Health Care Operations Management: Immunization and infection control

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Overview

Immunization

- In 2018, the childhood immunization program requires 7 clinical visits and up to 30 injections in the first 15 months.
- In 2012, the same program required 5 clinical visits and 19 injections over the first 18 months.
- The overall pattern shows that the number of clinical visits and dose injections are increasing with time.

Immunization

Such an increase creates a set of challenges which may result in noncompliance with the advised policy and ultimately expose the child and the society to harmful diseases.

- A child may be intolerant to multiple injection in a short time.
- Parents may not be able to make immunization visits in time.
- New vaccines are added to the already crowded schedule.
  - The volume of vaccines and the frequency of visits increase which in turn results in higher costs.
  - Analyzing the side effects becomes more complex whit overcrowded schedules.

Immunization

One way to address the problem is to combine the effective agent of multiple vaccines in a single injection. The question is to determine which vaccines can be combines such that

- The effective agents are biologically compatible
- The combination is as effective and not harmful
- The side effects are minimal
- The production process is economical

Organ allocation models

How does one prioritize patients when new organs become available?

- How one would determine the value of a transplant?
- Is it fair to assign a number to survival of a patient?

The National Organ Transplant Act (NOTA) enacted by U.S. congress established a national organ sharing system, the United Network Organ Sharing (UNOS), whose purpose is to maintain a national transplant waiting list and to coordinate the activities of the local agencies that procure organs for transplantation.

Organ allocation models

The system:

- The system is organized in three different geographical levels: locally, regionally, and nationally.
- If an organ is available and an appropriate candidate is not found by "**some algorithm**" locally, then the search includes broader regions. If no one is deemed a match, the search becomes national.

Organ allocation models

The algorithm:

Key ingredients:

- The patient must be blood-compatible.
- The patient should not have immunity problems.
- The patient must be in good physical conditions.
- It is desirable that the patient's tissue type matches with that of the donor.

Based on these (simplified) ingredients, UNOS has created a point system in which each patient receives a number of priority points.

Organ allocation models

The point system is created by medical experts who can estimate the probability of a successful transplant operation. However, this system creates its own problems.

- There are patients who do not have immunity problems and are in good physical conditions and have compatible blood types but they have rare tissue types.
- To address this issue, waiting time and rank in the list of patients are also considered in the point system.

Queueing theory: Customer behavior

Imagine the UNOS as a server for heart transplantation. The server performs 3244 transplant surgery per year. In each year, nearly 4000 patients add to the heart transplant waiting list.

- https://www.organdonor.gov/statistic-stories/statistics.html
- Approximate the system with a $M/M/1$ queueing model. Is the queue stable?

Utilization factor

Utilization factor is the fraction of time that a server is busy.

- Single server: $\rho = \frac{\lambda}{\mu}$
- m servers: $\rho = \frac{\lambda}{m\mu}$

Queueing theory: Customer behavior

If a utilization factor is greater than or equal to one, the servers are always busy, and thus the queue length goes to infinity.

- For UNOS, the utilization factor $\rho = \frac{4000}{3244} \geq 1$. Therefore, the organ transplant queue is unstable.

To approximate this queueing system more precisely, we need another element in the queueing theory. Not all customers stay in the queue forever. Some of them may die if the wait too long.

Queueing theory: Customer behavior

Waiting behavior

- Balking: Customers deciding not to join the queue if it is too long.
  - Patients in organ transplant waiting lists do not have a choice. They will join the queue to survive.
- Jockeying: Customers switch the queue if they think another queue will serve them faster.
  - This a multi server situation. In organ transplant queues, there is no other server.
- Reneging: Customers leave if they have to wait too long for service.
  - For patients in a heart transplant waiting list, this is not a choice but some of the patients may die if not operated soon.

Queueing theory: Customer behavior

Queueing theory with reneging

The arrival rate is λ, the service rate is μ, the utilization factor is $\rho = \frac{\lambda}{\mu} \geq 1$, and a customer in the queue leaves after a **fixed given** time τ.

- Probability of customer loss in the queue:

$$P_L = \frac{(1-\rho)e^{\frac{\tau(\rho-1)}{\mu}}}{1-\rho e^{\frac{\tau(\rho-1)}{\mu}}}$$

- The rate of customer loss:

$$R_L = \lambda P_L$$

Queueing theory: Customer behavior

Queueing theory with reneging (continued)

In our case, assume that $\tau = 200$ weeks, i.e., patients in need of heart transplant die after 200 weeks of waiting. Therefore,

- Probability of customer loss in the queue:

$$\lambda = \frac{4000}{52} \simeq 76.9, \quad \mu = \frac{3244}{52} \simeq 62.3,$$

$$\rho = \frac{76.9}{62.3} \simeq 1.23, \quad P_L \simeq 0.307.$$

- Rate of customer loss:

$$R_L = 76.9 \times 0.189 \simeq 23.61$$

Queueing theory: Customer behavior

Queueing theory with reneging (continued)

- Arrival rate - loss rate:

$$\lambda_{new} = 76.9 - 23.61 = 53.29$$

- New utilization factor:

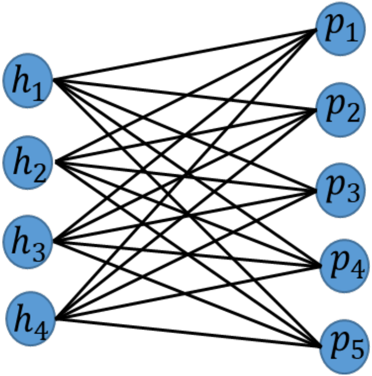
$$\rho_{new} = \frac{\lambda_{new}}{\mu} = \frac{53.29}{62.3} \simeq 0.85,$$

- Avg. Length of the queue:

$$L_q = \frac{\rho_{new}^2}{1 - \rho_{new}} \simeq 5 \text{ customer per week}$$

Assignment model

Suppose a local facility of UNOS has 5 patients in the waiting list for heart transplantations but the UNOS facility only has access to 4 healthy organs. Medical experts can determine the matching score of all possible allocations. What is the optimal allocation scheme to achieve the highest overall score?



Assignment model

Data and parameters

Part of the data (sets of hearts and patients) is given to us in the form of an **complete undirected bipartite graph**.

- Set of hearts: $H := \{1, \ldots, 4\}$,
- Set of patients: $P := \{1, \ldots, 5\}$,
- Allocation score:

s_{ij}	1	2	3	4	5
1	15	20	10	40	35
2	20	25	20	35	20
3	30	20	10	15	25
4	35	25	15	30	35

Assignment model

Decision variables

The goal is to find optimal allocation of hearts to patients such that the overall score is maximized. Therefore, we define our decision variables to denote the allocation of heart i to patient j.

$$x_{ij} = \begin{cases} 1 & \text{if heart } i \text{ is assigned to patient } j, \\ 0 & \text{otherwise,} \end{cases} \qquad \forall i \in H, j \in P.$$

Assignment model

Objective function

The goal is to find optimal heart to patient allocations such that the overall allocation score is maximized.

$$\max \sum_{i \in H} \sum_{j \in P} \boxed{s_{ij} x_{ij}}$$

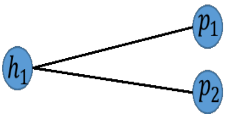
For example, if $x_{35} = 1$, then the third heart has been allocated to the fifth patient. This particular allocation has the score $s_{35} = 25$ which is now included in the objective function.

Assignment model

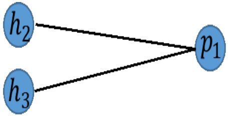
Constraints

There are no obvious and explicit constraints here. We have to think about unwanted assignments to limit picking any x_{ij} in the optimal solution.

- 1:One heart to two patients



- 2:Two hearts to one patient



Assignment model

Constraints (continued)

- Remove the first set of unwanted assignments

$$\sum_{j \in P} \boxed{x_{ij} = 1} \qquad \forall i \in H,$$

Ex., heart h_1 is only assigned to one patient p_j. Out of these assignments $x_{11}, x_{12}, x_{13}, x_{14}, x_{15}$, only one is equal to 1.

- Remove the second set of unwanted assignments

$$\sum_{i \in H} \boxed{xij \leq 1} \qquad \forall j \in P,$$

Ex., each patient p_j is only assigned to one heart h_i. Out of these assignments $x_{11}, x_{21}, x_{31}, x_{41}, x_{51}$, only one is equal to 1.

Assignment model: Formal formulation

Parameters

Set of hearts: $H := \{1, \dots, h\}$,

Set of patients: $P := \{1, \dots, p\}$,

Coverage: s_{ij}: expert assessed score of allocating heart i to patient j,

Model

$$
\begin{array}{rll}
\max & \sum_{i \in H} \sum_{j \in P} s_{ij}\, x_{ij}, & \\
s.t.\ 1: & \sum_{j \in P} x_{ij} = 1, & \forall i \in H, \\
2: & \sum_{i \in H} x_{ij} \leq 1, & \forall j \in P, \\
3: & x_{ij} \in \{0, 1\} & \forall i \in H, j \in P,
\end{array}
$$

Assignment model: AMPL

Code this assignment model in AMPL. The data file is given in CANVAS\Files\Ch 03\assignment.dat. Run with gurobi and report the results.

Assignment model

- Q1: Why the second assignment is less than or equal to 1? What would happen if the number of patients were less than the number of hearts?
- Q2: In this assignment problem, patients (or more realistically their doctors) should receive the hearts they have been assigned? Is there a way to take the patients preference into account?

Stable assignment model

In this case, the UNOS facility has sorted 4 patients for each of its 4 available hearts based on a point system. Patients' doctors have also sorted the hearts according to their own expertise. A pair of (hearts,patients) assignment is unstable if a patients finds another heart for which he/she was listed higher than its current assigned patient. Find an stable assignment which also maximized the UNOS total score.

Patients	Hearts			
p_1	h_3	h_1	h_4	h_2
p_2	h_1	h_3	h_4	h_2
p_3	h_4	h_3	h_1	h_2
p_4	h_3	h_2	h_1	h_4

Hearts	Patients			
h_1	p_4	p_3	p_1	p_2
h_2	p_3	p_4	p_1	p_2
h_3	p_3	p_2	p_1	p_4
h_4	p_4	p_2	p_3	p_1

Stable assignment model

Data and parameters

- Set of hearts: $H := \{1, \ldots, 4\}$,
- Set of patients: $P := \{1, \ldots, 4\}$,
- Allocation score:

s	h_1	h_2	h_3	h_4
p_1	20	25	20	15
p_2	10	20	25	30
p_3	30	35	30	20
p_4	40	30	15	35

- Another important part of data is the preference set of hearts and patients.

Stable assignment model

Decision variables

The goal is to find optimal allocation of hearts to patients such that the overall score is maximized and the pairing are stable. We define our decision variables to denote the allocation of patient i to heart j. Notice that the first index denotes the patients and the second index denotes the hearts.

$$x_{ij} = \begin{cases} 1 & \text{if patient } i \text{ is assigned to heart } j, \\ 0 & \text{otherwise,} \end{cases} \qquad \forall i \in P, j \in H.$$

Stable assignment model

Objective function

Similar to the assignment model discussed before, the goal is to find optimal heart to patient allocations such that the overall allocation score is maximized.

$$\max \sum_{i \in H} \sum_{j \in P} \boxed{s_{ij} x_{ij}}$$

For example, if $x_{31} = 1$, then the third patient has been allocated to the first heart. This particular allocation has the score $s_{31} = 30$ which is now included in the objective function.

Stable assignment model

Constraints

Recalling the assignment model, we can identify unwanted pairings and prevent them from happening. Assignment of two hearts to one patient, or two patients to one heart do not qualify for a feasible solution. Thus,

$$\sum_{i \in P} \boxed{x_{ij} = 1} \qquad \forall j \in H, \qquad\qquad \sum_{j \in H} \boxed{x_{ij} = 1} \qquad \forall i \in P,$$

Each heart is assigned to exactly one patient.

Each patient is assigned to exactly one heart.

Stable assignment model

Constraints (continued)

To write the stability constraints, consider the pairs (p_2, h_2) and (p_3, h_4). Notice that patient p_2 is motivated to pair with h_4 because h_4 is higher on his/her list. h_4 is also motivated to be paired with p_2 because patient p_2 is higher on its list. To prevent such an unstable pairing, we should not let (p_2, h_2) and (p_3, h_4) to happen at the same time. Note that (p_2, h_2) and (p_1, h_4) is also unstable by the same reasoning.

- To prevent the pairs (p_2, h_4) and (p_1, h_3) to happen because of other unstable pairings,

$$x_{34} + x_{14} + x_{22} \leq 1, \text{ considering pair } (p_2, h_4),$$
$$x_{43} + x_{11} + x_{14} + x_{12} \leq 1, \text{ considering pair } (p_1, h_3),$$

Stable assignment model: Formal formulation

Parameters

Set of hearts: $H := \{1, \ldots, h\}$,
Set of patients: $P := \{1, \ldots, p\}$,
Score: s_{ij}: Expert assessed score of allocating heart i to patient j,
Preference: An ordered list stating patients' preference for the hearts and hearts' score for each patient

Model

$$
\begin{array}{rll}
\max & \sum_{i\in H} \sum_{j\in P} s_{ij}\, x_{ij}, & \\
s.t.\ 1: & \sum_{j\in P} x_{ij} = 1, & \forall i \in H, \\
2: & \sum_{i\in H} x_{ij} = 1, & \forall j \in P, \\
3-19: & \text{Stability constraint for each pair}, & \\
20: & x_{ij} \in \{0,1\} & \forall i \in H, j \in P,
\end{array}
$$

Stable assignment model: AMPL

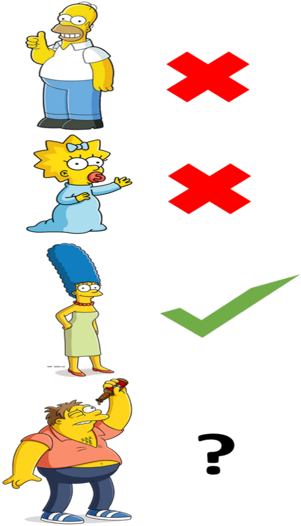
Code this assignment model in AMPL. The data file is given in CANVAS\Files\Ch 03\stable-assignment.dat. Run with gurobi and report the results.

Stable assignment model

- Q1: In realistic situations, organs become available sequentially, and if rejected by a patient, they will be assigned to the next patient in the waiting list. Is there a strategy to selecting or rejecting an organ when the arrival of a better organ is random and the patient can not go back to the organs previously rejected?

The secretary approach

A patient is waiting in the UNOS kidney transplant waiting list. He is in a good enough condition such that he can afford to not select the first few kidneys that become available. His doctor is waiting for the best possible match but he does not know when the best match will become available, and once he rejects an organ, he can not choose it again. What is the best strategy to stop waiting for better future organs?



The secretary approach

Objective

The best **strategy** in this case is to maximize the probability of success. Success is defined as stopping the random sequence, i.e., selecting a candidate, at a time when the best candidate is observed.

Data and randomness

- Data: What is observed so far is part of the data. Total number of candidates is known in advance.
- Random: The quality of the rest of candidates is unknown. The order of arrival is also random.

The secretary approach

Strategy

After experimenting a few times, it gets clear that the best strategy is to let $k-1$ candidates to go by, and then select the first candidate which is better than the all previous candidates. This strategy has the maximum probability of selecting the best candidate.

Example

Suppose three candidates are arriving in random order. Their quality is unknown until observed. How do we determine k to select the best candidate?

The secretary approach

Probabilities

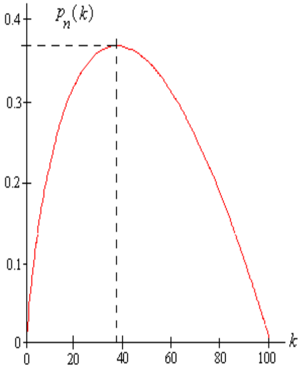
Suppose number 3 shows the best candidate.

	Selected candidate		
Permutation	$k=1$	$k=2$	$k=3$
$(1,2,3)$	1	2	3
$(1,3,2)$	1	3	2
$(2,1,3)$	2	3	3
$(2,3,1)$	2	3	1
$(3,2,1)$	3	1	1
$(3,1,2)$	3	2	2
Correct selections	2	3	2

The best strategy is to let 1 candidate to go by and select the first candidate that is better than the one observed, i.e., $k=2$. $P_3(k) = \frac{3}{6}$.

The secretary approach

As n increases, we can calculate $P_n(k)$ for different values of k. It turns out that as $n \to \infty$, there is an optimal number of candidates that should go by to maximize the probability of successfully choose the best candidate after them. The optimal number of candidates to go by tends to $k \to \frac{n}{e}$ where $e \simeq 2.718$. Roughly, let 38% of the candidates go by before selecting the first candidate that is better than all the previous ones.



The secretary approach

Probabilities

Let $S_{n,k}$ denote the event of success for strategy k when the number of candidates is n, i.e, $k-1$ candidates are observed and their score is recorded, then the first candidate from $k, \ldots, n$ which is better than all the previous candidates is selected. Denote the arrival time of the absolute best candidate with j.

$$\mathbb{P}(S_{n,k}|j) = \begin{cases} 0, & j \in \{1, 2, \ldots, k-1\}, \\ \frac{k-1}{j-1}, & j \in \{k, k+1, \ldots, n\}. \end{cases}$$

The strategy succeeds in selecting the best if and only if one of the first $k-1$ candidates is better than the first $j-1$ candidates.

The secretary approach

Probabilities (continued)

The probabilities in the last slide where conditioned on fixing the absolute rank of the best candidate. However the absolute rank is not known in random arrival. Therefore,

$$\mathbb{P}(S_{n,k}) = \begin{cases} \frac{1}{n}, & k = 1, \\ \frac{k-1}{n} \sum_{j=k}^{n} \frac{1}{j-1}, & k \in \{2, 3, \dots, n\}. \end{cases}$$

$$\mathbb{P}(S_{n,k}) = \sum_{j=1}^{n} \mathbb{P}(j)\mathbb{P}(S_{n,k}|j) = \sum_{j=k}^{n} \frac{1}{n} \frac{k-1}{j-1}$$

The secretary approach: Excel

Try the secretary approach for 4 candidates. An Excel file is given in CANVAS\Files\Ch 03\secretary.xlsx. Check permutation probabilities with that of the formula.

The secretary approach

- The secretary approach is robust, i.e., it considers a limited set of assumptions: Arrival of organs are random, quality of organs are random, patients have time to observe and reject the first few candidates.
- However, in most real cases, more information about the distribution of organ qualities and patients affordable waiting time is available.
- What if the distribution of organ qualities was known? In this case, the quality is still random but it comes from a known distribution?

The threshold approach

n patients are waiting in the UNOS kidney transplant waiting list. They are in a good enough condition to not select the first few kidney arrivals. UNOS waits for the best possible match and experience shows that the quality of organs follows $\phi(x)$ p.d.f. Again, once the doctor rejects an organ, it will not be available any more. What is the best strategy to stop waiting for better future organs?

The threshold approach

Objective

The **objective** is similar to the secretary approach. We want to maximize the probability of successfully selecting the best candidate for each patient when the arrival order of the candidates is random.

Data and randomness

- Data: What is observed so far is part of the data. Probability distribution function of the quality of organs is also known. Total number of candidates is also known.
- Random: The quality of the rest of candidates is unknown. The order of arrival is also random.

The threshold approach

Strategy

The best **Strategy** is to identify a set of thresholds $c_1 \geq \ldots \geq c_n$ such that if an organ quality is better than c_1, the organ is assigned to patient r_1. If the organ quality falls between (c_1, c_2), the organ is assigned to patient r_2. Note that UNOS ranks the patients $r_1, \ldots, r_n$ according to some numerical scoring system (e.g., the chance of successful operation)

Thresholds

$$\boxed{\mathbb{P}(X \geq c_i) = \frac{i}{n}}, \qquad \forall i = 1, \ldots, n$$

One must solve this equation for every i using the inverse of the cumulative distribution function

The threshold approach

- In the secretary approach, no information with regards to the randomness was present.
- In the threshold approach, the distribution of organ quality was known.
- What is the optimal policy when patients have deadlines? i.e., they can not wait more that a certain time.
  - If deadline times are deterministic, the threshold policy is still optimal. Why?
  - If deadlines are stochastic, the threshold policy is optimal but calculation of the threshold is a complex problem.

Class works

- 1: UNOS just got 5 new hearts to for its patients waiting for a heart transplantation. Out of 6 patients waiting in the line, UNOS has determined that patient 1, who has a rare tissue type and has waited a long time in the queue, must definitely get a heart in spite of his low medical score with respect to all assignment. Also, heart 2 is only compatible with patients 2 and 3 and has zero chance of a successful transplant with the rest of the patients. Find an optimal allocation such the overall assignment score is maximized.

Class works

- 2: A patient's doctor has advised her to go through a kidney transplant surgery. The chance of survival is very high and the doctor has assured her that they only proceed with the surgery if a kidney in excellent condition is found. UNOS has estimated that on average and in the next 6 month, 10 organs with different conditions would become available for her. What is the strategy that maximizes her chance of selecting the best kidney? What is the maximum probability of best selection?

Project

- UNOS has 6 patient waiting for heart and lung transplants in the queue. Just recently, 4 hearts and 3 lungs have become available, and UNOS wants to allocated these organs such that the overall allocation score is maximized. Patient 6 and patient 4 need simultaneous heart and lung transplants, otherwise they could not survive. Patients 1 and 2 only require heart transplants, however patient 2 can benefit from a lung transplant if a lung is available (i.e., it does not make sense to patient 2 to get a lung transplant without a heart transplant first). Patients 3 and 5 require lung transplants but patient 5 can use the benefit of a heart transplant too!

Sources:

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