

Problem Statement 1:

Blood glucose levels for obese patients have a mean of 100 with a standard deviation of 15. A researcher thinks that a diet high in raw cornstarch will have a positive effect on blood glucose levels. A sample of 36 patients who have tried the raw cornstarch diet have a mean glucose level of 108. Test the hypothesis that the raw cornstarch had an effect or not.

Solution 1:

The problem statement clear indicates it as an instance of two tailed test.

Therefore, our assumption would be on the null hypothesis (H_0) and alternative hypothesis (H_1)

Lets take μ as the population mean; $\mu=100$

Lets take σ as the standard deviation; $\sigma=15$

$\bar{x}=108$ (Mean of a random Sample)

$n=36$ (Sample size)

Assuming a significance level of 95%

We have $\alpha=0.25$

In standard normal distribution value of $Z_{\alpha/2}$ will be

$Z_{\alpha/2}= 1.96$

$H_0:\mu=100$

$H_1:\mu \neq 100$ ($\mu > 100$ | $\mu < 100$)

We will find Z value for sample mean ($Z_{\bar{x}}$)

$$\begin{aligned} Z_{\bar{x}} &= (\bar{x} - \mu) / (\sigma / \sqrt{n}) \\ &= (108 - 100) / (15 / \sqrt{36}) \\ &= 8 / 2.5 \\ &= 3.2 \end{aligned}$$

We can observe that $Z_{\bar{x}} > Z_{\alpha/2}$ lies in the rejection zone. Hence we would reject the null hypothesis (H_0) and accept the alternative hypothesis (H_1)

Therefore we can state that with a confidence interval of 95%, raw cornstarch had an effect on glucose level.

Problem Statement 2:

In one state, 52% of the voters are Republicans, and 48% are Democrats. In a second state, 47% of the voters are Republicans, and 53% are Democrats. Suppose a simple random sample of 100 voters are surveyed from each state. What is the probability that the survey will show a greater percentage of Republican voters in the second state than in the first state?

Solution 2:

Proportion of Republican voters in the first state $P_1 = .52$

Proportion of Republican voters in the second state $P_2 = .47$

Ps1=Proportion of Republican voters in the sample from the first state
Ps2=Proportion of Republican voters in the sample from the second state.
n1= 100, n2= 100.

We can observe that the distribution is Bernoulli distribution
Hence, Variance= $P(P-1)$

We need to make a distribution for the Sampling Distribution - Difference between Two Means

Therefore, Mean of the difference

$$\mu(Ps1 - Ps2) = P1 - P2 = 0.52 - 0.47 = 0.05.$$

So, Standard Deviation of the difference.

$$\begin{aligned}\text{Sigma } (Ps1 - Ps2) &= \sqrt{[\text{var}(p1) / n1] + [\text{var}(p2) / n2]} \\ &= \sqrt{[P1 (1 - P1) / n1] + [P2 (1 - P2) / n2]} \\ &= \sqrt{[0.52 (0.48) / 100] + [(0.47) (0.53) / 100]} \\ &= \sqrt{0.002496 + 0.002491} = \sqrt{0.004987} \\ &= 0.0706\end{aligned}$$

$$\begin{aligned}Z(Ps1 - Ps2) &= (x - \mu(p1 - p2)) / \text{Sigma}(Ps1 - Ps2) \\ &= (0 - 0.05) / 0.0706 \\ &= -0.7082\end{aligned}$$

The probability that Ps1 - Ps2 is less than zero is equal to the probability of a z-score being -0.7082 or less than -0.7082.

$$P(z < -0.7082) = p(z > 0.7082) = .5 - p(0 < z < 0.7082) = .5 - 0.2611 = 0.2389$$

Hence there is a 23.89% probability that the survey will show a greater percentage of Republican voters in the second state than the first state.

Problem Statement 3:

You take the SAT and score 1100. The mean score for the SAT is 1026 and the standard deviation is 209. How well did you score on the test compared to the average test taker?

Solution 3:

$$\begin{aligned}\mu &= 1026 \\ \text{Sigma} &= 209 \\ x &= 1100 \\ \text{Using formula to calculate } z \text{ from } x \\ z &= (x - \mu) / \text{sigma} \\ &= (1100 - 1026) / 209 = .354\end{aligned}$$

Fraction of people scored less than me = .1368 + .5000 = 0.6368
i.e. 63.68 % people scored marks less than me in SAT exam.

Therefore, I scored 13.68 % more marks than the average test taker.