In the Name of God

Communication Systems (25751-1)
Quiz 03

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Exam Duration: 75 minutes

Problem 1

Consider a cable transmission system with $L = 210\text{dB}$ has $m = 6$ equal-length repeater sections and $(\frac{S}{N})_D = 25\text{dB}$.
The effective noise temperature of the receiver as well as repeaters is $900K$. (Assume normal repeater design methodology whereby repeater gain is equal to cable loss and they are placed such that the distance between nodes are equal.)
Find the new value of $(\frac{S}{N})_D$ if

1. *(20 points)* m is increased to 9.
2. *(20 points)* m is decreased to 3.

Problem 2

Let $Z(t) = X(t)+jY(t)$ be a random process, where $X(t)$ and $Y(t)$ are real-valued, independent, zero-mean, and jointly stationary Gaussian random processes. We assume that $X(t)$ and $Y(t)$ are both band-limited processes with a bandwidth of W and a flat spectral density within their bandwidth, i.e.

$$\mathcal{S}_X(f) = \mathcal{S}_Y(f) = \begin{cases} N_0 & |f| \leq W \\ 0 & \text{otherwise} \end{cases}$$

1. *(15 points)* Find $\mathbb{E}[Z(t)]$ and $R_Z(t+\tau, t)$, and show that $Z(t)$ is WSS[1].
2. *(15 points)* Find the power spectral density of $Z(t)$.

[1] Wide-Sense Stationary

3. *(15 points)* Assume $\phi_1(t), \phi_2(t), \ldots, \phi_n(t)$ are orthonormal, i.e.

$$\int_{-\infty}^{\infty} \phi_j(t)\phi_k^*(t)\mathrm{d}t = \begin{cases} 1 & j = k \\ 0 & \text{otherwise} \end{cases}$$

and all $\phi_j(t)$'s are band-limited to $[-W, W]$. Define random variables Z_j as the projections of $Z(t)$ on the $\phi_j(t)$'s, i.e.

$$Z_j = \int_{-\infty}^{\infty} Z(t)\phi_j^*(t)\mathrm{d}t \quad , \quad j \in \{1, 2, \ldots, n\}$$

Determine $\mathbb{E}[Z_j]$ and $\mathbb{E}[Z_j Z_k^*]$ and conclude that the Z_j's are i.i.d. zero-mean Gaussian random variables. Also Find their common variance.
Hint: You may use the following identity:

$$\int_{-\infty}^{\infty} x(t)y^*(t)\mathrm{d}t = \int_{-\infty}^{\infty} X(f)Y^*(f)\mathrm{d}f$$

4. *(15 points)* Let us define

$$\hat{Z}(t) = Z(t) - \sum_{j=1}^{n} Z_j\phi_j(t)$$

to be the error in expansion of $Z(t)$ as a linear combination of $\phi_j(t)$'s. Show that $\mathbb{E}[\hat{Z}(t)Z_k^*]$=0 for all $k \in \{1, 2, \ldots, n\}$. In other words, show that the error $\hat{Z}(t)$ and all the Z_k's are uncorrelated. Can you say $\hat{Z}(t)$ and the Z_k's are independent?