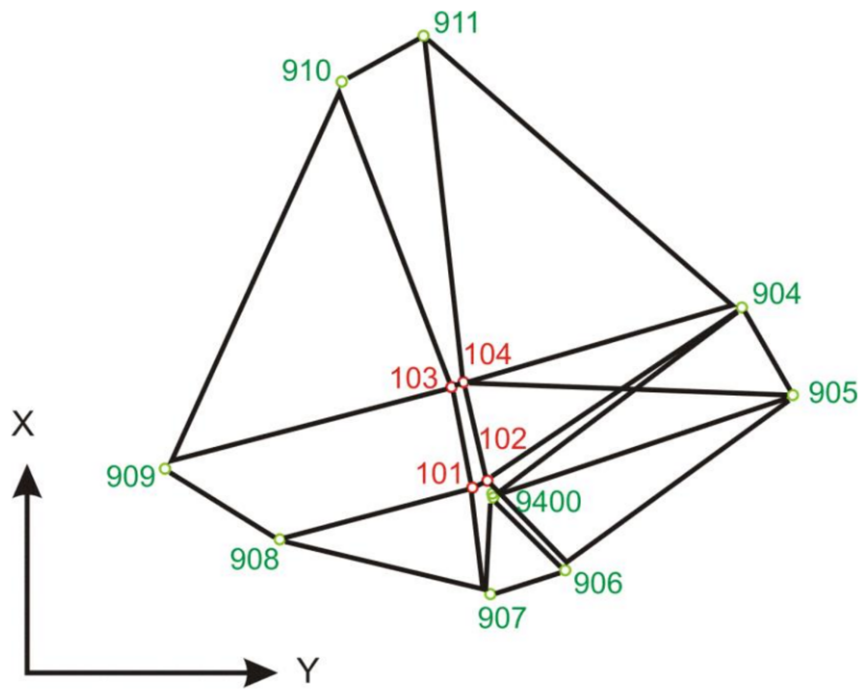

Figure 2: The figure illustrates several pillars within the geodetic network. It is important to note that for simplicity, not all coordinates of the network are depicted, but only a subset of them. These pillars play a crucial role in ensuring the stability and reliability of the network and are essential components of the overall structure.

2 Task1

2.1 Datum Regulation

In the two-dimensional (planar) network, there are four unknowns related to the defects of the datums. These include:

1- The unknowns of the network with respect to the two-dimensional coordinates: These are the unknown positions of points in the network in the x and y directions.

2- The unknown of the state of the network period relative to a specific direction: This refers to the unknown orientation or rotation of the network with respect to a chosen reference direction.

3- The unknown scale: This represents the unknown scaling factor that relates the distances in the network to their corresponding real-world measurements.

These four unknowns need to be determined in order to fully characterize and analyze the two-dimensional network.

Defect

Total Defects	Fixed Point X	Fixed Point Y	Fix Direction	Scale
3	1	1	1	0

Table 1: Table Defection in Datum of *HANNA* number of 1 means it has defect in that part and 0 is opposite.

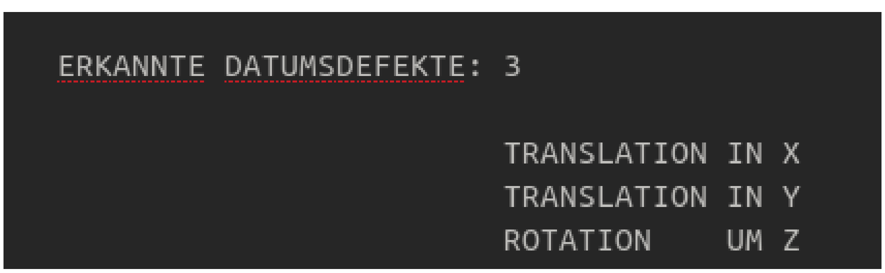

Figure 3: The figure indicates that in the file "HANNA.avdgem.out," there are three defects of the datum.

2.2 s0 a-posteriori standard deviation

Important parameter for calculating **Variance covariance matrix (VCM) of the coordinates**

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Figure 4

2.3 Degrees of freedom

According to the Hanna file, there are 210 observations of horizontal length and 94 observations of direction. The number of coordinate unknowns is equal to 38, and there are 25 azimuth unknowns. Therefore, the total number of diatom defects is 244.

Degrees of freedom

number of Unknowns(u)	Unknowns X	Unknowns Y	Unknowns Azimuth
63	18	18	25
number of Observations (n)		Length	Direction
304		210	94
Defects (d)			
3			

Table 2: Table of Degrees freedom.

$$df = n - u + d = 244$$

2.4 Indication of Accuracy

The variance-covariance matrix of each epoch is as follows:

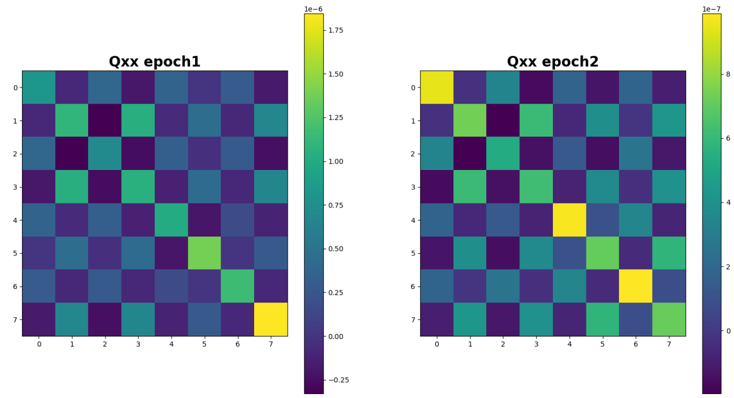

Figure 5

From the covariance variance matrix, you can calculate the ellipse parameters of the error and standard deviation between the points and the x and y parameters of each point.

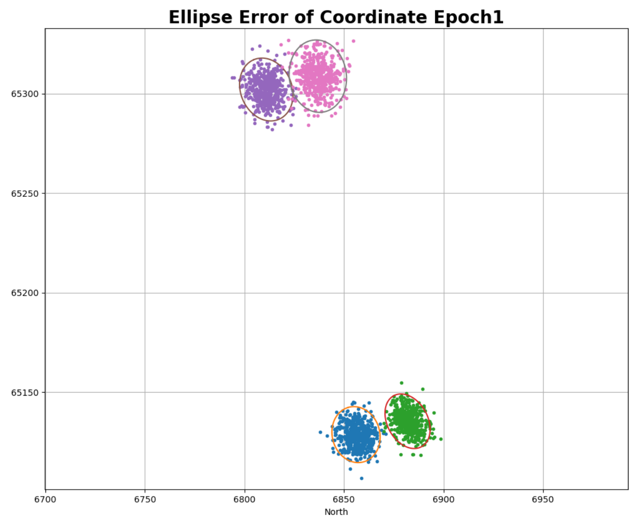

Figure 6: The semidiameters of the error ellipse of this point at the confidence level of 95 percent are equal to 2.65 and 1.86 It is millimeters.

2.5 Redundancy component

The sum of the observations is equal to the degree of freedom of the network redundancy, it is directly related to the weight and accuracy of the network, a good redundancy indicates a high level of confidence of the network, if the minimum redundancy is equal to the maximum, in this case, the network has the best strength from a geometric point of view, the network redundancy is in the file Hanna is placed for each observation.

Redundancy

	Mean Redundancy	Max Redundancy	Min Redundancy
epoch1	0.595	0.794	0.022
epoch2	0.895	0.984	0.633

3 Task 2

3.1 Translation

3.1.1 Preparation of Data each epoch

Adjusted coordiante from epochs:

$$\hat{x}_{0,1} = \begin{bmatrix} \hat{y}_{0101} \\ \hat{x}_{0101} \\ . \\ . \\ . \\ \hat{x}_{0104} \end{bmatrix} \rightarrow Epoch1 \quad \hat{x}_{0,2} = \begin{bmatrix} \hat{y}_{0101} \\ \hat{x}_{0101} \\ . \\ . \\ . \\ \hat{x}_{0104} \end{bmatrix} \rightarrow Epoch2$$

Variance covariance matrix(VCM) of coordinates

$$Sigma_{xx,0,1} = Q_{xx}\hat{\sigma}_0^2$$

3.1.2 Calculation of translation

Calculation of the centers of gravity for the two epochs and the corresponding VCM:

$$\hat{x}_{SP,0,1} = \frac{1}{n}F.\hat{x}_{0,1} \rightarrow \hat{\Sigma}_{x_{SP}x_{SP},0,1} = (\frac{1}{n})^2.F.\hat{\Sigma}_{xx,0,1}.F^T$$
$$\hat{x}_{SP,0,2} = \frac{1}{n}F.\hat{x}_{0,2} \rightarrow \hat{\Sigma}_{x_{SP}x_{SP},0,2} = (\frac{1}{n})^2.F.\hat{\Sigma}_{xx,0,2}.F^T$$

Difference of center of garvities between the two epochs

$$\Delta x\hat{_{SP}} = \hat{x}_{SP,0,2} - \hat{x}_{SP,0,1} \rightarrow \Sigma_{\Delta\hat{x}_{SP}\Delta\hat{x}_{SP}} = \Sigma_{\Delta\hat{x}_{SP,0,1}} + \Sigma_{\Delta\hat{x}_{SP,0,2}}$$

with

$$F = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \rightarrow selectionmatrix$$

result :

$$\Delta_{x_{sp}} = \begin{bmatrix} 0.0018 \\ 0.006225 \end{bmatrix}$$

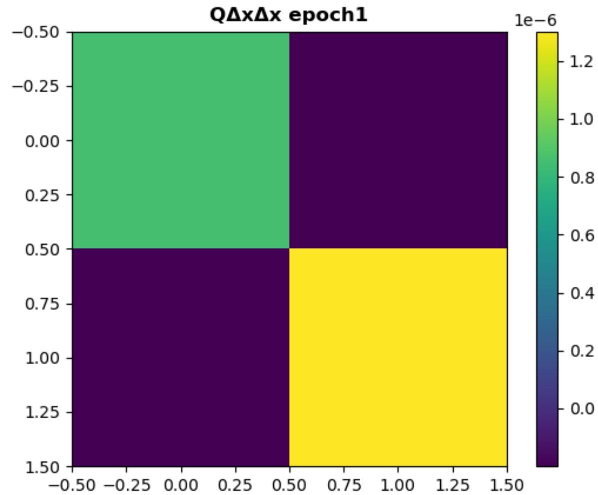

Figure 7

3.1.3 Significant test

The zero hypothesis is that the transition between two datum is 0.

$$H_0 = E[\Delta_{\hat{x}_{SP}}] = 0 \quad H_A = E[\Delta_{\hat{x}_{SP}}] \neq 0$$

Test value

$$\hat{F} = \Delta x_{SP}^T . \Sigma^{-}1_{\Delta x_{SP} \Delta x_{SP}} \sim 2F_{f_1,f_2,1-\alpha}$$

Rotation hypothesis

$\hat{F}$	$2F_{f_1,f_2,1-\alpha}$	
38.9597	6.606	rejected

hypothesis zero is rejected and this indicates a transition between two epochs.

3.2 Rotation

3.2.1 Preparation of Data each epoch

Transforming adjusted coordinates to a local coordinate system with center of gravity as origin using the transformation matrix G :

$$\hat{\xi}_{0,1} = G\hat{x}_{0,1} = \begin{bmatrix} \hat{\eta}_{0101} \\ \hat{\xi}_{0101} \\ . \\ . \\ . \\ \hat{\xi}_{0104} \end{bmatrix} \rightarrow Epoch1 \quad \hat{\xi}_{0,2} = G\hat{x}_{0,2} = \begin{bmatrix} \hat{\eta}_{0101} \\ \hat{\xi}_{0101} \\ . \\ . \\ . \\ \hat{\xi}_{0104} \end{bmatrix} \rightarrow Epoch2$$

with $G = I - \frac{1}{n}F^T F$
Variance covariance matrix(VCM) of coordinates

$$\Sigma_{\xi\xi,0,1} = G\Sigma_{xx0,1}G^T \text{ and } \Sigma_{\xi\xi,0,2} = G\Sigma_{xx0,2}G^T$$

3.2.2 Calculation of rotation

Defining the direction angles with respect to the origin (center of the gravity)

$$t_{i,1} = \arctan \frac{\eta_i}{\xi_i}$$

Variance covariance matrix (VCM) of the direction angles

$$\Sigma_{tt,1} = H_1\Sigma_{\xi\xi,0,1}H_1^T \text{ and } \Sigma_{tt,2} = H_2\Sigma_{\xi\xi,0,2}H_2^T$$

Rotation is derived by differentiating the calculated orientation angles between the two epochs

$$\omega = t_{i,2} - t_{i,1}$$

VCM of rotation

$$\Sigma_{\omega\omega} = \Sigma_{tt,1} + \Sigma_{tt,2}$$

Mean of the rotation angles is used to evaluate rotation of the rigid body

$$\omega_0 = \frac{1}{n}e^T\omega \rightarrow withe^T = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$$

Standard deviation of the mean rotation

$$\sigma_{\omega_0} = \sqrt{(\frac{1}{n})^2 e^T \Sigma_{\omega\omega} e}$$

Rotation Parameters

ω_0	σ_{ω_0}
6.39386085e-06	1.73645963e-06 -7.20925359e-05 -5.48341567e-05 -8.69117884e-06

3.2.3 Significance test

The zero hypothesis is that the transition between two datum is 0.

$$H_0 = E[\omega_0] = 0 \quad H_A = E[\omega_0] \neq 0$$

Test value

$$\hat{T} = \frac{\omega_0}{\sigma_{\omega_0}} \sim t_{f_1,1-\alpha}$$

Rotation hypothesis

$\hat{T}$	$t_{f_1,1-\alpha}$	
5.234	1.651	rejected

hypothesis zero is rejected and this indicates a rotation between two epochs.

4 Conclusion

From the two rejected assumptions, we conclude that both rotation and transfer have occurred in the structure.