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Analog IC Design

Project

Universal Bi-quadratic Filter
(Parameters Design and Simulations)

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Universal Biquadratic Filter

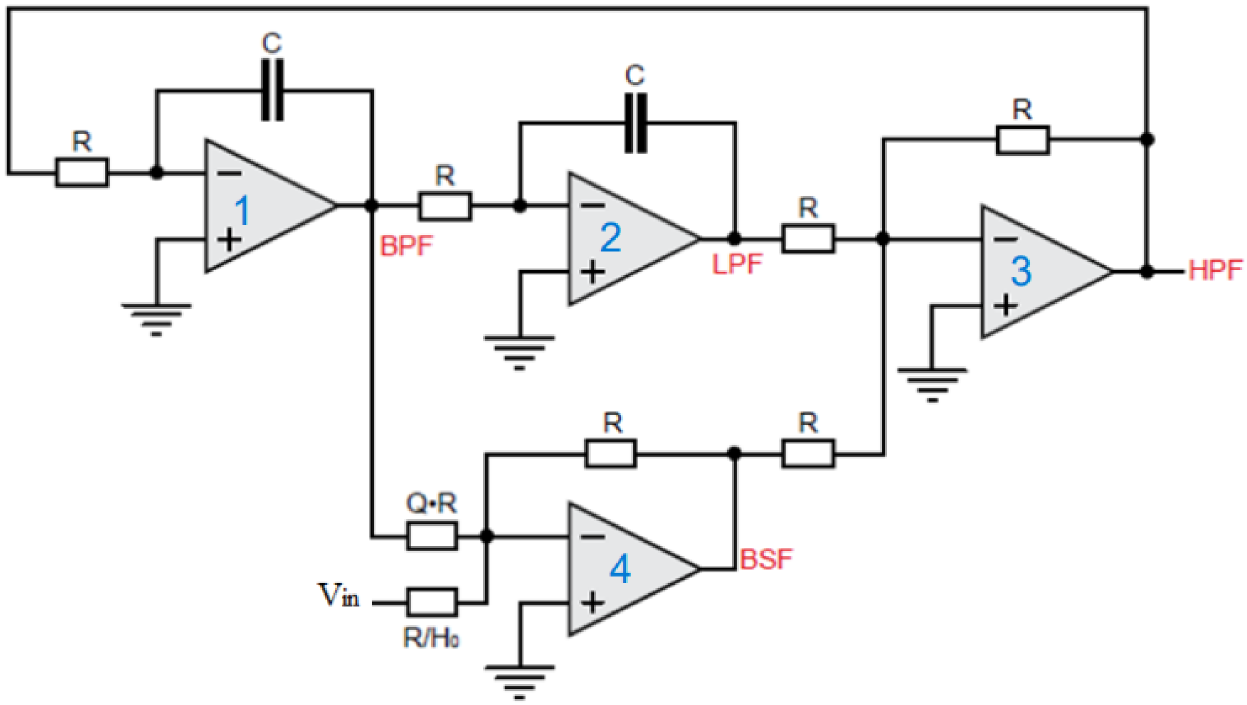

Figure 1: Circuit Diagram from the project statement, I numbered op-amps in blue to make Derivations

1- Derivation and Components Design.

Assume all op-amps are ideal and in negative feedback. Output of each op-amp is given as: $V_{out} = A_v\,(V_{in+} - V_{in-})$. Since the output is finite, While $A_v \to \infty$, then $(V_{in+} = V_{in-})$. We approach the derivation by doing KCL at input nodes using s-domain.

KCL for op-amp 1: $\frac{V_{HPF}}{R} = -V_{BPF}\,(sC).$ $\quad\therefore V_{HPF} = (-sCR)\,V_{BPF}$ →1

KCL for op-amp 2: $\frac{V_{BPF}}{R} = -V_{LPF}\,(sC).$ $\quad\therefore V_{BPF} = (-sCR)\,V_{LPF}$ →2

KCL for op-amp 3: $\frac{V_{LPF}}{R} + \frac{V_{BSF}}{R} = \frac{-V_{HPF}}{R}.$ $\quad\therefore V_{HPF} = -V_{LPF} - V_{BSF}$ →3

KCL for op-amp 4: $\frac{V_{in}}{R/H} + \frac{V_{BPF}}{R.Q} = \frac{-V_{BSF}}{R}.$ $\quad\therefore \text{H.}V_{in} + \frac{1}{Q}\,V_{BPF} = -V_{BSF}$ →4

From 3, $-V_{BSF} = V_{HPF} + V_{LPF}$, Substitute in 4. $\quad\therefore \text{H.}V_{in} + \frac{1}{Q}\,V_{BPF} = V_{HPF} + V_{LPF}$

From 2, $V_{BPF} = (-sCR)\,V_{LPF}$, From 1, $V_{HPF} = (-sCR)\,V_{BPF} = (sCR)^2\,V_{LPF}$

Substitute to make the equation in terms of V_{LPF} $\quad\therefore \text{H.}V_{in} + \frac{-(sCR)}{Q}\,V_{LPF} = (sCR)^2\,V_{LPF} + V_{LPF}$

$$\therefore \text{H.}V_{in} = \left(s^2C^2R^2 + \frac{(sCR)}{Q} + 1\right)V_{LPF} \qquad \therefore \frac{V_{LPF}}{V_{in}} = \frac{H}{s^2C^2R^2 + \frac{sCR}{Q} + 1} = \frac{\left(\frac{H}{C^2R^2}\right)}{s^2 + \frac{s}{QRC} + \left(\frac{1}{C^2R^2}\right)}$$

Now substitute in 2, $V_{BPF} = (-sCR)\ V_{LPF}$, $\therefore \frac{V_{BPF}}{V_{in}} = \frac{-H.(sCR)}{s^2C^2R^2 + \frac{sCR}{Q} + 1} = \frac{-H\ (\frac{s}{RC})}{s^2 + \frac{s}{QRC} + (\frac{1}{C^2R^2})}$

Now substitute in 1, $V_{HPF} = (-sCR)\ V_{BPF}$, $\therefore \frac{V_{HPF}}{V_{in}} = \frac{H.\ s^2C^2R^2}{s^2C^2R^2 + \frac{sCR}{Q} + 1} = \frac{H.s^2}{s^2 + \frac{s}{QRC} + (\frac{1}{C^2R^2})}$

Now substitute in 4, $-V_{BSF} = V_{HPF} + V_{LPF}$, $\therefore \frac{V_{BSF}}{V_{in}} = \frac{-H.\ (s^2C^2R^2+1)}{s^2C^2R^2 + \frac{sCR}{Q} + 1} = \frac{-H.\ (s^2+(\frac{1}{C^2R^2}))}{s^2 + \frac{s}{QRC} + (\frac{1}{C^2R^2})}$

Below is a list of transfer functions' standard forms of second order filters. The blue highlighted parameter represents an additional gain (and phase in case of negative sign) to the standard form:

$H_{LPF}(s) = \frac{{\omega_o}^2}{(s^2 + \frac{\omega_o}{Q}s + {\omega_o}^2)} \equiv \frac{H.(\frac{1}{C^2R^2})}{s^2 + \frac{s}{QRC} + (\frac{1}{C^2R^2})}$. $H_{HPF}(s) = \frac{s^2}{(s^2 + \frac{\omega_o}{Q}s + {\omega_o}^2)} \equiv \frac{H.\ s^2}{s^2 + \frac{s}{QRC} + (\frac{1}{C^2R^2})}$

$H_{BPF}(s) = \frac{\frac{\omega_o}{Q}s}{(s^2 + \frac{\omega_o}{Q}s + {\omega_o}^2)} \equiv \frac{-HQ\ (\frac{s}{QRC})}{s^2 + \frac{s}{QRC} + (\frac{1}{C^2R^2})}$. $H_{BSF}(s) = \frac{s^2 + {\omega_o}^2}{(s^2 + \frac{\omega_o}{Q}s + {\omega_o}^2)} \equiv \frac{-H\ .\ (s^2+(\frac{1}{C^2R^2}))}{s^2 + \frac{s}{QRC} + (\frac{1}{C^2R^2})}$

Fast intuition on parameters:

H: DC gain (the frequency response should be shifted vertically upwards by 20 log |H|).

Q: Quality factor of filter, it can represent $(\frac{\omega_o}{BW})$ graphically.

ω_o: Cut-off frequency in rad/sec. $\omega_o = \frac{1}{RC}$.

Design required:

f_o=1 MHz (ω_o= $2\pi f_o$ = $6.283\times 10^6\ rad/sec$) Q=2.2 H=1

Using the cut-off frequency we get, RC = $1/\omega_o$ = 1.59155×10^{-7} sec. This is the Design Curve.

In IC design, component values get translated into area. Capacitance, C = 1fF/ $\mu\boldsymbol{m}^2$ and typically capacitor size ranges between 10 pF (100 µm x 100 µm) to 100 fF (10 µm x 10 µ). For resistor values, a huge resistor will increase thermal noise power level. S (f) = 4KTR. However, a small resistor will lead to high current and power consumption. Resistors are in kilo ohms typically.

The following figure shows our design space:

Green area: 1 kΩ < R < 100 kΩ
Pink area: 1fF < C < 10 pF

Design point chosen **A**

C = 5.4 pF R = 29.4 kΩ
R/H = 29.4 kΩ R*Q = 64.5 kΩ

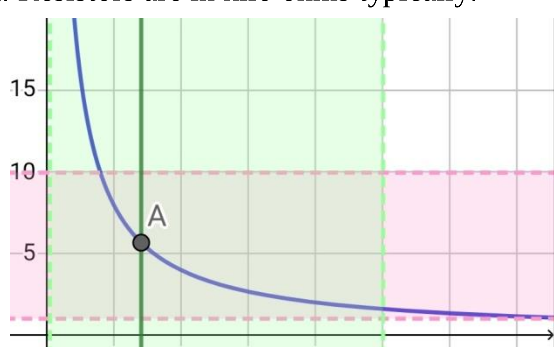

Figure 2: x-axis represents resistance in KΩ
y-axis represents the Capacitance in pF

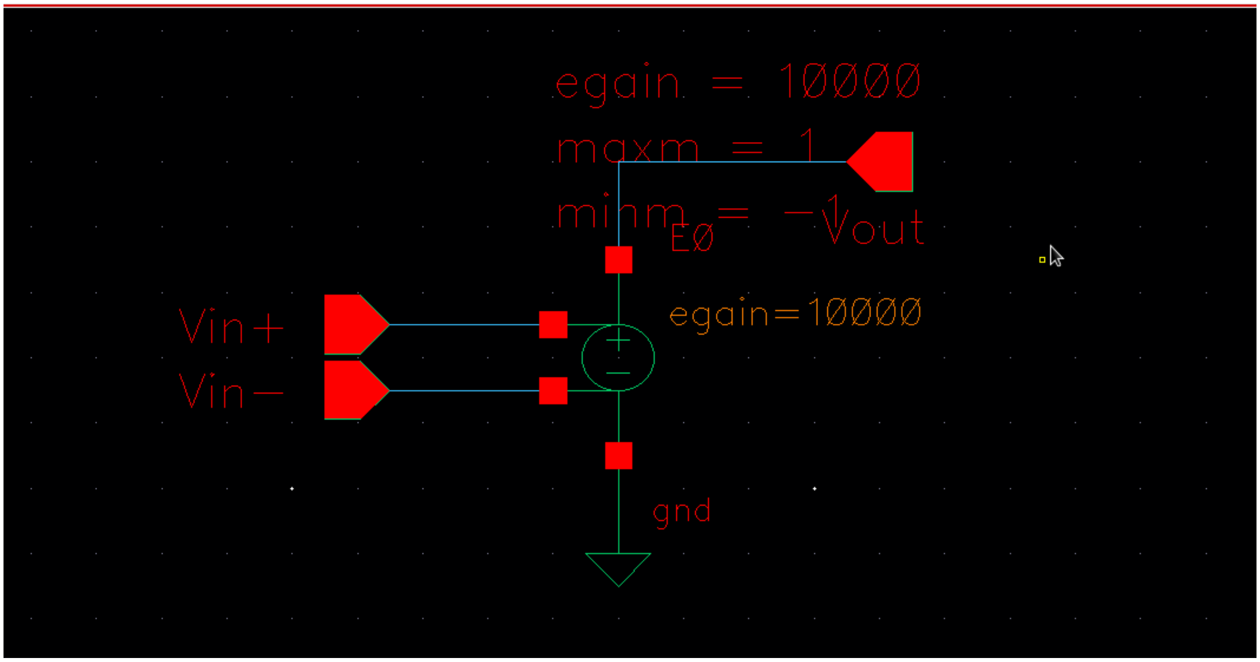

Figure: 3 Schematic of ideal op-amp

We modeled it as a voltage-controlled voltage source with volt gain = 10,000. And a maximum $|V_{out}| < 1$ volt. Note that we don't have any pole. Also we don't need a buffer stage. Because the resistance seen (Rout = 0). Volt source is modeled as a short circuit in small signal model.

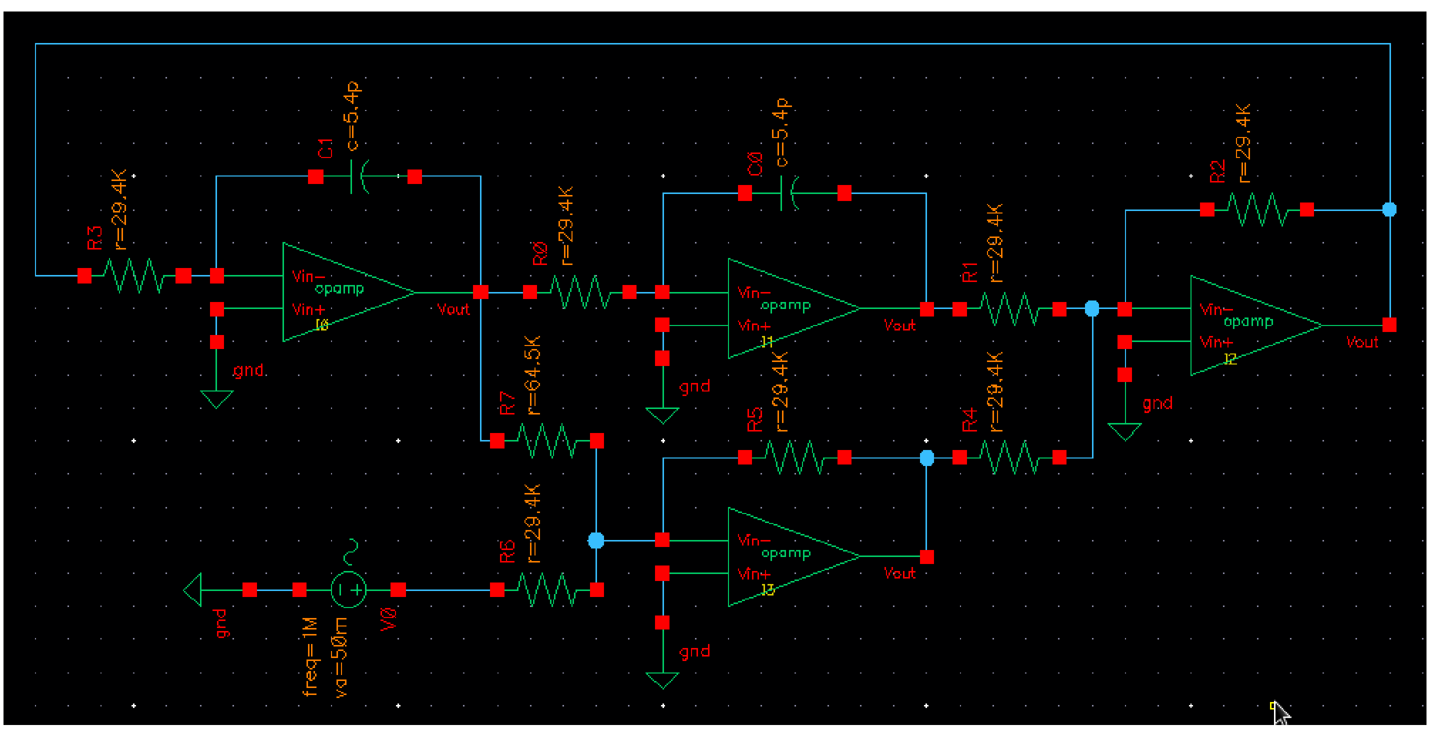

Figure: 4 Schematic of Biquadratic Filter with components values visible on schematic

2- **Frequency Response.**

2.1. Low Pass Filter Frequency Response: (s = jω)

$$H_{LPF}(s) = \frac{{\omega_o}^2}{(s^2 + \frac{\omega_o}{Q}s + {\omega_o}^2)}, \quad H_{LPF}(\omega) = \frac{{\omega_o}^2}{({\omega_o}^2 - \omega^2) + j\frac{\omega\omega_o}{Q}}, \quad |H_{LPF}(\omega)| = \frac{{\omega_o}^2}{\sqrt{({\omega_o}^2 - \omega^2)^2 + (\frac{\omega\omega_o}{Q})^2}}.$$

$@(\omega \to 0) \quad \therefore |H_{LPF}(\omega)| = 1, \quad < \theta = 0°$

$@(\omega \to \infty) \quad \therefore |H_{LPF}(\omega)| = 0, \quad < \theta = -180°$

$@(\omega \to \omega_o) \quad \therefore H_{LPF}(\omega) = \frac{{\omega_o}^2}{j\frac{{\omega_o}^2}{Q}} = \frac{Q}{j}$ $\quad \therefore |H_{LPF}(\omega_o)| = Q, \quad < \theta_{@\omega_o} = -90°$

$@(\omega \to \omega_{peak})$, max magnitude $\quad \therefore \frac{d|H(\omega)|}{d\omega} = 0, \quad \therefore 2({\omega_o}^2 - \omega^2)(-2\omega) + 2(\frac{\omega\omega_o}{Q})(\frac{\omega_o}{Q}) = 0$

Let $\omega \neq 0$, then divide by 2 ω $\quad \therefore 2(\omega^2 - {\omega_o}^2) = -(\frac{\omega_o}{Q})^2 \quad \therefore \omega^2 = {\omega_o}^2(1 - \frac{1}{2Q^2})$

$\omega_{peak} = \omega_o\sqrt{1 - \frac{1}{2Q^2}}$ $\quad \rightarrow$ will give real value only if Q > $\frac{1}{\sqrt{2}}$

$$|H_{LPF}(\omega_{peak})| = \frac{{\omega_o}^2}{\sqrt{({\omega_o}^2 - {\omega_{peak}}^2)^2 + (\frac{\omega_{peak}\omega_o}{Q})^2}} = \frac{{\omega_o}^2}{\sqrt{\frac{{\omega_o}^4}{4Q^4} + \frac{{\omega_o}^4}{Q^2}(1 - \frac{1}{2Q^2})}} = \frac{Q^2}{\sqrt{Q^2 - \frac{1}{4}}}$$ slightly > Q

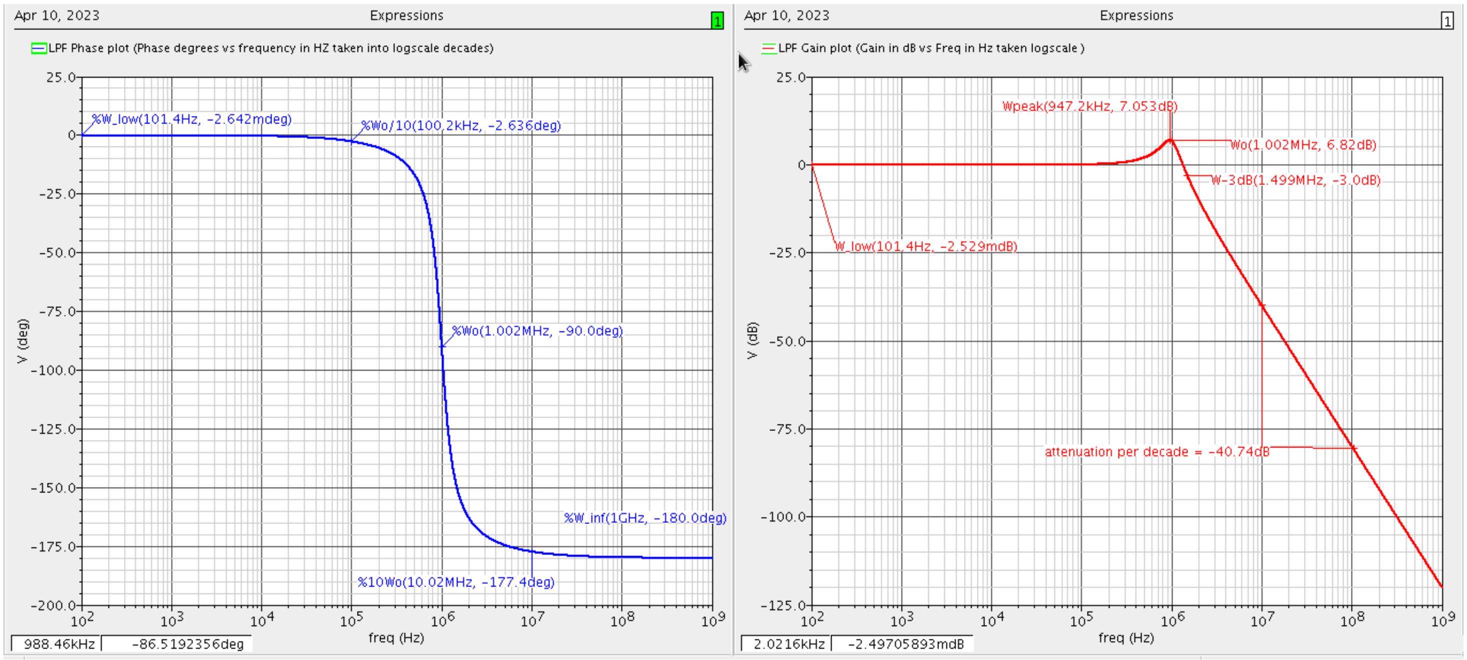

Figure 5: LPF frequency response: magnitude and phase plots

Now, let’s compare the measured quantities vs. the theoretical quantities by derivations for LPF

	Measured at Cadence	Theoretically
Cut-off frequency (ω_o)	From phase plot At phase = - 90° f_o = 1.002 MHz, by marker ω_o= $6.296 \times 10^6\ rad/sec$	C = 5.4 pF R = 29.4 kΩ Q = 2.194 $H = 1$ $H_{LPF}(s) = \frac{H.(\frac{1}{C^2R^2})}{s^2 + \frac{s}{QRC} + (\frac{1}{C^2R^2})} = \frac{H.\omega_o{}^2}{(s^2 + \frac{\omega_o}{Q}s + \omega_o{}^2)}$ $\omega_o = \frac{1}{RC} = 6.3\times 10^6\ rad/sec$ f_o = 1.0024 MHz
DC Gain (H)	At low frequency, $\lvert H_{LPF}(\omega_{\_low})\rvert$ = 20 log |H| H = 0 dB = 1	Defined as a design parameter (ratio between resistors) H = $\left(\frac{29.4\ k\Omega}{29.4 k\Omega}\right)$ = 1
Quality Factor (Q)	From magnitude plot $@\omega_o,\ \lvert H_{LPF}(\omega_o)\rvert$ = Q = 6.82 dB Q = 6.82 dB = 2.1928	Defined as a design parameter (ratio between resistors) Q = $\left(\frac{64.5\ k\Omega}{29.4 k\Omega}\right)$ = 2.194
Peaking Frequency (ω_{peak})	f_{peak} = 947.2 kHz, by marker A_{peak} = 7.053 dB, by marker A_{peak} = 2.2524	$f_{peak} = f_o\sqrt{1 - \frac{1}{2Q^2}}$ = 948.9 kHz A = $Q^2/\sqrt{Q^2 - \frac{1}{4}}$ = 2.2533

2.2. High Pass Filter Frequency Response: (s = jω)

$$H_{HPF}(s) = \frac{s^2}{(s^2 + \frac{\omega_o}{Q}s + \omega_o{}^2)}, \quad H_{HPF}(\omega) = \frac{-\omega^2}{(\omega_o{}^2 - \omega^2) + j\frac{\omega\omega_o}{Q}}, \quad |H_{HPF}(\omega)| = \frac{\omega^2}{\sqrt{(\omega_o{}^2 - \omega^2)^2 + (\frac{\omega\omega_o}{Q})^2}}.$$

$@(\omega \to 0)$ $\therefore |H_{HPF}(\omega)| = 0,$ $< \theta = 180°$ (divide by ω^2 *and take limits* $= \frac{-1}{\infty}$)

$@(\omega \to \infty)$ $\therefore |H_{HPF}(\omega)| = 1,$ $< \theta = 0°$ $\omega_{peak} = \omega_o\sqrt{1 + \frac{1}{2Q^2}}$

$@(\omega \to \omega_o)$ $\therefore H_{HPF}(\omega) = \frac{-\omega_o{}^2}{j\frac{\omega_o{}^2}{Q}} = \frac{-Q}{j}$ $\therefore |H_{HPF}(\omega_o)| = Q,$ $< \theta_{@\omega_o} = 90°$

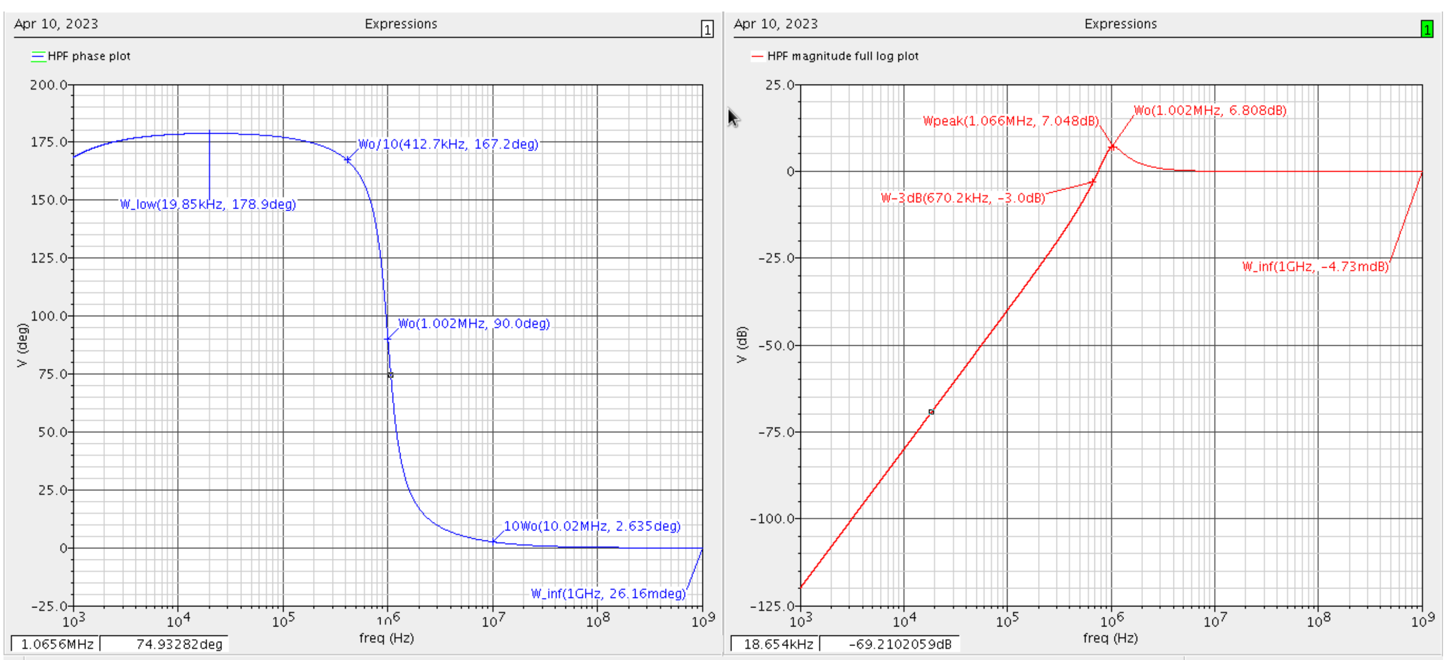

Figure 6: HPF frequency response: magnitude and phase plots

	Measured at Cadence	Theoretically
Cut-off frequency (ω_o)	From phase plot At phase = 90° f_o = 1.002 MHz, by marker ω_o= 6.296 × 10^6 rad/sec	C = 5.4 pF R = 29.4 kΩ Q = 2.194 $H = 1$ H_{HPF}(s) = $\frac{H.s^2}{s^2+\frac{s}{QRC}+(\frac{1}{C^2R^2})}$ = $\frac{H.s^2}{(s^2+\frac{\omega_o}{Q}s+\omega_o{}^2)}$ $\omega_o = \frac{1}{RC}$ = 6.3× 10^6 rad/sec f_o = 1.0024 MHz
Gain (H)	At high frequency, $\lvert H_{HPF}(\omega\_inf)\rvert$ = 20 log |H| H = 0 dB = 1	Defined as a design parameter (ratio between resistors) H = $\left(\frac{29.4\ k\Omega}{29.4k\Omega}\right)$ = 1
Quality Factor (Q)	From magnitude plot $@\omega_o$, $\lvert H_{HPF}(\omega_o)\rvert$ = Q = 6.808 dB Q = 6.808 dB = 2.19	Defined as a design parameter (ratio between resistors) Q = $\left(\frac{64.5\ k\Omega}{29.4k\Omega}\right)$ = 2.194

2.3. Band Pass Filter Frequency Response: (s = jω)

$$H_{BPF}(s) = \frac{\frac{\omega_o}{Q}s}{(s^2+\frac{\omega_o}{Q}s+\omega_o{}^2)}, \qquad H_{BPF}(\omega) = \frac{j\frac{\omega\omega_o}{Q}}{(\omega_o{}^2-\omega^2)+j\frac{\omega\omega_o}{Q}}, \qquad |H_{BPF}(\omega)| = \frac{\frac{\omega\omega_o}{Q}}{\sqrt{(\omega_o{}^2-\omega^2)^2+(\frac{\omega\omega_o}{Q})^2}}.$$

$@(\omega \to 0)$ $\therefore |H_{BPF}(\omega)| = 0,$ $<\theta$ = 90° (divide by $and\ take\ limits = \frac{j}{\infty}$)

$@(\omega \to \infty)$ $\therefore |H_{BPF}(\omega)| = 1,$ $<\theta$ = - 90° ($take\ limits = \frac{j}{-\infty}$)

$@(\omega \to \omega_o)$ $\therefore H_{BPF}(\omega) = \frac{j\frac{\omega\omega_o}{Q}}{j\frac{\omega\omega_o}{Q}} = 1$ $\therefore |H_{BPF}(\omega_o)| = 1,$ $<\theta_{@\omega_o} = 0°$

$@(\omega \to \omega_{1,2}, -3dBs)$ $\therefore |H_{BPF}(\omega)| = \frac{1}{\sqrt{2}} = \frac{\frac{\omega\omega_o}{Q}}{\sqrt{(\omega_o{}^2-\omega^2)^2+(\frac{\omega\omega_o}{Q})^2}}$. (Solve for BW and 3dBs)

$2(\frac{\omega\omega_o}{Q})^2 = (\omega_o{}^2 - \omega^2)^2 + (\frac{\omega\omega_o}{Q})^2$ $\therefore (\omega_o{}^2 - \omega^2)^2 = (\frac{\omega\omega_o}{Q})^2$ (accept only +ve freq)

$\omega^2 + \frac{\omega\omega_o}{Q} - \omega_o{}^2 = 0$ $\therefore\ \omega_{1,2} = \frac{\pm\frac{\omega_o}{Q}+\sqrt{(\frac{\omega_o}{Q})^2+4\omega_o{}^2}}{2} = \pm\frac{\omega_o}{2Q} + \omega_o\sqrt{1+\frac{1}{4Q^2}}$

$\therefore$ BW = $\omega_2 - \omega_1 = \frac{\omega_o}{Q}$ $\therefore\ \omega_o{}^2 = \omega_2\,\omega_1$

Note that these standard derivations are for a function that looks like: $H_{BPF}(s) = \frac{\frac{\omega_o}{Q}s}{(s^2+\frac{\omega_o}{Q}s+\omega_o{}^2)}$

our function looks like: $\frac{-H\,(\frac{s}{RC})}{s^2+\frac{s}{QRC}+(\frac{1}{C^2R^2})} = \frac{-HQ\,(\frac{s}{QRC})}{s^2+\frac{s}{QRC}+(\frac{1}{C^2R^2})}$. This affects derivations by:

Magnitude shifts up by 20 log |QH| ≅ 6.824 dB; Phase shifts down by -180°.

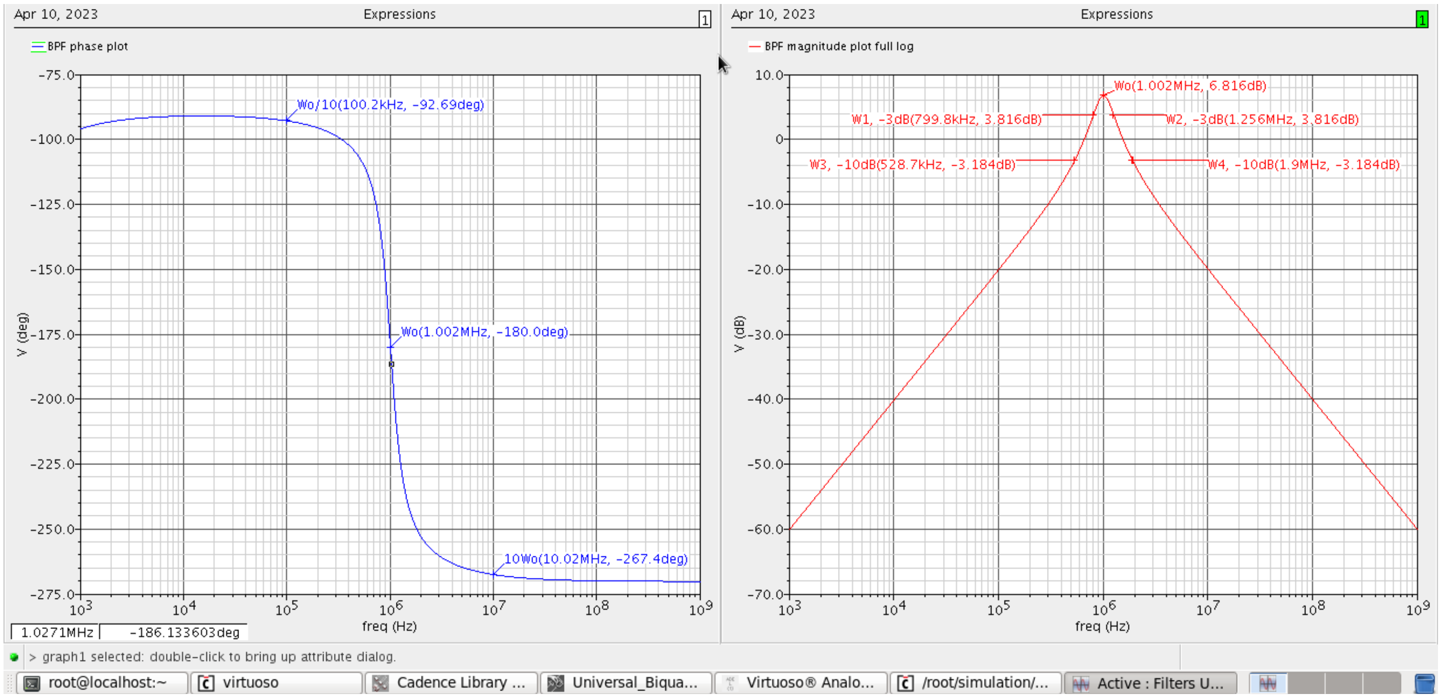

Figure 7: BPF frequency response: magnitude and phase plots

	Measured at Cadence	Theoretically
Cut-off frequency (ω_o)	From phase plot At phase = -180° f_o = 1.002 MHz, by marker ω_o= $6.296 \times 10^6\ rad/sec$	C = 5.4 pF R = 29.4 kΩ Q = 2.194 H = 1 $H_{BPF}(s)=\frac{-HQ\,(\frac{s}{QRC})}{s^2+\frac{s}{QRC}+(\frac{1}{C^2R^2})}. = \frac{-HQ.(\frac{\omega_o}{Q}s)}{(s^2+\frac{\omega_o}{Q}s+\omega_o^2)}$ $\omega_o = \frac{1}{RC}$ = 6.3× $10^6\ rad/sec$ f_o = 1.0024 MHz
3dB points $\omega_{1,2}$	f_1 = 799.8 kHz f_2 = 1.256 MHz Check: $\sqrt{f_1 f_2}$ = 1.00227 MHz	$\omega_{1,2} = \pm\frac{\omega_o}{2Q} + \omega_o\sqrt{1+\frac{1}{4Q^2}}$ f_1 = 799.34 kHz f_2 = 1.256 MHz
Bandwidth BW	BW = f_2 - f_1 = 456.2 kHz	BW =f_2 - f_1= $\frac{f_o}{Q}$ = 456.7 kHz
Quality Factor (Q)	BW = $\frac{\omega_o}{Q}$ Q = $\frac{\omega_o}{BW} = \frac{1.002}{0.4562}$ = 2.196	Defined as a design parameter (ratio between resistors) Q = $\left(\frac{64.5\ k\Omega}{29.4 k\Omega}\right)$ = 2.194
Gain (H) check on H and Q	From magnitude plot $\|H_{BPF}(\omega_o)\|$ = HQ = 6.816 dB 20 log|H| + 20 log|Q| = 6.816 dB 20 log|H| = -0.012 dB H = 1.001 ≅ 1 (note that the error in approx.. is due to Q, but H is 1)	Defined as a design parameter (ratio between resistors) H = $\left(\frac{29.4\ k\Omega}{29.4 k\Omega}\right)$ = 1

2.4. Band Stop Filter Frequency Response: (imaginary zeroes are used to create notch at ω_o)

$$H_{BSF}(s) = \frac{s^2+\omega_o{}^2}{(s^2 + \frac{\omega_o}{Q}s + \omega_o{}^2)}, \qquad H_{BSF}(\omega) = \frac{(\omega_o{}^2 - \omega^2)}{(\omega_o{}^2 - \omega^2) + j\frac{\omega\omega_o}{Q}}, \qquad |H_{BSF}(\omega)| = \frac{(\omega_o{}^2 - \omega^2)}{\sqrt{(\omega_o{}^2 - \omega^2)^2 + (\frac{\omega\omega_o}{Q})^2}}.$$

$@(\omega \to 0)$ $\therefore |H_{BSF}(\omega)| = 1,$ $< \theta = 0°$

$@(\omega \to \infty)$ $\therefore |H_{BSF}(\omega)| = 1,$ $< \theta = 0°$

$@(\omega \to \omega_o)$ $\therefore H_{BSF}(\omega) = 0,$ $\therefore |H_{BSF}(\omega_o)| = 0 = -\infty\ dB, \quad < \theta_{@\omega_o} = 0°$

$@(\omega \to \omega_o{}^-)$ $\therefore H_{BSF}(\omega) = 0,$ $< \theta_{@\omega_o{}^-} = -90°$ (limits $= \frac{+1}{j*\infty}$)

$@(\omega \to \omega_o{}^+)$ $\therefore H_{BSF}(\omega) = 0,$ $< \theta_{@\omega_o{}^+} = 90°$ (limits $= \frac{-1}{j*\infty}$)

Note that 3dB points $\omega_{1,2}$ are same derivation as in BPF $\therefore\ \omega_o{}^2 = \omega_2\,\omega_1$

$$\therefore\ \omega_{1,2} = \frac{\pm\frac{\omega_o}{Q} + \sqrt{(\frac{\omega_o}{Q})^2 + 4\omega_o{}^2}}{2} = \pm\frac{\omega_o}{2Q} + \omega_o\sqrt{1 + \frac{1}{4Q^2}}$$

$$\therefore \mathrm{BW} = \omega_2 - \omega_1 = \frac{\omega_o}{Q}$$

Our function looks like: $\dfrac{-H\,(s^2 + \frac{1}{C^2R^2})}{s^2 + \frac{s}{QRC} + (\frac{1}{C^2R^2})}$. This affects derivations by -180° to all phases.

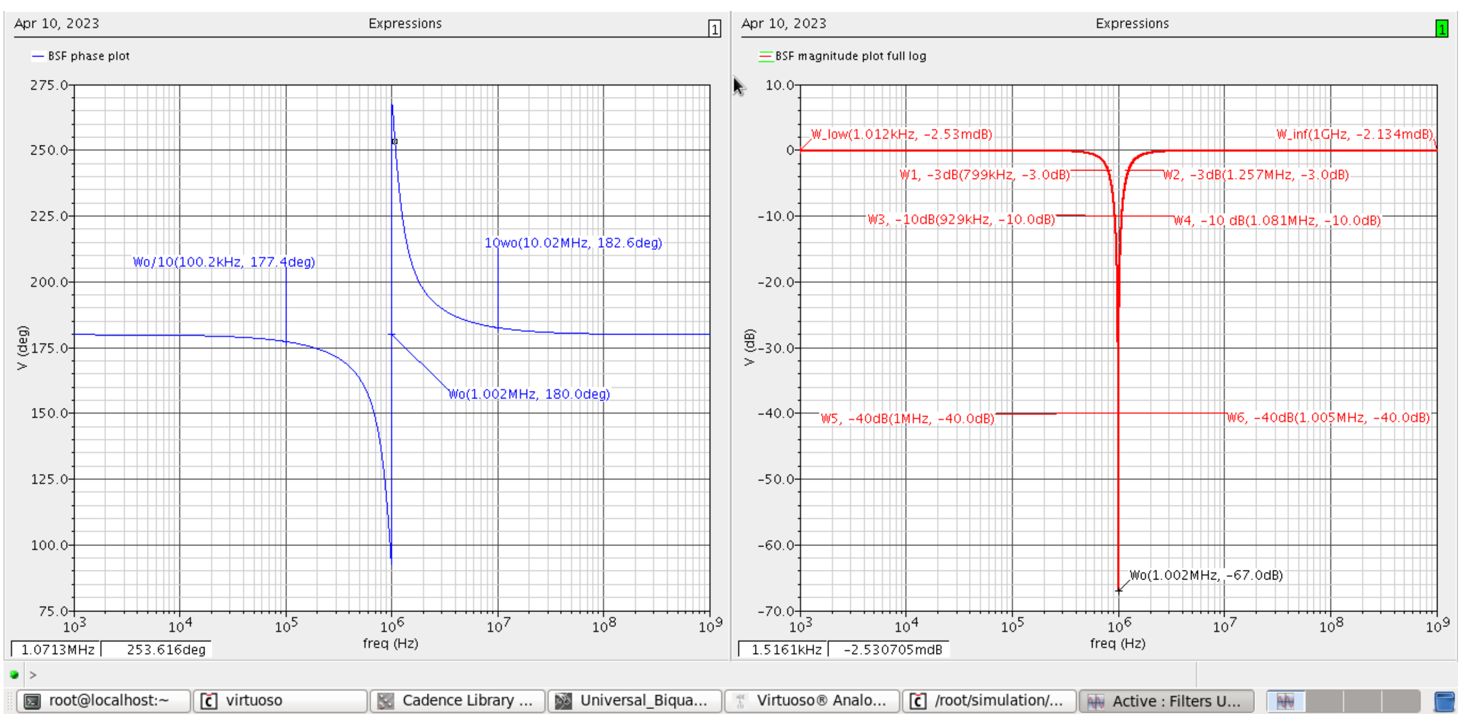

Figure 8: BSF frequency response: magnitude and phase plots

	Measured at Cadence	Theoretically
Cut-off frequency (ω_o)	From Magnitude Plot (notch) At Mag = -67 dB (ideally -∞ dB) f_o = 1.002 MHz, by marker ω_o= 6.296 × 10^6 rad/sec	C = 5.4 pF R = 29.4 kΩ Q = 2.194 $H = 1$ $H_{BSF}(s) = \frac{-H\,(s^2+\frac{1}{C^2R^2})}{s^2+\frac{s}{QRC}+(\frac{1}{C^2R^2})}. = \frac{-H.(s^2+\omega_o{}^2)}{(s^2+\frac{\omega_o}{Q}s+\omega_o{}^2)}$ $\omega_o = \frac{1}{RC}$ = 6.3× 10^6 rad/sec f_o = 1.0024 MHz
3dB points $\omega_{1,2}$	f_1 = 799 kHz f_2 = 1.257 MHz Check: $\sqrt{f_1 f_2}$ = 1.00217 MHz	$\omega_{1,2} = \pm\frac{\omega_o}{2Q} + \omega_o\sqrt{1+\frac{1}{4Q^2}}$ f_1 = 799.34 kHz f_2 = 1.256 MHz
Bandwidth BW	BW = f_2 - f_1 = 458 kHz	BW =f_2 - f_1= $\frac{f_o}{Q}$ = 456.7 kHz
Quality Factor (Q)	BW = $\frac{\omega_o}{Q}$ Q = $\frac{\omega_o}{BW} = \frac{1.002}{0.458}$ = 2.188	Defined as a design parameter (ratio between resistors) Q = $\left(\frac{64.5\ k\Omega}{29.4 k\Omega}\right)$ = 2.194
Gain (H) check on H and Q	From magnitude plot $\lvert H_{BSF}(\omega_{\_low})\rvert$ = H = 0 dB 20 log|H| = 0 dB H = 1	Defined as a design parameter (ratio between resistors) H = $\left(\frac{29.4\ k\Omega}{29.4 k\Omega}\right)$ = 1

There are other definitions of BW like 10 dB bandwidth. Ideally the magnitude at notch is $-\infty$ dB. We can’t get BW in terms of (notch rejection + x dB) in this example.

3- Transient Analysis. (Sine Wave)

Using "Vsin" with following specs:

Amplitude = 100 mV (to avoid output voltage clipping)

Frequency = 1 MHz (f =f_o) → $V_{out} = V_{in} * H(\omega_o)$

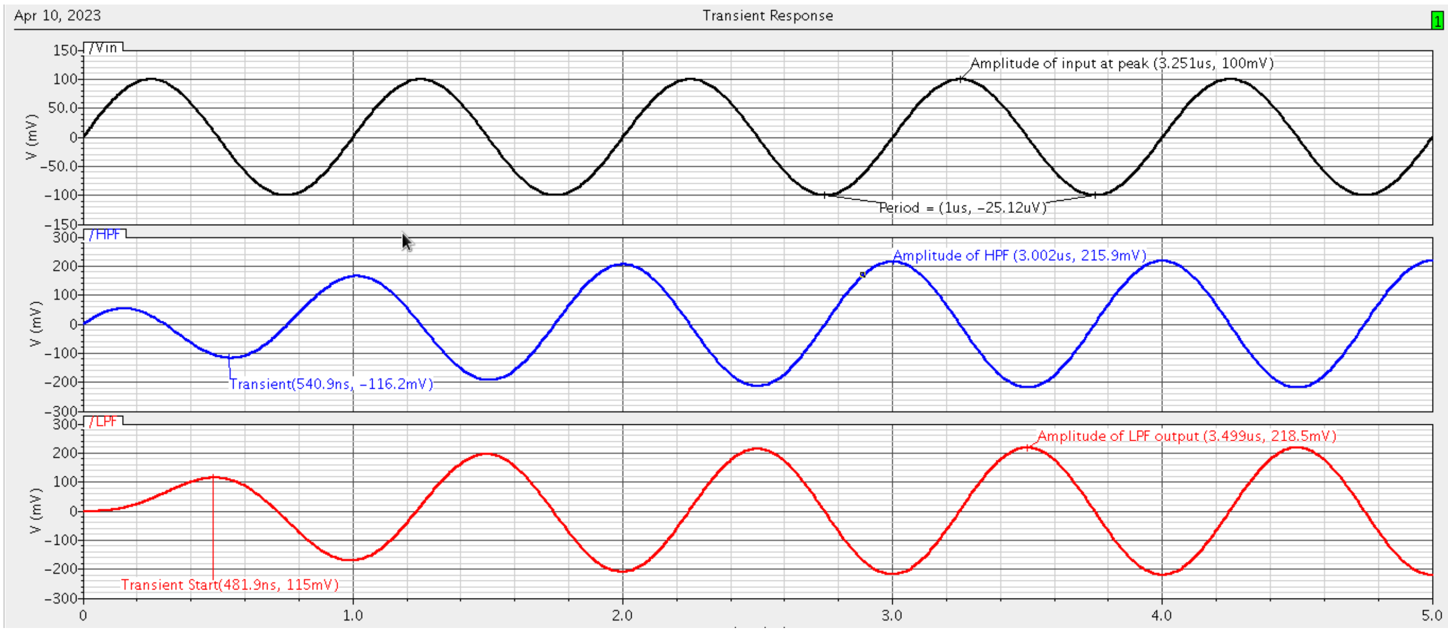

Figure 9: Transient analysis due to sine wave: LPF and HPF outputs

	Measured at Cadence	Theoretically
Amplitudes	$V_{in} = 100\ mv$ $V_{LPF} = 218.5\ mv$ $V_{HPF} = 215.9\ mv$ at steady state values approach 220 mv = Q*100 mv	∴ $\lvert H_{HPF}(\omega_o)\rvert$ = Q ∴ $\lvert H_{LPF}(\omega_o)\rvert$ = Q Q = 2.194 $V_{LPF} = 219.4\ mv\ (Amplitude)$ $V_{HPF} = 219.4\ mv\ (Amplitude)$
Delay (Phase Shift)	V_{HPF} and V_{LPF} are 180° out of phase V_{HPF} peak at 3.002 µsec V_{in} peak at 3.251 µsec V_{LPF} peak at 3.499 µsec T = 1 µsec = λ = 360°. Then if: ΔT_{peak} = 0.25 µsec = $\frac{\lambda}{4}$ = 90° difference in phase ΔT_{peak} = 0.5 µsec = $\frac{\lambda}{2}$ = 180° difference in phase V_{HPF} leads V_{in} by 90°, V_{LPF} lags V_{in} by 90° V_{HPF} leads V_{LPF} by 180°	∴ $\lvert H_{HPF}(\omega_o)\rvert$ = Q,< $\theta_{@\omega_o}$ = 90° ∴ $\lvert H_{LPF}(\omega_o)\rvert$ = Q,< $\theta_{@\omega_o}$ = - 90° V_{HPF} = |Q| V_{in} <90° V_{LPF} = |Q| V_{in} <-90° There is 180° phase difference between V_{HPF} and V_{LPF}

4- Transient Analysis. (Pulse Wave).

Using "Vpulse" with following specs

V1 (V_High) = 100 mv (to avoid output voltage clipping), V2 (V_Low) = 0 v

Frequency = 1 MHz (f =f_o)

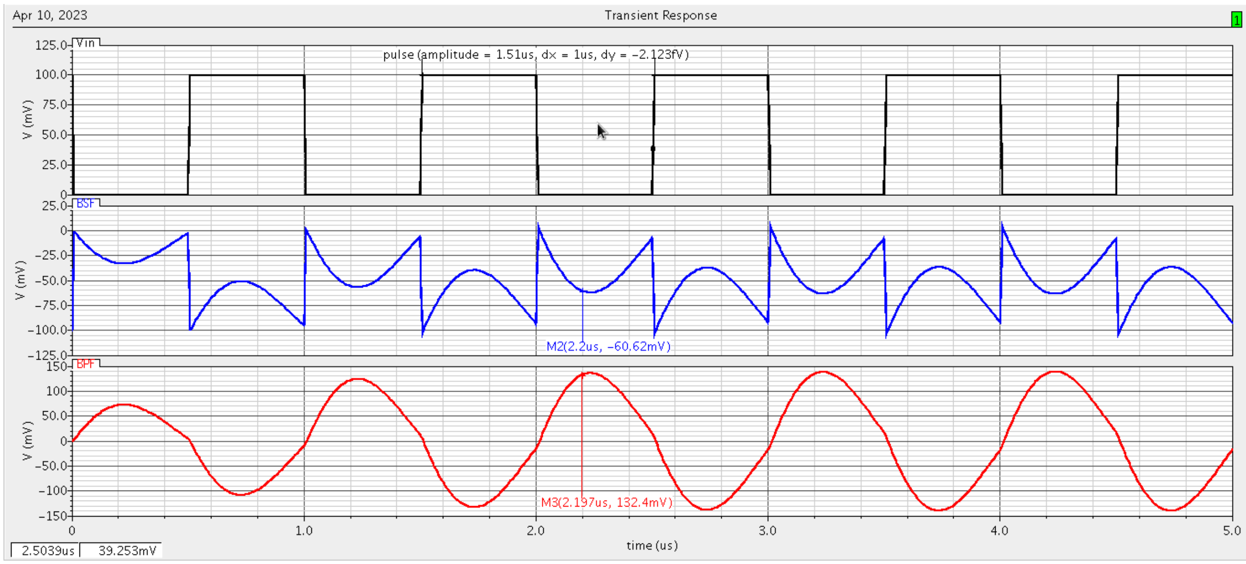

Figure 10: Transient analysis due to Pulse wave: BPF and BSF outputs

Square Pulse can be analyzed using Fourier Coefficients. We use the trigonometric Fourier: $a_o = \frac{Area}{T} = 50\ mV\ (DC\ component)$ $b_n = 0$ (even signal)

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\omega_o t)\,dt = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right) \quad \text{(Only gives value for odd n)}$$

$V_{in}(t) = 100\text{ mv}\left[\frac{1}{2} + \sum \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos(n\omega_o t)\right]$ → Fourier synthesis of Square Wave

$$V_{in}(t) = 100\ mv\left[\frac{1}{2} + \frac{2}{\pi}\left(\cos(\omega_o t) - \frac{1}{3}\cos(3\omega_o t) + \frac{1}{5}\cos(5\omega_o t) - \cdots\right)\right]$$

$$V_{out}(t) = 100\ mv\left[\frac{1}{2}H(0) + \frac{2}{\pi}\left(\cos(\omega_o t)H(\omega_o) - \frac{1}{3}\cos(3\omega_o t)H(3\omega_o) + \frac{1}{5}\cos(5\omega_o t)H(5\omega_o) - \cdots\right)\right]$$

This means frequency components pass, get amplified, or get rejected.

@DC ($\omega = 0$)	$H_{BPF}(0) = 0$ (rejection)	$H_{BSF}(0) = 1$ (pass)
@ω_o	$H_{BPF}(\omega_o) = Q$ (amplified and pass)	$H_{BSF}(\omega_o) = 0$ (rejection/blocked)
@$3\omega_o$	$H_{BPF}(3\omega_o)$ → rejection by some dB	$H_{BSF}(3\omega_o) = 1$ (pass)
@$5\omega_o, \ldots$	All higher harmonics are rejected	All higher harmonics pass as well
	f_o component passes, others rejected	All components pass except f_o.

Conclusions:

We notice that BPF signal is like a sine wave (slightly distorted sine wave). This is because the BPF amplified the component which is exactly at f $=f_o$, Then all other harmonics faced high rejection ratio.

On the other hand the BSF has blocked the frequency component at its notch f $=f_o$. Yet it allowed all other components to pass without rejection. So its output appears as if we subtracted a sine wave (point by point subtraction) from the rectangular pulse.

DFT analysis of output waveforms:

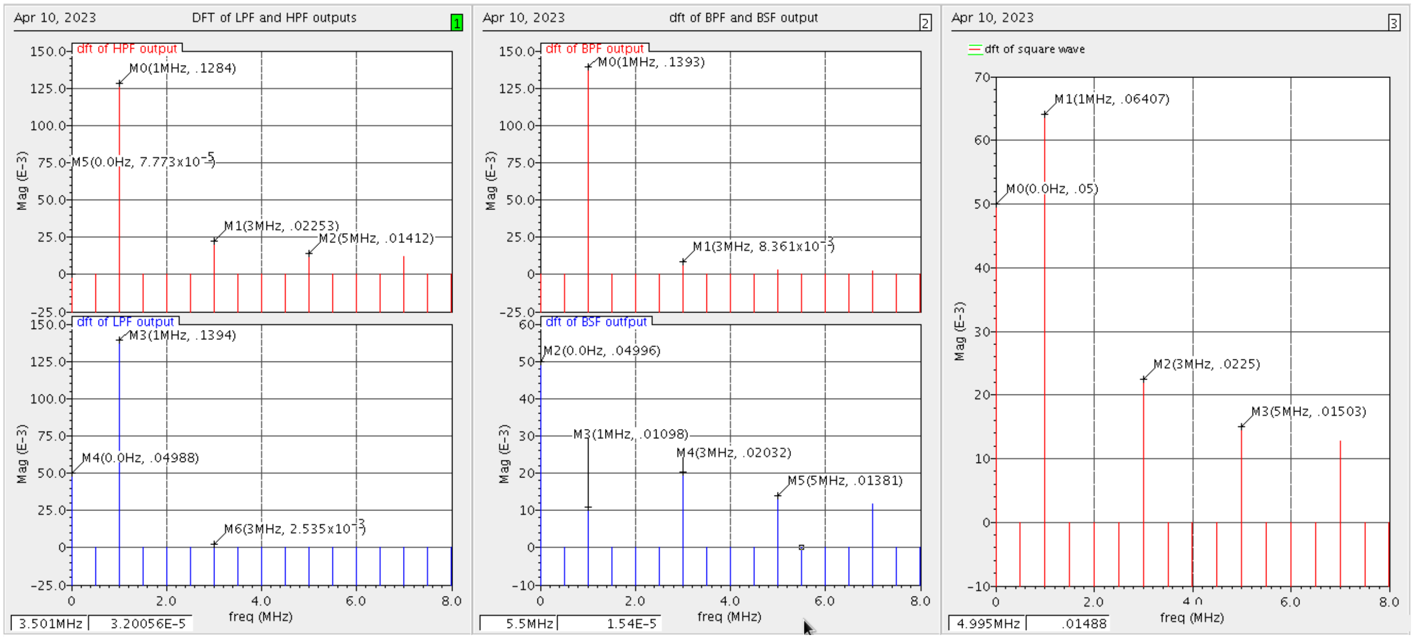

Figure 11: DFT analysis BPF and BSF outputs

Comment:

DFT verifies the Fourier coefficients of rectangle wave. For BPF, highest component is at 1MHz (f_o) while higher harmonics get rejected. For BSF, All components pass nearly without rejection except the component at the notch (f_o).