In the Name of God

Communication Systems (25751-1)
Problem Set 03

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Due on ////////////////// at 7:30 a.m.

1 Spectral Density

Determine whether these signals are energy-type or power-type. In each case, find the energy spectral density and the energy content or the power spectral density and the power content of the signal.

1. $x_1(t) = e^{-\alpha|t|}\sin(\beta t) \qquad (\alpha, \beta > 0)$
2. $x_2(t) = \text{sinc}^2(3t)$
3. $x_3(t) = \sum_{n=-\infty}^{\infty} \Lambda(t - 3n)$
4. $x_4(t) = 2u(-t)$

2 Energy Spectral Density

The voltage $v(t) = Ate^{-\alpha t}u(t)$ is developed across a R *ohm* resistor. $(\alpha > 0)$

1. Calculate the total energy dissipated in the resistor.
2. What fraction of the energy is contained within a low pass bandwidth of W Hz?
3. What fraction of the energy is contained within a bandwidth of W Hz with a center frequency of f_c Hz?

3 Autocorrelation

Determine the autocorrelation function of each of the signals:

1. $x_1(t) = e^{-\alpha t}u(t) \qquad (\alpha > 0)$
2. $x_2(t) = \text{rect}(\frac{t}{T})$
3. $x_3(t) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

4 Power Spectral Density Estimation

1. Let $x(t)$ be periodic with period T_0. Show that $R_x(\tau)$ has the same periodicity.
2. Show that if $x_T(t)$ denotes the truncated signal corresponding to the power-type signal $x(t)$; that is,

$$x_T(t) = \begin{cases} x(t) & -\frac{T}{2} < t \leq \frac{T}{2} \\ 0 & \text{otherwise} \end{cases}$$

and if $\mathcal{S}_{x_T}(f)$ denotes the energy spectral density of $x_T(t)$, then $\mathcal{S}_x(f)$, the power-spectral density of $x(t)$, can be expressed as

$$\mathcal{S}_x(f) = \lim_{T \to \infty} \frac{\mathcal{S}_{x_T}(f)}{T}$$

5 Order Statistics

Let $X_1, X_2, \cdots, X_n$ denote i.i.d. random variables, each with PDF $f_X(x)$.

1. If $Y = \min\{X_1, X_2, \cdots, X_n\}$, find the PDF of Y.
2. If $Y = \max\{X_1, X_2, \cdots, X_n\}$, find the PDF of Y.

6 Conditional Expectation

Let $X_1, X_2, \ldots, X_n$ denote i.i.d. random variables. Find $\mathbb{E}[X_1 | X_1 + X_2 + \cdots + X_n = k]$, where k is a constant. (Find the answer in terms of k and $\mathbb{E}[X_i]$)
(*Hint:* You may solve this problem without any further information about the distribution of X_is, or even about whether they are continuous or discrete.)

7 Variance and Conditional Expectation

Let X and Y be two arbitrary random variables for which the following property holds:

$$\mathbb{E}[Y|X] = 1$$

Show that:

$$\text{var}[XY] \geq \text{var}[X]$$

(*Hint:* Note that $\mathbb{E}[Y|X]$ is a function of X.)

8 A Coin Problem

A coin having probability p of coming up heads is successively flipped until two of the most recent three flips are heads. Let N denote the number of flips. (Note that if the first two flips are heads, then $N = 2$.) Find $\mathbb{E}[N]$.