The z-Transform

Amirhossein Afsharrad

Signals and Systems
Tutorial Session 2

March 9, 2020

Overview

1. Introduction
2. Properties of the ROC
3. Properties of the z-Transform
4. z-Transform and LTI Systems

Introduction

Introduction

Definition

The z-transform of a discrete-time signal $x[n]$ is:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad , \quad z \in \mathbb{C}$$

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$$x_1[n] = a^n u[n]$$
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Summary - Examples #1 and #2

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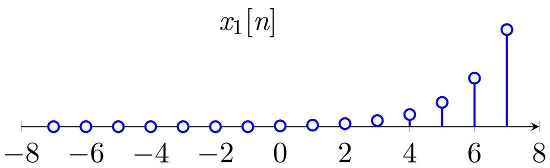

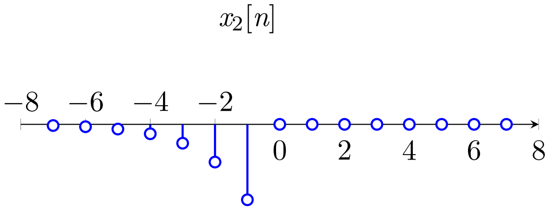

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Note that $X_1(z) = X_2(z)$ while $x_1[n] \neq x_2[n]$.

The z-transform is not unique without specifying where $X(z)$ is defined.

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The z-transform is not unique without specifying where $X(z)$ is defined.

The Region of Convergence

Definition

The region of the complex plane for which the z-transform $X(z)$ is defined is called the **Region Of Convergence (ROC)**.

Remark

The z-transform of a discrete-time signal is a pair $(X(z), \mathrm{ROC})$. To answer the question "Find the z -transform of the signal $x[n]$", reporting $X(z)$ without specifying the ROC is **an incomplete answer**.

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The Inverse z-Transform

The following equation for the inverse z-transform is *rarely* used.

Formula

$$x[n] = \frac{1}{2\pi j} \oint_{|z|=r} X(z) z^{n-1} \mathrm{d}z$$

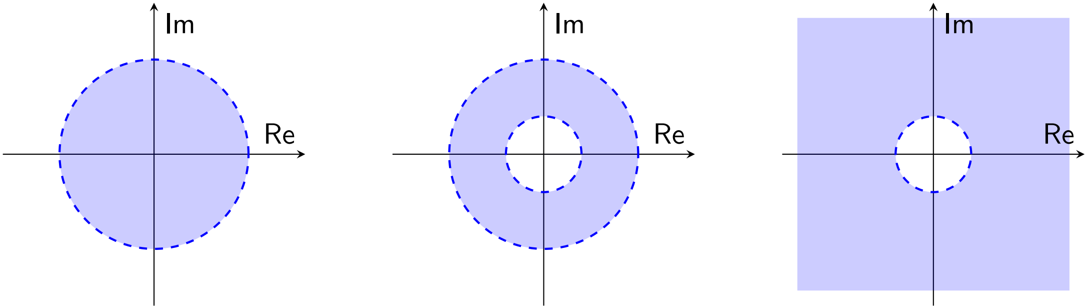
The value of r can be chosen as any value for which $X(z)$ converges – i.e., any value such that the circular contour of integration $|z| = r$ is in the ROC.

Properties of the ROC

Properties of the ROC

Property 1

The ROC of $X(z)$ consists of a ring in the z-plane centered about the origin.



Properties of the ROC

Property 2

The ROC does not contain any poles.

Properties of the ROC

Property 3

If $x[n]$ is of finite duration, then the ROC is the entire z-plane, except possibly $z = 0$ and/or $z = \infty$.

$$
\begin{aligned}
&x_1[n] = \delta[n] && X_1(z) = 1 && \mathrm{ROC} = \mathbb{C} \\
&x_2[n] = \delta[n] + \delta[n-1] && X_2(z) = 1 + z^{-1} && \mathrm{ROC} = \mathbb{C} - \{0\} \\
&x_3[n] = \delta[n+1] + \delta[n] && X_3(z) = z + 1 && \mathrm{ROC} = \mathbb{C} - \{\infty\} \\
&x_4[n] = \delta[n+1] + \delta[n] + \delta[n-1] && X_4(z) = z + 1 + z^{-1} && \mathrm{ROC} = \mathbb{C} - \{0, \infty\}
\end{aligned}
$$

Properties of the ROC

Property 4

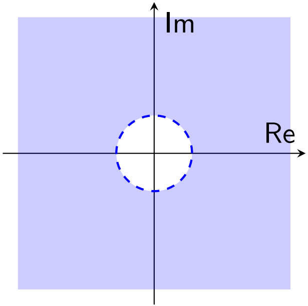
If $x[n]$ is a *right-sided* sequence, and if the circle $|z| = r_0$ is in the ROC, then all finite values of z for which $|z| > r_0$ will also be in the ROC.

Definition

The signal $x[n]$ or $x(t)$ is called **right-sided** if:

$$\exists n_0 \, , \, \forall n < n_0 \; : \; x[n] = 0$$

$$\exists t_0 \, , \, \forall t < t_0 \; : \; x(t) = 0$$



Properties of the ROC

Property 5

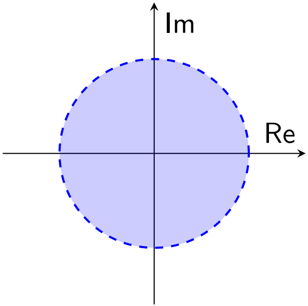
If $x[n]$ is a *left-sided* sequence, and if the circle $|z| = r_0$ is in the ROC, then all values of z for which $0 < |z| < r_0$ will also be in the ROC.

Definition

The signal $x[n]$ or $x(t)$ is called **left-sided** if:

$$\exists n_0 \;,\; \forall n > n_0 \;:\; x[n] = 0$$

$$\exists t_0 \;,\; \forall t > t_0 \;:\; x(t) = 0$$



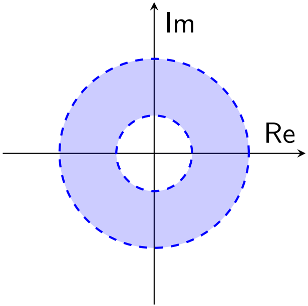
Properties of the ROC

Property 6

If $x[n]$ is two sided, and if the circle $|z| = r_0$ is in the ROC, then the ROC will consist of a ring in the z-plane that includes the circle $|z| = r_0$.

Definition

The signal $x[n]$ or $x(t)$ is called **two sided** if it is neither left-sided nor right-sided.



Properties of the ROC

Property 7

If the z-transform $X(z)$ of $x[n]$ is rational, then its ROC is bounded by poles or extends to infinity.

Properties of the ROC

Property 8

If the z-transform $X(z)$ of $x[n]$ is rational, and if $x[n]$ is right sided, then the ROC is the region in the z-plane outside the outermost pole – i.e., outside the circle of radius equal to the largest magnitude of the poles of $X(z)$. Furthermore, if $x[n]$ is causal (i.e., if it is right sided and equal to 0 for $n < 0$), then the ROC also includes $z = \infty$.

Properties of the ROC

Property 9

If the z-transform $X(z)$ of $x[n]$ is rational, and if $x[n]$ is left sided, then the ROC is the region in the z-plane inside the innermost nonzero pole – i.e., inside the circle of radius equal to the smallest magnitude of the poles of $X(z)$ other than any at $z = 0$ and extending inward to and possibly including $z = 0$. In particular, if $x[n]$ is *anticausal* (i.e., if it is left sided and equal to 0 for $n > 0$), then the ROC also includes $z = 0$.

Example #3

$$X(z) = \frac{z^2 - \frac{3}{2}z}{z^2 - \frac{5}{6}z + \frac{1}{6}} \Rightarrow x[n] = ?$$

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$$\begin{aligned} X(z) &= \frac{z(z - \frac{3}{2})}{(z - \frac{1}{3})(z - \frac{1}{2})} \\ &= \frac{7}{1 - \frac{1}{3}z^{-1}} - \frac{6}{1 - \frac{1}{2}z^{-1}} \end{aligned}$$

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Do you remember the results from examples #1 and #2?

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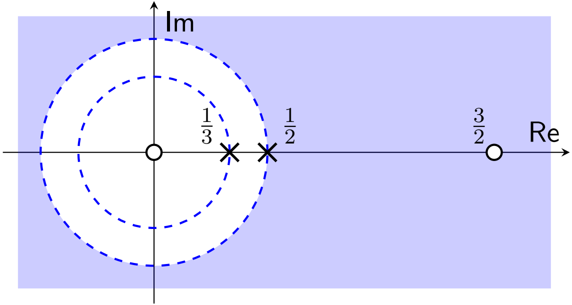
$$X(z) = \frac{z(z-\frac{3}{2})}{(z-\frac{1}{3})(z-\frac{1}{2})} \Rightarrow x[n] = 7\left(\frac{1}{3}\right)^n u[n] - 6\left(\frac{1}{2}\right)^n u[n]$$



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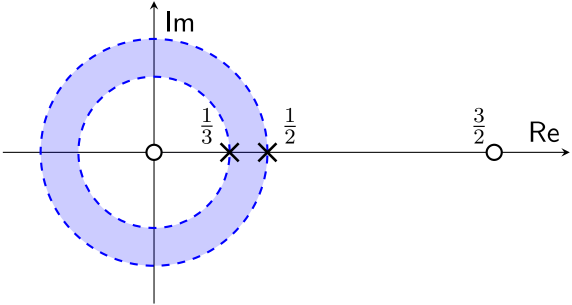

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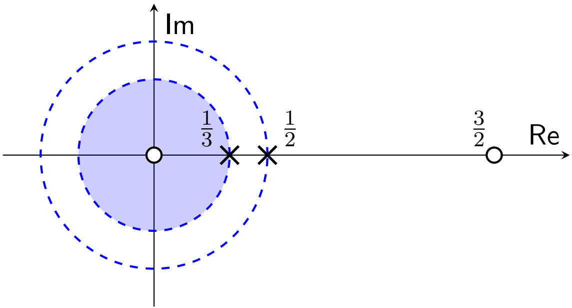

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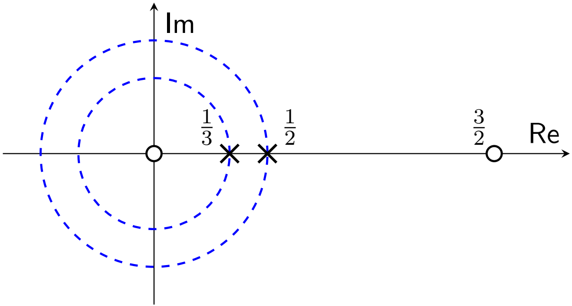

Properties of the z-Transform

Linearity

If

$$x_1[n] \xleftrightarrow{\mathcal{Z}} X_1(z), \text{ with ROC} = R_1$$

and

$$x_2[n] \xleftrightarrow{\mathcal{Z}} X_2(z), \text{ with ROC} = R_2$$

then

Property 1

$$ax_1[n] + bx_2[n] \xleftrightarrow{\mathcal{Z}} aX_1(z) + bX_2(z)$$

with ROC containing $R_1 \cap R_2$

Time Shifting

If

$$x[n] \xleftrightarrow{\mathcal{Z}} X(z), \text{ with ROC} = R$$

then

Property 2

$$x[n-n_0] \xleftrightarrow{\mathcal{Z}} z^{-n_0}X(z)$$

with $\mathrm{ROC} = R$, except for the possible addition or deletion of the origin or infinity.

Scaling in the z-domain

If

$$x[n] \xleftrightarrow{\mathcal{Z}} X(z), \text{ with ROC} = R$$

then

Property 3

$$z_0^n x[n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{z}{z_0}\right)$$

$$\text{with ROC} = |z_0| R$$

Time Reversal

If

$$x[n] \xleftrightarrow{\mathcal{Z}} X(z), \text{ with ROC} = R$$

then

Property 4

$$x[-n] \xleftrightarrow{\mathcal{Z}} X\left(\frac{1}{z}\right)$$

$$\text{with ROC} = \frac{1}{R}$$

Time Expansion

If

$$x[n] \overset{\mathcal{Z}}{\longleftrightarrow} X(z), \text{ with ROC} = R$$

and

$$x_{(k)}[n] = \begin{cases} x[n/k] & \text{if } n \text{ is a multiple of } k \\ 0 & \text{if } n \text{ is not a multiple of } k \end{cases}$$

then

Property 5

$$x_{(k)}[n] \overset{\mathcal{Z}}{\longleftrightarrow} X(z^k)$$

$$\text{with ROC} = R^{1/k}$$

Conjugation

If

$$x[n] \xleftrightarrow{\mathcal{Z}} X(z), \text{ with ROC} = R$$

then

Property 6

$$x^*[n] \xleftrightarrow{\mathcal{Z}} X^*(z^*)$$

$$\text{with ROC} = R$$

The Convolution Property

If

$$x_1[n] \xleftrightarrow{\mathcal{Z}} X_1(z), \text{ with } \mathrm{ROC} = R_1$$

and

$$x_2[n] \xleftrightarrow{\mathcal{Z}} X_2(z), \text{ with } \mathrm{ROC} = R_2$$

then

Property 7

$$x_1[n] * x_2[n] \xleftrightarrow{\mathcal{Z}} X_1(z)X_2(z)$$

with ROC containing $R_1 \cap R_2$

Differentiation in the z-Domain

If

$$x[n] \xleftrightarrow{\mathcal{Z}} X(z), \text{ with ROC} = R$$

then

Property 8

$$nx[n] \xleftrightarrow{\mathcal{Z}} -z\frac{\mathrm{d}X(z)}{\mathrm{d}z}$$

$$\text{with ROC} = R$$

Difference

If

$$x[n] \xleftrightarrow{\mathcal{Z}} X(z), \text{ with ROC} = R$$

then

Property 9

$$x[n] - x[n-1] \xleftrightarrow{\mathcal{Z}} (1 - z^{-1})X(z)$$

with ROC containing $R \cap \{|z| > 0\}$

Accumulation

If

$$x[n] \xleftrightarrow{\mathcal{Z}} X(z), \text{ with ROC} = R$$

then

Property 10

$$\sum_{k=-\infty}^{n} x[k] \xleftrightarrow{\mathcal{Z}} \frac{X(z)}{1 - z^{-1}}$$

with ROC containing $R \cap \{|z| > 1\}$

The Initial-Value Theorem

Property 11

If $\forall n < 0 \ : \ x[n] = 0$ then:

$$x[0] = \lim_{z \to \infty} X(z)$$

Also we can find $x[n]$ for larger values of n similarly:

$$x[1] = \lim_{z \to \infty} z\left(X(z) - x[0]\right)$$

$$x[2] = \lim_{z \to \infty} z^2\left(X(z) - x[0] - x[1]z^{-1}\right)$$

$$x[3] = \lim_{z \to \infty} z^3\left(X(z) - x[0] - x[1]z^{-1} - x[2]z^{-2}\right)$$

$$x[4] = \lim_{z \to \infty} z^4\left(X(z) - x[0] - x[1]z^{-1} - x[2]z^{-2} - x[3]z^{-3}\right)$$

$$\dots$$

The Final-Value Theorem

Property 12

If $x[n]$ is right-handed and $\lim_{n\to\infty} x[n]$ exists, then:

$$\lim_{n\to\infty} x[n] = \lim_{z\to 1}(1 - z^{-1})X(z)$$

Example #4

$$X(z) = \frac{3z^{-3}}{\left(1 - \frac{1}{4}z^{-1}\right)^2} \Rightarrow x[n] = ? \quad , \quad x[n] \text{ is left-sided}$$

Example #4

$$X(z) = \frac{3z^{-3}}{\left(1 - \frac{1}{4}z^{-1}\right)^2} \Rightarrow x[n] = ? \quad , \quad x[n] \text{ is left-sided}$$

$$\frac{1}{1 - \frac{1}{4}z^{-1}} \xleftrightarrow{\mathcal{Z}} -\left(\frac{1}{4}\right)^n u[-n-1]$$

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$$\frac{1}{1 - \frac{1}{4}z^{-1}} \xleftrightarrow{\mathcal{Z}} -\left(\frac{1}{4}\right)^n u[-n-1]$$

$$-z\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{1 - \frac{1}{4}z^{-1}}\right) = \frac{\frac{1}{4}z^{-1}}{\left(1 - \frac{1}{4}z^{-1}\right)^2} \xleftrightarrow{\mathcal{Z}} -n\left(\frac{1}{4}\right)^n u[-n-1]$$

Example #4

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$$-z\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{1 - \frac{1}{4}z^{-1}}\right) = \frac{\frac{1}{4}z^{-1}}{\left(1 - \frac{1}{4}z^{-1}\right)^2} \xleftrightarrow{\mathcal{Z}} -n\left(\frac{1}{4}\right)^n u[-n-1]$$

$$12z^{-2}\left(-z\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{1}{1 - \frac{1}{4}z^{-1}}\right)\right) = \frac{3z^{-3}}{\left(1 - \frac{1}{4}z^{-1}\right)^2} \xleftrightarrow{\mathcal{Z}} -12(n-2)\left(\frac{1}{4}\right)^{(n-2)} u[-n+1]$$

z-Transform and LTI Systems

System Transfer Function

The input-output relationship for an LTI system with impulse response $h[n]$:

$$y[n] = h[n] * x[n]$$

Using the convolution property of the z-transform:

$$Y(z) = H(z)X(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

Definition

$H(z)$ is called the **transfer function** (or **system function**) of the system.

Note that $H(z)$ is the z-transform of the impulse response of the system.

Causality of LTI Systems

Theorem

A discrete-time LTI system is causal if and only if the ROC of its system function is the exterior of a circle, *including infinity*.

Theorem

A discrete-time LTI system with rational system function $H(z)$ is causal if and only if:

(a) the ROC is the exterior of a circle outside the outermost pole.

(b) with $H(z)$ expressed as a ratio of polynomials in z, the order of the numerator cannot be greater than the order of the denominator.

Stability of LTI Systems

Theorem

An LTI system is stable if and only if the ROC of its system function $H(z)$ includes the unit circle, $|z| = 1$.

Example #5

Find the impulse response of the system described by the difference equation

$$y[n] - \frac{5}{2}y[n-1] + y[n-2] = 2x[n] - \frac{5}{2}x[n-1]$$

if the system is:

(a) causal.

(b) stable.

(c) non-causal and unstable.

Example #5

$$y[n] - \frac{5}{2}y[n-1] + y[n-2] = 2x[n] - \frac{5}{2}x[n-1]$$

$$Y(z) - \frac{5}{2}z^{-1}Y(z) + z^{-2}Y(z) = 2X(z) - \frac{5}{2}z^{-1}X(z)$$

Example #5

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$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - \frac{5}{2}z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}$$

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$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - \frac{5}{2}z^{-1}}{1 - \frac{5}{2}z^{-1} + z^{-2}}$$

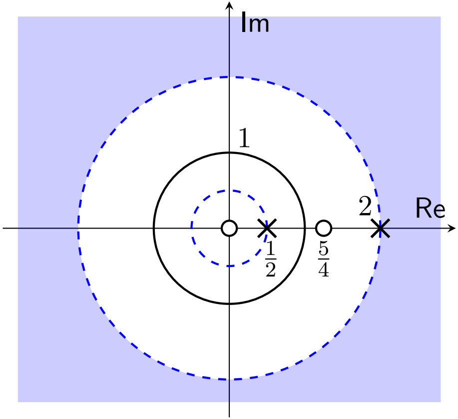
$$= \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 - 2z^{-1}}$$

Example #5

For a causal system:

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 - 2z^{-1}}$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] + 2^n u[n]$$

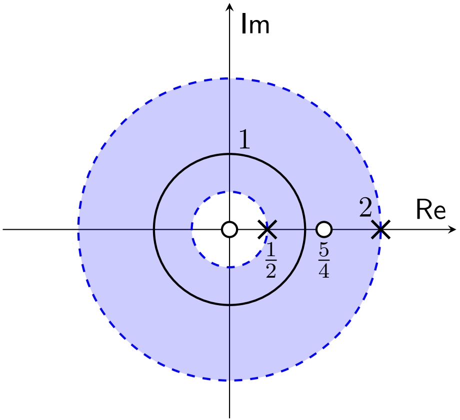


Example #5

For a stable system:

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 - 2z^{-1}}$$

$$h[n] = \left(\frac{1}{2}\right)^n u[n] - 2^n u[-n-1]$$



Example #5

For a non-causal and unstable system:

$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 - 2z^{-1}}$$

$$h[n] = -\left(\frac{1}{2}\right)^n u[-n-1] - 2^n u[-n-1]$$