In the Name of God

Communication Systems (25751-1)
Quiz 02

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Exam Duration: 75 minutes

Problem 1

Let S be a signal generated by a random source with the following probability distribution:

$$\begin{cases} \mathbb{P}[S=+1] = p \\ \mathbb{P}[S=-1] = 1-p \end{cases}$$

Suppose S is sent through a noisy communication channel, so the received signal, Y is of the form below:

$$Y = S + N$$

where N is an additive zero-mean Gaussian noise:

$$N \sim \mathcal{N}(0, \sigma^2)$$

You may assume that N and S are independent.

1. *(10 points)* Find $\mathbb{E}[Y]$ and $\mathrm{Var}[Y]$.
2. *(20 points)* In order to decode the received signal at the receiver, we use the following decision rule:
$$\begin{cases} Y \geq 0 \longrightarrow \hat{S} = +1 \\ Y < 0 \longrightarrow \hat{S} = -1 \end{cases}$$
   where $\hat{S}$ is our estimation of the original sent signal S.
   Find the probability of error, $\mathbb{P}[\text{error}]$, i.e. $\mathbb{P}[\hat{S} \neq S]$.
   You may express your answer in terms of the function $Q(x)$ defined below:
$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-t^2/2} \mathrm{d}t$$
3. *(10 points)* For this part suppose that the decision rule is modified as below:
$$\begin{cases} Y \geq \theta \longrightarrow \hat{S} = +1 \\ Y < \theta \longrightarrow \hat{S} = -1 \end{cases}$$
   Find θ^*, the value of θ for which the probability of error is minimized:
$$\theta^* = \underset{\theta}{\operatorname{argmin}}\, \mathbb{P}[\text{error}]$$

Problem 2

Let $x(t) = \dfrac{\sin t}{t}$ be the input to the circuit in figure 1.

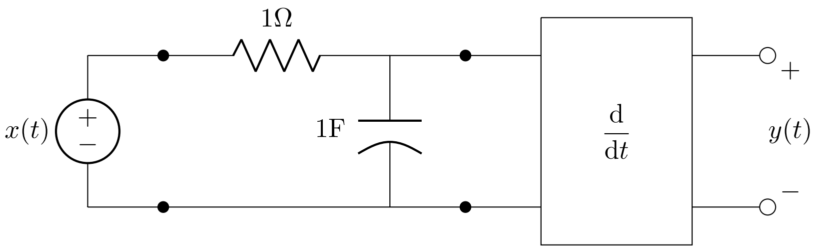

Figure 1

1. *(10 points)* Find $R_x(\tau)$, the autocorrelation function of $x(t)$.
2. *(15 points)* Find $S_y(f)$, the spectral density function of the output $y(t)$.
3. *(10 points)* Find E_y, the energy content of $y(t)$.

Problem 3

Let $X(t)$ be a stochastic process defined below:

$$X(t) = ae^{j(2\pi Vt+\Phi)}$$

where a is a constant, V is a random variable with the pdf function $f_V(v)$, and Φ is a uniformly distributed random variable over the interval $[-\pi, \pi]$, i.e. $\Phi \sim U[-\pi, \pi]$. Also assume that V and Φ are independent.

1. *(10 points)* Find $\mathbb{E}[X(t)]$.
2. *(15 points)* Find $S_X(f)$, the power spectral density function of $X(t)$.