Denoising of Signals and Photo with Wavelet

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If our ear uses a certain technique to analyze a signal, then if you use that same mathematical technique, you will be doing something like our ear. You might miss important things, but you would miss things that our ear would miss too.

Ingrid Daubechies

The mathematical analysis of the frequency content of signals is called Fourier analysis. analysis as it applies to discrete signals and use it to analyze the frequency content of wavelets. A deeper understanding of wavelets can be gained from studying their frequency content, and by examining how this frequency content relates to wavelet transforms of signals.
To understand better the mathematics as simple as possible we shall focus on 1D signals.then we concetrate on photos(2D signals).The main Philosophy of generating of wavelet is defects in Statical feature of time or space Fourier analysis.
The aim of doing this project is to remove noise as possible as can.

1 1D Wavelet:

First, we need to create pure siganl without any noise with some asumpptions that i will say as follows :
1- $f = 2cos(8\pi x/N) + 4\pi x/N$
2- N = Numbers of sample of signals we have : 2^n
we consider n = 8.

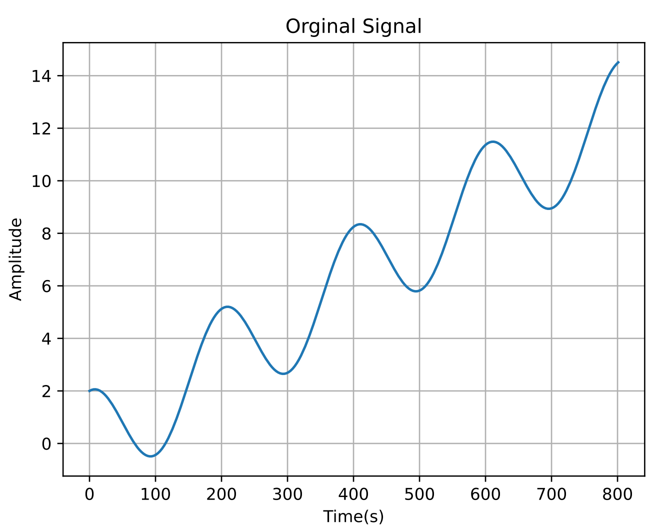

Figure 2: Orginal 1D signals $2cos(8\pi x/N) + 4\pi x/N$

Then we add sum noise(Random Gaussian Noise) to make a noisy signals : main strucure

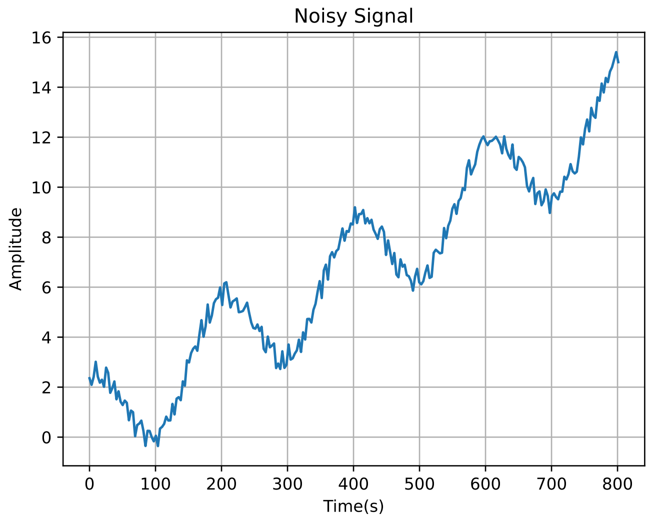

Figure 3: Noisy 1D signals $2cos(8\pi x/N) + 4\pi x/N + random_{noise}$

of wavelet is finding some cofficeint to recreate our signal without noise :

$$f = \sum_{i=1} \langle f\ ,\ v_i^1 \rangle v_i^1 + \sum_{j=1}^{\frac{N}{2^j}} \sum_{i=1} \langle f\ ,\ w_i^j \rangle w_i^j$$

Here we use some type of wavewlet like Harr ,Daubechies and Mexician Hat which is in order write as follows :

1.1 Harr 1D:

firse we write scale functions and creator function of Harr and then we seprate signals in to Avrage part and Details part with these function.

$$\phi(x) = \begin{cases} 1, & \text{if } 0 \le x < 1. \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

$$\psi(x) = \begin{cases} 1, & \text{if } 0 \le x < 1/2. \\ -1, & \text{if } 1/2 \le x < 1. \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

and the result is :

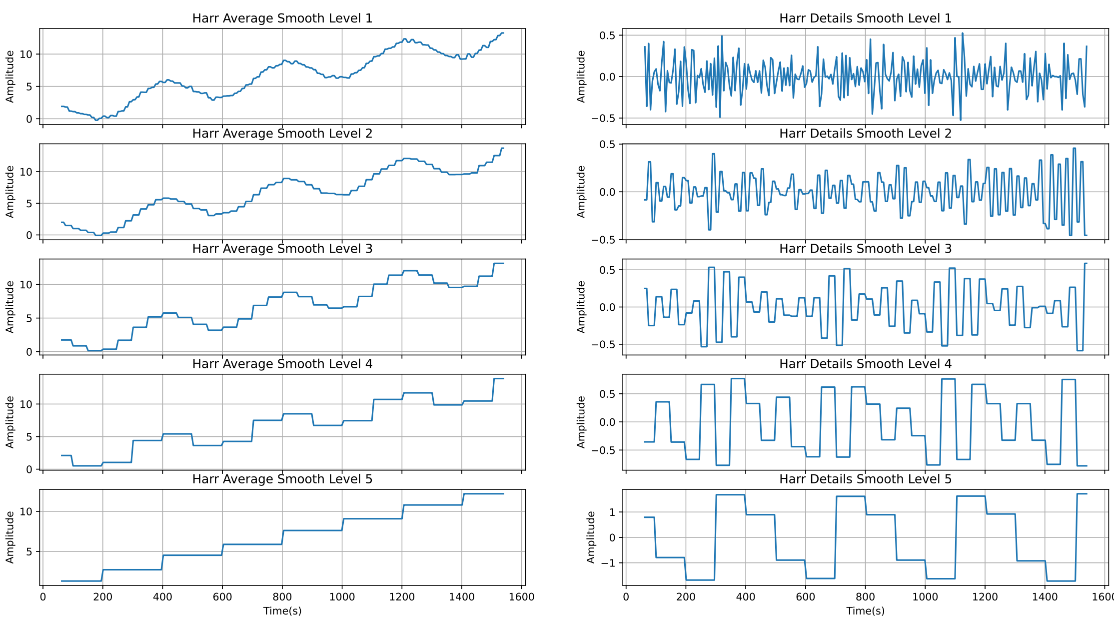

Figure 4: Harr Wavelet level1 to level5

1.2 Daubechies4 1D:

cofficent is root of recreating function with wavelet method and for Daubechies4 is given as follows :

$$\alpha_1 = \tfrac{1+\sqrt{3}}{4\sqrt{2}} \quad \alpha_2 = \tfrac{3+\sqrt{3}}{4\sqrt{2}} \quad \alpha_3 = \tfrac{3-\sqrt{3}}{4\sqrt{2}} \quad \alpha_4 = \tfrac{1-\sqrt{3}}{4\sqrt{2}}$$

$$v_1^1 = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, 0, 0, 0, 0..., 0)$$

$$v_1^2 = (0, 0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, 0, 0, ...0)$$

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$$v_{\frac{N}{2}}^2 = (\alpha_3, \alpha_4, 0, 0, 0, 0, ...0, \alpha_1, \alpha_2)$$

$$\beta_1 = \tfrac{1+\sqrt{3}}{4\sqrt{2}} \quad \beta_2 = \tfrac{3+\sqrt{3}}{4\sqrt{2}} \quad \beta_3 = \tfrac{3-\sqrt{3}}{4\sqrt{2}} \quad \beta_4 = \tfrac{1-\sqrt{3}}{4\sqrt{2}}$$

$$w_1^1 = (\beta_1, \beta_2, \beta_3, \beta_4, 0, 0, 0, 0..., 0)$$

$$w_1^2 = (0, 0, \beta_1, \beta_2, \beta_3, \beta_4, 0, 0, ...0)$$

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$$w_{\frac{N}{2}}^2 = (\beta_3, \beta_4, 0, 0, 0, 0, ...0, \beta_1, \beta_2)$$

$$v_m^{n+1} = \alpha_1 v_{2m-1}^n + \alpha_2 v_{2m}^n + \alpha_3 v_{2m+1}^n + \alpha_4 v_{2m+2}^n$$

$$w_m^{n+1} = \beta_1 w_{2m-1}^n + \beta_2 w_{2m}^n + \beta_3 w_{2m+1}^n + \beta_4 w_{2m+2}^n$$

$$v_n^1 . v_m^1 = \begin{cases} 1, & \text{if } m = n. \\ 0, & \text{if } m \neq n \end{cases} \tag{3}$$

$$w_n^1 . w_m^1 = \begin{cases} 1, & \text{if } m = n. \\ 0, & \text{if } m \neq n \end{cases} \tag{4}$$

$$w_n^1 . w_m^1 = 0, n, m \in Z \tag{5}$$

and the result is :

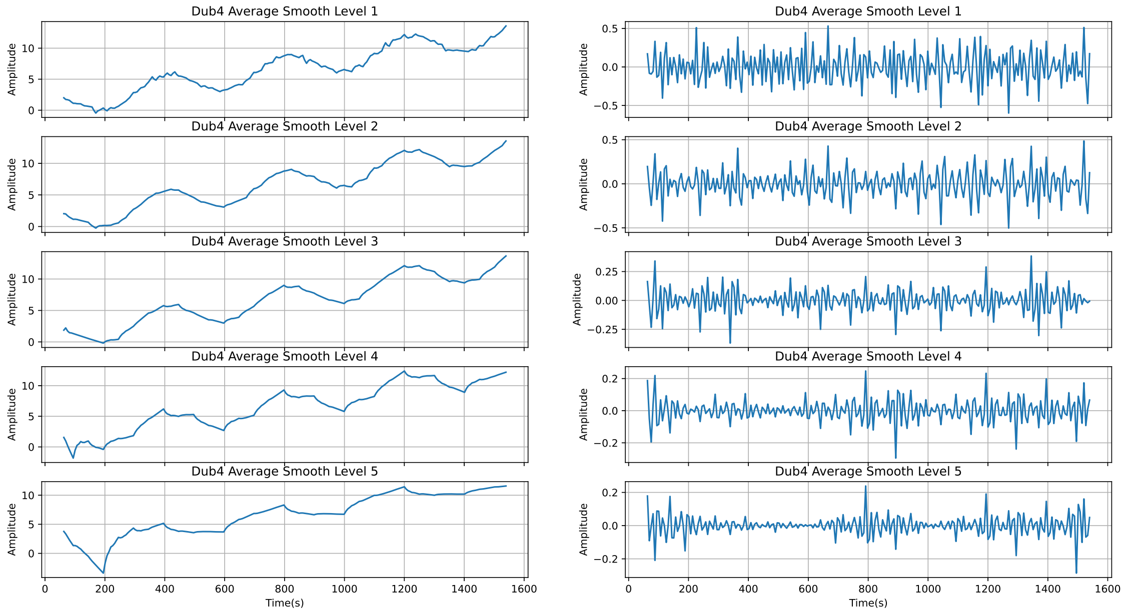

Figure 5: Daubechies4 Wavelet level1 to level5

1.3 Daubechies6 1D:

cofficent is root of recreating function with wavelet method and for Daubechies4 is given as follows :

$$\alpha_1 = 0.33267 \quad \alpha_2 = 0.80689 \quad \alpha_3 = 0.45987 \quad \alpha_4 = -0.13501 \quad \alpha_5 = -0.08544 \quad \alpha_6 = 0.03522$$

$$v_1^1 = (\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, 0, 0, 0, 0..., 0)$$
$$v_1^2 = (0, 0, \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6, 0, 0, ...0)$$
$$.$$
$$.$$
$$.$$
$$v_{\frac{N}{2}}^2 = (\alpha_3, \alpha_4, \alpha_5, \alpha_6 0, 0, 0, 0, ...0, \alpha_1, \alpha_2)$$
$$\beta_1 = \alpha_6 \quad \beta_2 = -\alpha_5 \quad \beta_3 = \alpha_4 \quad \beta_4 = -\alpha_3 \quad \beta_5 = \alpha_2 \quad \beta_6 = -\alpha_1$$
$$w_1^1 = (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, 0, 0, 0, 0..., 0)$$
$$w_1^2 = (0, 0, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6, 0, 0, ...0)$$
$$.$$
$$.$$
$$.$$
$$w_{\frac{N}{2}}^2 = (\beta_3, \beta_4, \beta_5, \beta_6, 0, 0, 0, 0, ...0, \beta_1, \beta_2)$$

$$v_m^{n+1} = \alpha_1 v_{2m-1}^n + \alpha_2 v_{2m}^n + \alpha_3 v_{2m+1}^n + \alpha_4 v_{2m+2}^n + \alpha_5 v_{2m+3}^n + \alpha_6 v_{2m+4}^n$$
$$w_m^{n+1} = \beta_1 w_{2m-1}^n + \beta_2 w_{2m}^n + \beta_3 w_{2m+1}^n + \beta_4 w_{2m+2}^n + \beta_5 w_{2m+3}^n + \beta_6 w_{2m+4}^n$$

$$v_n^1.v_m^1 = \begin{cases} 1, & \text{if } m = n. \\ 0, & \text{if } m \neq n \end{cases} \tag{6}$$

$$w_n^1.w_m^1 = \begin{cases} 1, & \text{if } m = n. \\ 0, & \text{if } m \neq n \end{cases} \tag{7}$$

$$w_n^1.w_m^1 = 0, n, m \in Z \tag{8}$$

and the result is :

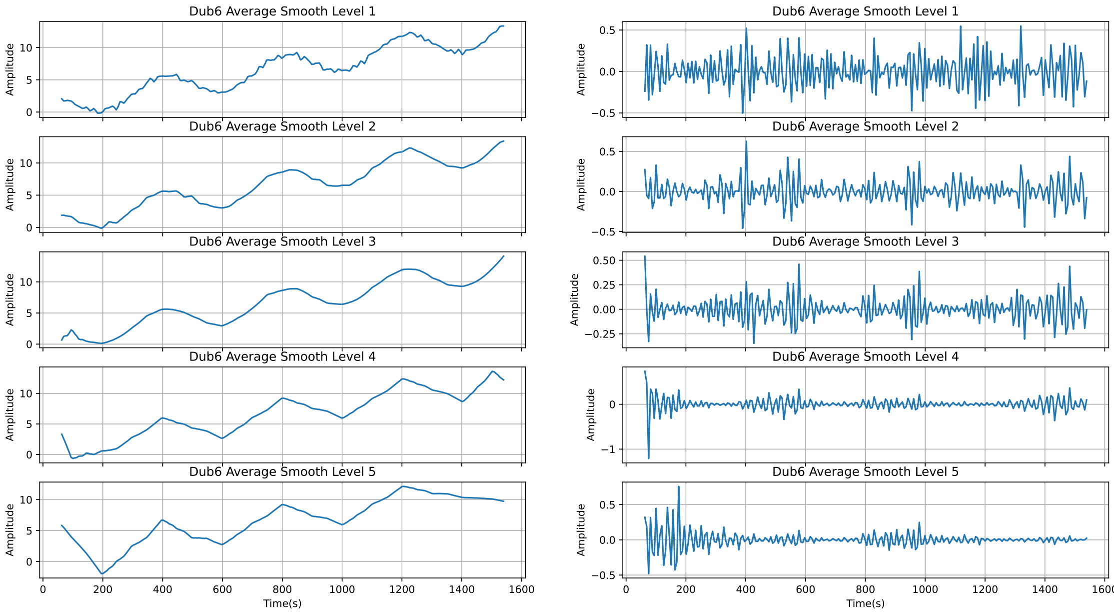

Figure 6: Daubechies6 Wavelet level1 to level5

1.4 Mexician Hat 1D:

There is any different between Daubechies6 and Mexician Hat except , difference in cofficent :

$$\alpha_1 = \frac{1-\sqrt{7}}{16\sqrt{2}} \quad \alpha_2 = \frac{5+\sqrt{7}}{16\sqrt{2}} \quad \alpha_3 = \frac{14+2\sqrt{7}}{16\sqrt{2}} \quad \alpha_4 = \frac{14-2\sqrt{7}}{16\sqrt{2}} \quad \alpha_5 = \frac{1-\sqrt{7}}{16\sqrt{2}} \quad \alpha_6 = \frac{-3+\sqrt{7}}{16\sqrt{2}}$$

$$\beta_1 = \alpha_6 \quad \beta_2 = -\alpha_5 \quad \beta_3 = \alpha_4 \quad \beta_4 = -\alpha_3 \quad \beta_5 = \alpha_2 \quad \beta_6 = -\alpha_1$$

and the result is :

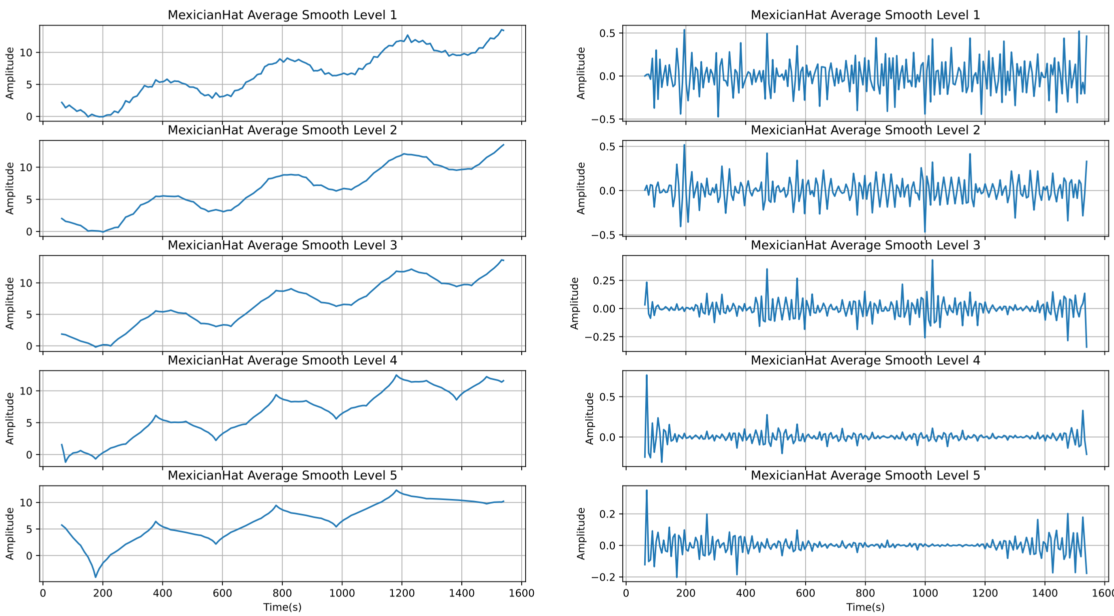

Figure 7: Mexician Hat Wavelet level1 to level5

1.5 Conclution 1D Wavelet :

$$e = \Sigma(f^i - f_w^i)^2 \tag{9}$$

Table 1: Table of Wavelet Error.

Wavelet			
Harr	Dub6	Dub4	Mexicain Hat
67.6701	64.8833	71.8755	61.5979
67.7504	60.8257	66.38971	56.0953
81.7307	62.0543	66.4861	54.8403
153.601	94.8377	104.948	86.0585
584.675	176.142	317.073	183.6669

Level three of Mexician Hat is the best in our sample.

2 2D Wavelet:

Asump an image N*N . level of this image represents below :

$$f \to \begin{pmatrix} aa^1 & ad^1 \\ da^1 & dd^1 \end{pmatrix} \to ... \to \begin{pmatrix} aa^m & ad^m \\ da^m & dd^m \end{pmatrix}$$

In here the base element of our space caculte from this formula :

$$V_{11}^1 \otimes W_{11}^1 = (V_{11}^1)^T W_{11}^1 \tag{10}$$

so we have :

$$f = aa_{11}^j V_{11}^1 \otimes V_{11}^1 + ... + aa_{N/2M/2}^j V_{N/2M/2}^1 \otimes V_{N/2M/2}^1 + dd_{11}^j W_{11}^1 \otimes W_{11}^1 + ... + dd_{N/2M/2}^j W_{N/2M/2}^1 \otimes W_{N/2M/2}^1 \tag{11}$$

now we consider an image here :

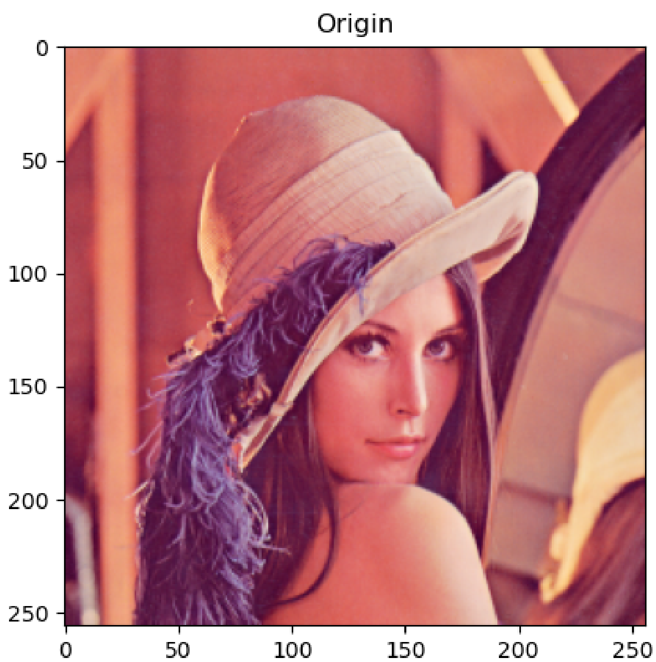

Figure 8: The Orginal image without any noise

Then we put some noise in it :

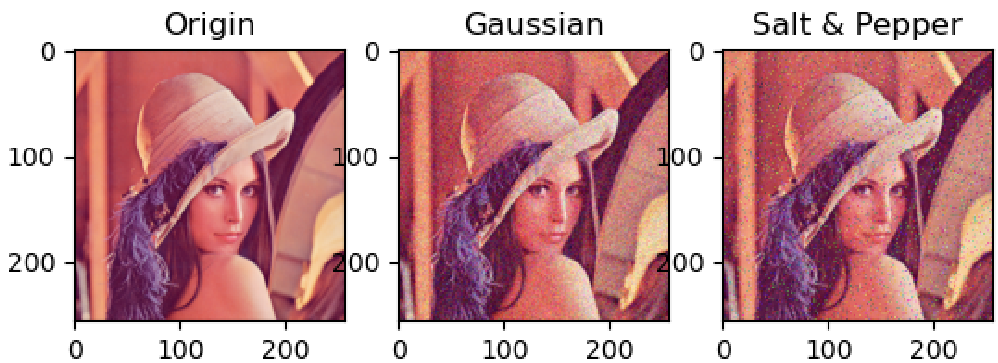

Figure 9: The Orginal image pluse gaussian noise and salt and pepper noise

2.1 Harr 2D :

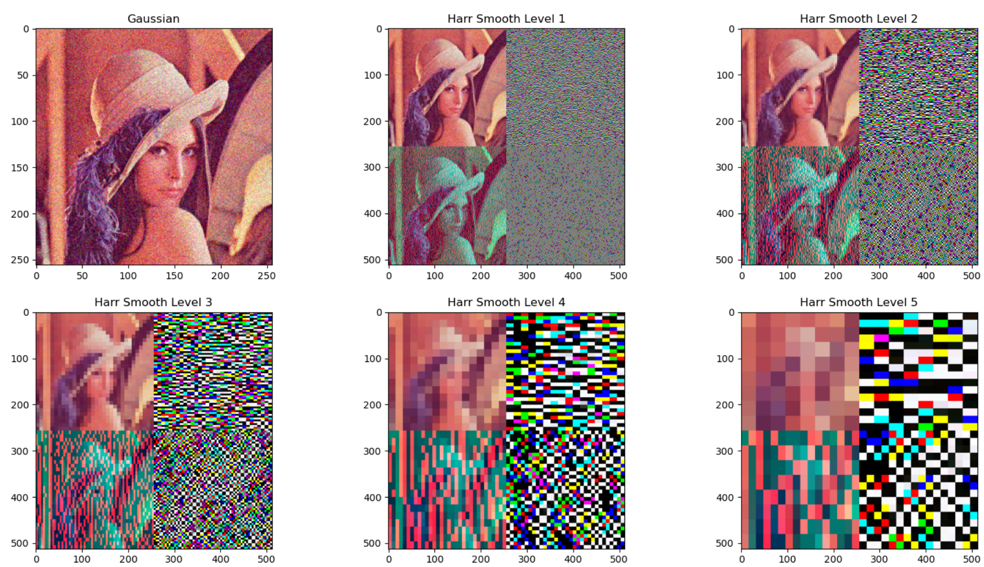

Figure 10: Harr 2D Wavelet level1 to level5

2.2 Daubechies4 2D :

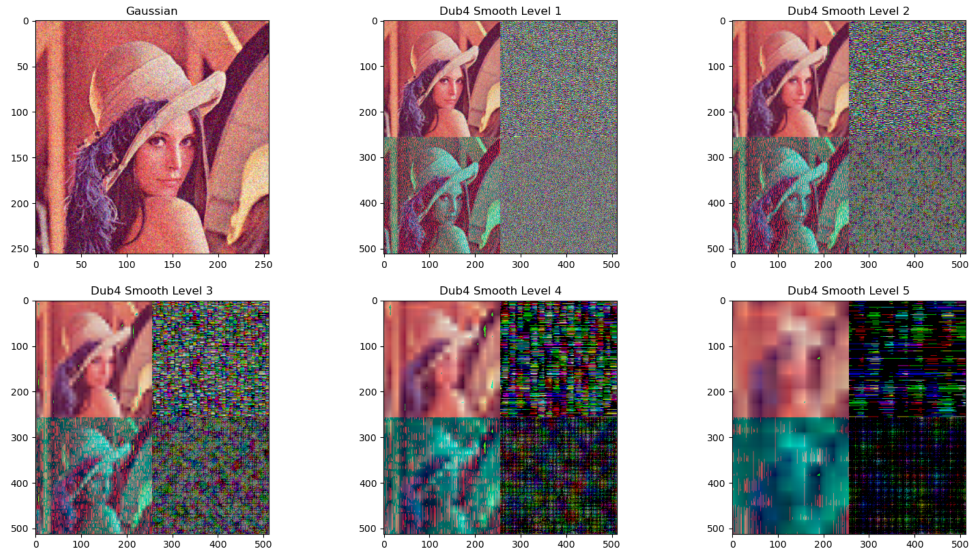

Figure 11: Daubechies4 2D Wavelet level1 to level5

2.3 Daubechies6 2D :

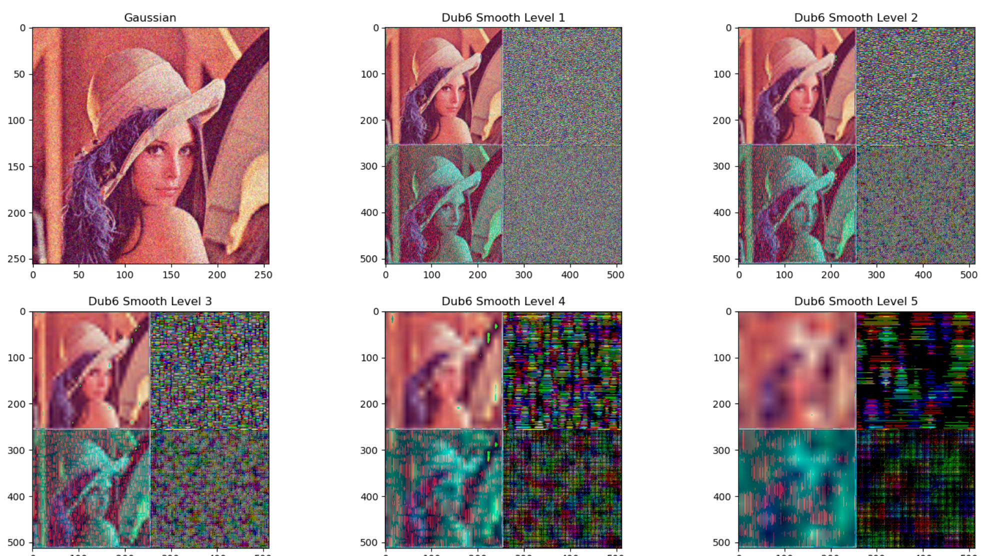

Figure 12: Daubechies6 2D Wavelet level1 to level5

2.4 Mexician Hat 2D :

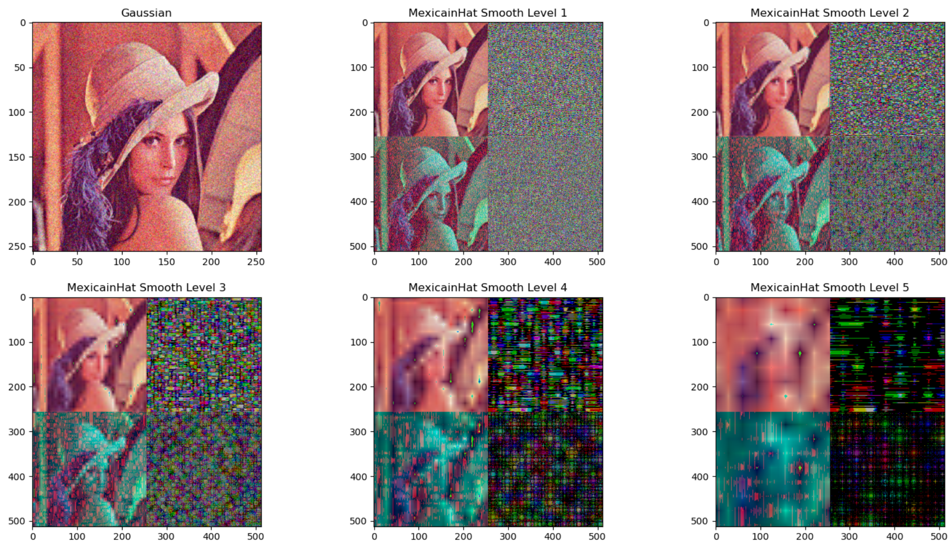

Figure 13: Mexician Hat 2D Wavelet level1 to level5