In the Name of God

Communication Systems (25751-1)
Quiz 05

Department of Electrical Engineering
Sharif University of Technology

Instructor: Dr. M. Pakravan
Exam Duration: 75 minutes

Problem 1

Wide-band FM can be generated by first generating a narrow-band FM signal and then using frequency multiplication to spread the signal bandwidth. Figure 1 illustrates such a scheme, which is called an *Armstrong-type FM modulator*. The narrow-band FM signal has a maximum angular deviation of 0.10 radians in order to keep distortion under control.

1. (*15 points*) If the message signal has a bandwidth of 15kHz and the output frequency from the oscillator is 100kHz, determine the frequency multiplication (n_1 and n_2) that is necessary to generate an FM signal at a carrier frequency of $f_c = 104$MHz and a frequency deviation of $f = 75$kHz.
2. (*15 points*) If the carrier frequency for the wide-band FM signal is to be within ± 2Hz, determine the maximum allowable drift of the 100kHz oscillator. Assume that the mixer in the system is an up-converter.

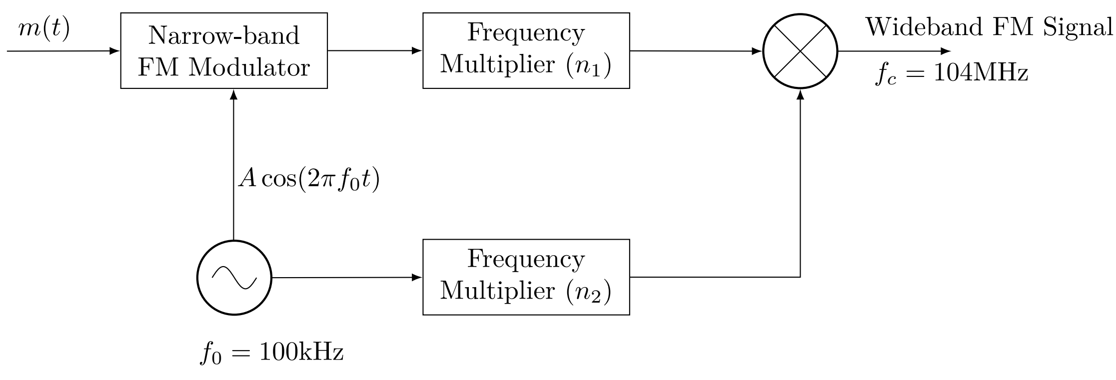

Figure 1: Armstrong-type FM Modulator

Problem 2

(*30 points*) Figure 2 shows a block diagram of a real-time *spectrum analyzer* working on the principle of frequency modulation.

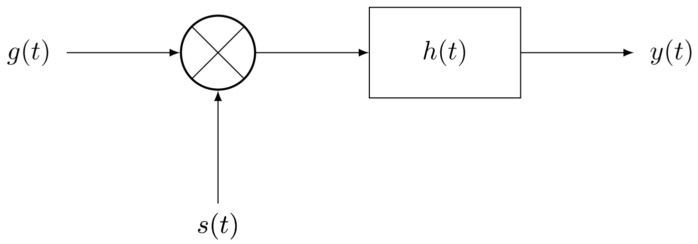

Figure 2

The given signal $g(t)$ and a frequency-modulated signal $s(t)$ are applied to a multiplier and the output $g(t)s(t)$ is fed into a filter of impulse response $h(t)$. The $s(t)$ and $h(t)$ are *linear FM signals* whose instantaneous frequencies vary linearly with time at opposite rates, as shown by

$$s(t) = \cos(2\pi f_c t - \pi k t^2)$$

$$h(t) = \cos(2\pi f_c t + \pi k t^2)$$

where k is a constant. Find $y(t)$, the output of the system.
Hint: Consider the complex envelope of the signals.

Problem 3

Consider a narrow-band FM signal approximately defined by

$$s(t) = A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t)$$

1. (*15 points*) Determine the envelope of this modulated signal. What is the ratio of the maximum to the minimum value of this envelope? Plot this ratio versus β, assuming that β is restricted to the interval $0 \leq \beta \leq 0.3$.
2. (*15 points*) Determine the average power of the narrow-band FM signal, expressed as a percentage of the average power of the unmodulated carrier wave. Plot this result versus β, assuming that β is restricted to the interval $0 \leq \beta \leq 0.3$.
3. (*10 points*) By expanding the angle $\theta_i(t)$ of the narrow-band FM signal $s(t)$ in the form of a power series, and restricting the modulation index β to a maximum value of 0.3 radians, show that

$$\theta_i(t) \approx 2\pi f_c t + \beta \sin(2\pi f_m t) - \frac{\beta^3}{3} \sin^3(2\pi f_m t)$$

Hint: $\tan^{-1}(x) \approx x - \dfrac{x^3}{3}$