Health Care Operations Management: Blood inventory management

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Overview

Introduction

The flow of blood and blood products from the donor to the patient is classified as both an inventory and a distribution problems in the supply chain literature.

The big picture

- Collect whole blood from donors.
- Process it into its components at a regional blood center or a community blood center.
- Delivers the components to hospitals where they are transfused into patients.

Introduction

Who is responsible for providing blood products to hospitals?

- Regional Blood Center (RCB): Blood banks supporting regional, national, and even global demands which are paired with several satellite community blood centers
- Community Blood Centers (CBC): Supporting a few hospitals and health care institutions

Blood supply chain operations

The supply chain operation is a scheduling problem designed to prevent the worst in blood inventory management, i.e., shortage:

- A schedule of donor drawing locations is made some months in advance.
- Donors are solicited to give blood at the identified locations as the drawing time nears.
- Mobile Phlebotomy vans with medical and service personnel and equipments are sent to the sites on the scheduled days.
- Decisions are made to prepare various components from the whole blood so the appropriate bags are used when drawing the blood.

Blood supply chain operations

- The drawn whole blood is returned to a processing location where it is recorded, tested for viruses and diseases, and the components are prepared.
- The resulting components are then inventoried and appropriate shipments are made to the hospitals based on their inventory needs.
- The hospital staffs then make decisions on how and when to use the blood components.

Blood supply chain operations

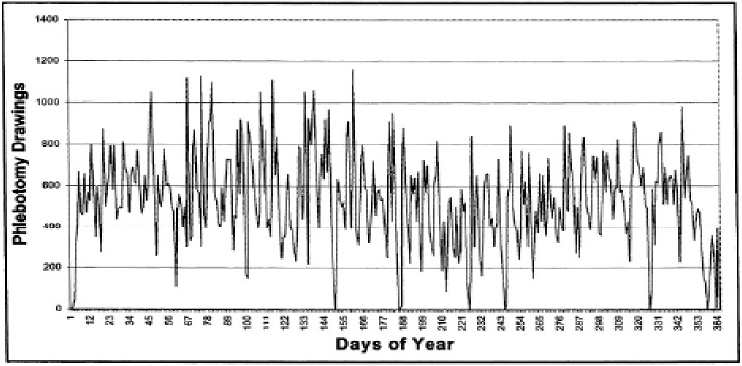
Challenges

The following distinguish blood management from a simple inventory and distribution problem:

- A perishable commodity with many components, each of which has a different shelf life and involves different costs.
- Random supplies with huge variances
- Screen out processes for a growing list of viruses, conditions and diseases which introduces additional risk and randomness
- Random demand in both amount and frequency
- A multi-objective problem to minimize shortage, minimize cost, and minimize out-dates and wastes.

Blood supply chain operations

- An example of randomness in blood drawing throughout a year



Economic Order of Quantity (EOQ)

Assumptions

- Demand is Deterministic and has a constant rate
- The cost of obtaining a unit is similar in every order
- The **lead time** is zero
  - Lead time: The time between order placement and order arrival
- No **backlogged demand** is allowed
  - Backlogged demand: Accepting delivery at a later date

Economic Order of Quantity (EOQ)

Data and parameters

- Order cost: K independent of order size
- Demand: D units
- Purchase cost: p for every unit of demand
- Holding cost: h for every unit of inventory
- Order quantity: q

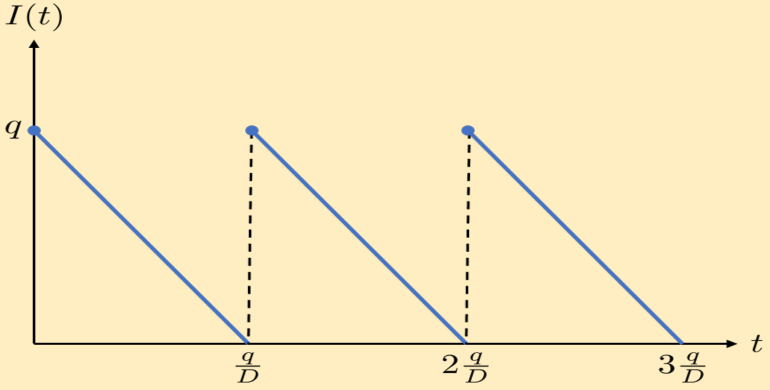
Objective

To minimize the total cost of the operation:

- Total cost, $TC(q)$: Total cost of order placement + Total cost of unit purchase + Total cost of holding inventory

Economic Order of Quantity (EOQ)

EOQ behavior



Economic Order of Quantity (EOQ)

Derivation of EOQ

- Total order cost:

$$\frac{KD}{q}$$

- Total purchase cost:

$$pD$$

- Total holding cost:

$$\frac{hq}{2}$$

Therefore,

$$TC(q) = \frac{KD}{q} + pD + \frac{hq}{2} \rightarrow q^* = \sqrt{\frac{2KD}{h}}$$

Economic Order of Quantity (EOQ)

A Community blood center faces an average demand of 1000 blood packages in the coming year. To prepare a blood drawing bus, CBC spends \$1000 each time its blood bank becomes empty. However, CBC estimates that each blood drawing from a volunteer would cost around \$10 in personnel cost. Past data also shows that average holding cost is also equal to \$50 a year for each blood package.

CBC does not allow any shortages but is interested in minimization of its managerial costs. To satisfy the estimated demand without any shortage, what is the optimal amount of blood drawings each time blood banks become empty?

Solution:

$$q^* = \sqrt{\frac{2KD}{h}} = 200 \text{ units.}$$

Economic Order of Quantity (EOQ)

- Q: In almost every inventory managing situation, the lead time is not zero (what is a realistic setting for 0 lead time?). In blood inventory management, the lead time for blood drawing is not zero. How should we modify the EOQ model to reflect a non-zero lead time?

Non-zero deterministic EOQ

The effect of non-zero lead time

In case of no shortages, lead time ($L > 0$) does not change ordering cost or holding cost. Therefore, the EOQ still minimizes total cost. However, instead of ordering when the inventory level reaches zero, each order time should ensure that when the order arrives, the inventory level has reached zero.

Reorder point

This is not a point in time. Reorder point is an inventory level at which an order should be placed.

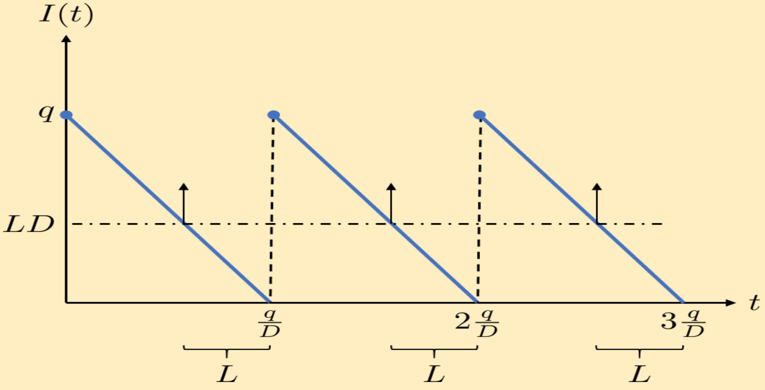
Non-zero deterministic EOQ

Derivation of reorder points

- Case 1: $LD \leq q^*$
  - How many units would be consumed in the lead time? LD. Therefore, when the inventory level equals LD, ordering q^* would take L units of time (and LD units of inventory) to be satisfied.
- Case2: $LD > q^*$
  - A similar principle to Case 1 is used in evaluating the reorder point. How many units would be consumed in the lead time? LD. This time, ordering at inventory level LD is not possible because the inventory level will never reach LD.

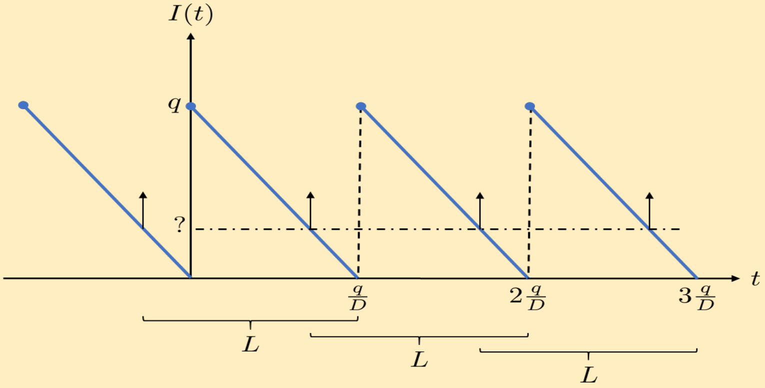
Non-zero deterministic EOQ

Lead time behavior, $LD \leq q^*$



Non-zero deterministic EOQ

Lead time behavior, $LD > q^*$



Non-zero deterministic EOQ

Derivation of reorder points (continued)

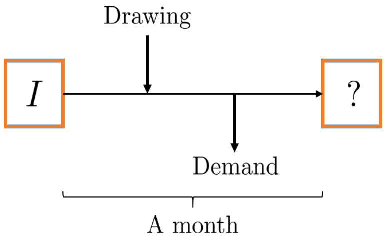
- Ex. Consider the problem in zero lead time EOQ section ($D = 1000, K = \$1000, h = \$50$). Suppose the lead time is 3 months. Therefore, the amount of blood packages needed in the lead time is $1000 \times \frac{1}{4} = 250$. Since q^* is 200, the inventory level does not ever reaches 250.

  In this example, ordering at inventory level 50 would prevent shortages until the order arrives. In general, the reorder point is **the remainder of LD divided by q.** Why?

Inventory control model

CBC has access to estimates of monthly blood package demands for a year. It is imperative that all demands are satisfied in time (no backlogged demand). Each time CBC has to organize a blood drawing bus, it costs $k_t, t = 1, \ldots, 12$ depending on the month. Drawing blood from each volunteer costs an additional c_t per unit per month. CBC can carry each unit in inventory for an entire year but the holding cost is h_t per unit inventory each month.

Model a yearly plan such that the cost is minimized, i.e., when to send blood drawing buses, how many packages to collect, and how many to hold in inventory? Initially, CBC has 150 units of inventory.



Inventory control model

Data and parameters

- Demands: $d_t, t = 1, \ldots, 12$
- Bus setup cost: $k_t, t = 1, \ldots, 12$
- Blood drawing cost: $c_t, t = 1, \ldots, 12$, per unit
- Inventory holding cost $h_t, t = 1, \ldots, 12$, per unit

Inventory control model

Decision variables

The objective is to find out if a blood drawing bus should be sent,

$$y_t := \begin{cases} 1, & \text{if a bus is sent out in month } t, \\ 0, & \text{otherwise,} \end{cases}$$

how many blood packages to collect,

$$x_t : \text{Number of blood packages collected in month } t,$$

and how many units to hold in inventory?

$$I_t : \text{Blood packages held in inventory at the end of month } t,$$

where all decision variables are defined for $t = 1, \ldots, 12$.

Inventory control model

Objective function

The objective is to model a yearly plan such that the cost is minimized. The plan's total cost consists of **the cost of sending blood drawing buses + the cost of blood drawings + the cost of holding blood packages in inventory**.

$$min \sum_{t=1}^{12} \boxed{k_t y_t} + \boxed{c_t x_t} + \boxed{h_t I_t}$$

The cost of sending blood drawing buses

The cost of blood drawings

The cost of holding inventory

Inventory control model

Constraints

Recall that no shortages are allowed. Therefore,

$$I_t \geq 0, \qquad \forall t = 1, \ldots, 12.$$

Note that

$$x_t \geq 0, \qquad \forall t = 1, \ldots, 12,$$

is derived by definition.

Inventory control model

Constraints (continued)

What is the relationship between decision variable y_t and x_t? In other words, is it possible to draw blood when no bus is sent out?

$$x_t \leq \boxed{M} y_t, \qquad \forall t = 1, \ldots, 12,$$

A very big number

- Suppose $y_3 = 0$, then no blood drawing can be done and $x_3 = 0$. Otherwise, $y_3 = 1$ and x_3 can be any number less than M.
- What is wrong with choosing a very very big number? Can we determine the smallest M?

Inventory control model

Constraints (continued)

What are the factors affecting the amount of inventory in each month?($I_0 = 150$)

$$\begin{aligned} \text{month } 1: \quad & I_1 = x_1 - d_1 + I_0, \\ \vdots \qquad & \qquad \vdots \\ \text{month } t: \quad & I_t = x_t - d_t + I_{t-1}, \end{aligned}$$

Note that the amount of inventory at the end of each month is equal to the amount of blood drawn in that month minus the demand of the month plus the inventory at the end of last month.

Inventory control model: Formal formulation

Model

Objective function: Minimize the total cost of the yearly plan.

$$\min \quad \sum_{t=1}^{12} k_t y_t + c_t x_t + h_t I_t,$$

Subject to,

x and y relationship:	$x_t \leq M y_t,$	$\forall t = 1, \ldots, 12,$
Inventory 1:	$I_1 = x_1 - d_1 + I_0,$	
Inventory t:	$I_t = x_t - d_t + I_{t-1},$	$\forall t = 2, \ldots, 12,$
	$x_t, I_t \geq 0,$	$\forall t = 1, \ldots, 12,$
	$y_t \in \{0, 1\}$	$\forall t = 1, \ldots, 12.$

Inventory control model: AMPL

Find this model's data file from CANVAS\IE 4910\Files\Ch 04\inventory-control.dat, code the model file in AMPL and report the results.

Inventory control model

- Q1: The blood packages are perishable items. They can not be held in inventory infinitely. Suppose there's a $J = 3$ months limit on keeping inventory. Modify the model accordingly.

Perishable inventory control model

Decision variables

The decision variables x_t and y_t are similar to the inventory control model. However, because blood packages have a limited shelf life, we need to keep track of inventory age

$$I_{j,t} : \text{Inventory of age } j \text{ at the end of month } t, \quad \forall j = 1, \ldots, J, \forall t = 1, \ldots, 12.$$

Perishable inventory control model

Objective function

The motivation behind the objective function is similar to inventory control model. However, because the inventory decision variables has changed, the total cost will become:

$$min \sum_{t=1}^{12} \left(k_t y_t + c_t x_t + h_t \sum_{j=1}^{J} I_{j,t} \right).$$

Constraint

Constraints such as

$$x_t \leq M y_t, \quad \forall t = 1, \ldots, 12,$$
$$x_t, I_t \geq 0, \quad \forall t = 1, \ldots, 12,$$

remain the same whereas inventory related constraints change.

Perishable inventory control model

Constraints (continued)

- Blood inventory of age 1 at the end of each month is built up from the blood drawings in that month. It excludes the amount of inventory in hold from previous months.
  - If inventories from previous months exceed demand, one can satisfy the demand only by previous inventories. Therefore,

$$I_{1,t} = x_t,$$

  - Otherwise, the entire monthly demand cannot be satisfied by previous inventories.

$$I_{1,t} = x_t - \boxed{\left(d_t - \sum_{j=1}^{J-1} I_{j,t-1}\right).}$$

Portion of demand not satisfied by previous inventories

Perishable inventory control model

Constraints (continued)

In other words,

$$\begin{aligned} &\text{If } d_t - \textstyle\sum_{j=1}^{J-1} I_{j,t-1} \leq 0 \rightarrow \quad I_{1,t} = x_t, \\ &\text{If } d_t - \textstyle\sum_{j=1}^{J-1} I_{j,t-1} > 0 \rightarrow \quad I_{1,t} = x_t - \Big(d_t - \textstyle\sum_{j=1}^{J-1} I_{j,t-1}\Big), \end{aligned}$$

Or,

$$I_{1,t} = x_t - \max\left\{0, \left(d_t - \sum_{j=1}^{J-1} I_{j,t-1}\right)\right\}.$$

Perishable inventory control model

Constraints (continued)

- Blood inventory of age $j = 2, \ldots, J$ at the end of each month is built up from the blood inventory of age $j-1$ minus the amount used to satisfy demand d_t in each month.
  - If $d_t - \sum_{i=j}^{J-1} I_{i,t-1} \leq 0$,
    $$I_{j,t} = I_{j-1,t-1},$$
  - If $d_t - \sum_{i=j}^{J-1} I_{i,t-1} > 0$,
    $$I_{j,t} = \max\left\{0, I_{j-1,t-1} - \left(d_t - \sum_{i=j}^{J-1} I_{i,t-1}\right)\right\},$$

Therefore,

$$I_{j,t} = \max\left\{0, I_{j-1,t-1} - \max\left\{0, \left(d_t - \sum_{i=j}^{J-1} I_{i,t-1}\right)\right\}\right\},$$

Perishable inventory control model

Linearize max constraint

To linearize max constraints, one of the following two methods:

- If the equality is holding at optimality c,

$$c = \max(a, b) \rightarrow \begin{cases} c \geq a, \\ c \geq b, \end{cases}$$

- If the equality is relaxed to $c \geq \max(a, b)$,

$$c \geq \max(a, b) \rightarrow \begin{cases} -M(1-\delta_1) \leq c - a \leq M(1-\delta_1), \\ c \leq b + M(1-\delta_1), \\ -M(1-\delta_2) \leq c - b \leq M(1-\delta_2), \\ c \leq a + M(1-\delta_2), \\ \delta_1 + \delta_2 = 1, \delta_1, \delta_2 \in \{0, 1\} \end{cases}$$

Perishable inventory control model: Formal formulation

Model

Objective: Minimize the total cost of the yearly plan

$$min \sum_{t=1}^{12} \left(k_t y_t + c_t x_t + h_t \sum_{j=1}^{J} I_{j,t} \right),$$

Subject to, (for every t and j)

$$x_t \leq M y_t,$$
$$I_{1,t} = x_t - \max\left\{0, \left(d_t - \textstyle\sum_{j=1}^{J-1} I_{j,t-1}\right)\right\},$$
$$I_{j,t} = \max\left\{0, I_{j-1,t-1} - \max\left\{0, \left(d_t - \textstyle\sum_{i=j}^{J-1} I_{i,t-1}\right)\right\}\right\},$$
$$x_t, I_{j,t} \geq 0,$$
$$y_t \in \{0, 1\},$$

The News vendor problem

Original statement. How many newspaper should be ordered each day form the newspaper plant? If demand (the number of buyers) is random, the order could turn out too be many or too few. Order too and you end up with expired papers, whereas shortages result in opportunity costs. We are only analyzing a single decision period.

Blood inventory statement. Consider a single decision period equivalent to the length of time a blood package expires. If demand for blood is random, how should a manager decide to balance the expiration cost versus the shortage cost? One may consider a higher than usual cost to discourage the model from allowing shortages.

The News vendor problem

Assumptions

- The model only considers a single decision period.
- The lead time is considered to be zero (is that a problem?)
- Shortage is allowed.
- Demand is probabilistic but the capacity of supplier is infinite.

Data and parameters

- Demand: D, a random variable with distribution $\mathbb{P}(d)$
- overstocking cost, c_o
- understocking cost, c_u

The News vendor problem

Solution, the discrete case

- The realized demand d is less than q, $d \leq q$

$$\text{overstocking cost, } c(d, q) = c_o q,$$

  where ordering one more unit will cost c_o.
- The realized demand d is more than q, $d \geq q + 1$

$$\text{understocking cost, } c(d, q) = -c_u q,$$

  where ordering one more results in being short one less unit.

The News vendor problem

Solution, the discrete case (continued)

Marginal analysis: What is the effect of ordering one more unit on cost?

Expected (cost of ordering $q+1$ units − cost of ordering q units)

- Probability that one more unit of order costs c_o: $\mathbb{P}(D \leq q)$.
- Probability that one more unit of order benefits c_u: $\mathbb{P}(D \geq q)$

$$
\begin{aligned}
\mathbb{E}(q+1) - \mathbb{E}(q) &= c_o \mathbb{P}(D \leq q) - c_u[1 - \mathbb{P}(D \leq q)] \\
&= (c_o + c_u)\mathbb{P}(D \leq q) - c_u
\end{aligned}
$$

When ordering one more unit is beneficial?

The News vendor problem

Solution, the discrete case (continued)

Ordering one more unit is beneficial when $\mathbb{E}(q+1) - \mathbb{E}(q) \geq 0$. Therefore,

$$\begin{aligned}
&(c_o + c_u)\mathbb{P}(D \leq q) - c_u \geq 0, \\
&\mathbb{P}(D \leq q) \geq \frac{c_u}{c_u + c_o}, \\
&F(q) \geq \frac{c_u}{c_u + c_o}.
\end{aligned}$$

Starting from $q = 0$, the smallest number (the minimum q) that achieves the above condition is the optimal expected quantity of order.

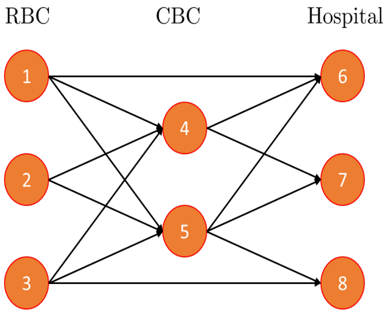
The News vendor problem

- Does the solution change when demand is continuous?
- Suppose probabilistic demand of a single decision period is given according to the following table. To discourage the model from allowing shortages, the understocking cost is chosen to be \$1000 by the blood bank manager. The overstocking cost is the holding cost, \$150 paid while a unit of blood package was in the inventory plus the disposing cost of \$50 per package. What is the optimal inventory level to satisfy the expected demand?

D	50	20	30	45	70
$\mathbb{P}$	0.15	0.2	0.1	0.4	0.15

The transshipment model

3 Regional Blood Centers are the main source for the final 3 demand locations. Each source can only supply a certain amount. The suppliers can ship the inventory to the end demand points in some cases, but in others the shipment has to go through a Community Blood Center. CBC locations have limited capacities. However, the final demand at each hospital must be met. Taking each route results in a specific cost and the goal is to minimize the total cost of shipment.



The transshipment model

Data and parameters

- RBC nodes: $S := \{1, 2, 3\}$,
- Hospital nodes: $D := \{6, 7, 8\}$,
- CBC nodes: $I := \{4, 5\}$,
- Supply at source: $s_i, i \in S$,
- Demand at hospital: $d_j, j \in D$,
- Capacity in inventory: $c_k, k \in I$,
- h_{ij} : cost of taking one unit of inventory in a route $i \to j$.

Feasibility check

If the supply is less than the demand the model is infeasible, so

$$\sum_{i \in S} s_i \geq \sum_{j \in D} d_j,$$

The transshipment model

Decision variables

The goal is to come up with a shipment plan such that the cost is minimized. Therefore, the decision variable should reflect the plan chosen, i.e., the routes taken from suppliers to demands.

$$x_{ij} = \text{The amount of inventory taken from node } i \text{ to node } j,$$

$$\forall i \in S \cup I, j \in I \cup D,$$

The transshipment model

Objective function

The goal is to minimize the total shipment cost. The cost associated with any routes taken is included in the objective by evaluating the cost of shipping x_{ij} inventory units in the route $i \to j$,

$$\min \sum_{i \in S \cup I} \sum_{j \in I \cup D} h_{ij} x_{ij}.$$

Note that no routes can start from demand nodes and no routes end in the supply nodes.

The transshipment model

Constraints

- The explicit constraint states that all demands must be met. So, the amount of shipments ending up in a hospital node must be greater than or equal to its demand.

$$\sum_{i \in A(S \cup I)} x_{i,j} \geq d_j, \qquad \forall j \in D,$$

- We can not ship more than the supplier can produce

$$\sum_{j \in B(I \cup D)} x_{i,j} \leq s_i, \qquad \forall i \in S,$$

The transshipment model

Constraints (continued)

- We can not ship to the CBC nodes more than their capacities

$$\sum_{i \in S} x_{i,k} \leq c_k, \qquad \forall k \in I,$$

- We can not ship from the CBC nodes more then their capacities

$$\sum_{j \in D} x_{k,j} \leq c_k, \qquad \forall k \in I,$$

The transshipment model

Constraints (continued)

- The amount of inventory going to a CBC node must be equal to (or greater) then the amount coming out.

$$\sum_{i \in S} x_{ik} = \sum_{j \in D} x_{kj} \qquad \forall k \in I,$$

**Note that the objective and the constraints are written assuming that all possible links starting from node i to node j actually exist.

The transshipment model: Formal formulation

Model

Objective: Minimize total shipment costs

$$\min \sum_{i\in S\cup I} \sum_{j\in I\cup D} h_{ij}x_{ij},$$

Subject to,

Demand:	$\sum_{i\in S\cup I} x_{i,j} \geq d_j,$	$\forall j \in D,$
Supply:	$\sum_{j\in I\cup D} x_{i,j} \leq s_i,$	$\forall i \in S,$
Ship to CBC:	$\sum_{i\in S} x_{i,k} \leq c_k,$	$\forall k \in I,$
Ship from CBC:	$\sum_{j\in D} x_{k,j} \leq c_k,$	$\forall k \in I,$
in and out of CBD:	$\sum_{i\in S} x_{ik} = \sum_{j\in D} x_{kj}$	$\forall k \in I,$
	$x_{i,j} \geq 0$	$\forall i \in S \cup I, j \in I \cup D.$

The transshipment model: AMPL

Code the transshipment model in AMPL. Use the data for the given configuration in the data file given in CANVAS\IE 4910\Files\Ch 04\transshipment.dat.

Class works

- 1: A blood distribution center has access to monthly blood package demands for a year. Preparing a blood drawing bus costs k_t in each month t. Drawing blood from each donor costs an additional c_t per unit in each month. However, the screening and testing process would take about a month, so the units drawn in month t are going to be available for inventory and use in month $t+1$. Holding each unit of blood products in inventory costs h_t. Note that the distribution center has to satisfy all demands completely. Optimize a yearly plan such that the total cost is minimized when faced with demand d_t. Use the data for the Inventory control model.

Class works

- 2: Blood products usually expire after 3 months. A blood bank is faced with stochastic demands according to normal distribution with a mean of 100 units of blood products, and a standard deviation of 10 units, i.e., $d \sim \mathcal{N}(100, 10^2)$. To prevent shortages, the management has decided to put a penalty of \$1000 on any unit short of the demand. Overstocking is not encouraged since for every unit of expired blood products, the blood banks pays \$100 total in safe disposing and holding costs. Find the optimal inventory level for every 3 months cycle?

Project

- Consider a similar transshipment configuration as in the slides. This time, holding a unit of inventory in nodes 4 or 5 for each decision period costs \$10 and \$8, respectively. Suppliers do not have infinite capacity and the plan may end up with shortages. Each unit short of the demand is equivalent to \$30, \$50, and \$25 cost in demand points $6, 7$, and 8, respectively. Find an optimal transshipment plan such that the cost is minimized. Use the new data file uploaded in Canvas.

Sources:

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