Health Care Operations Management: Immunization and infection control

Amir A. Nasrollahzadeh

Clemson University

snasrol@g.clemson.edu

Fall 2018

Overview

1. Immunization
   - Introduction
   - Vaccine delivery model
2. Infection control
   - Introduction
   - Resource allocation: The knapsack model
   - Infection monitoring: p-median, p-center (minimax)
   - Infection spread: Minimum spanning tree
3. Exercises
   - Class works
   - Project

Introduction

- In 2018, the childhood immunization program requires 7 clinical visits and up to 30 injections in the first 15 months.
- In 2012, the same program required 5 clinical visits and 19 injections over the first 18 months.
- The overall pattern shows that the number of clinical visits and dose injections are increasing with time.

Introduction

Such an increase creates a set of challenges which may result in noncompliance with the advised policy and ultimately expose the child and the society to harmful diseases.

- A child may be intolerant to multiple injection in a short time.
- Parents may not be able to make immunization visits in time.
- New vaccines are added to the already crowded schedule.
  - The volume of vaccines and the frequency of visits increase which in turn results in higher costs.
  - Analyzing the side effects becomes more complex whit overcrowded schedules.

Introduction

One way to address the problem is to combine the effective agent of multiple vaccines in a single injection. The question is to determine which vaccines can be combined such that

- The effective agents are biologically compatible
- The combination is as effective and not harmful
- The side effects are minimal
- The production process is economical

Furthermore, sometimes multiple doses can be injected in a single package depending on safety.

Vaccine delivery model

A child care hospital is tasked to run a monthly program of vaccination immunizing children with 6 antigen agents which protect them from a range of diseases. Each child should be injected with specific dosages of each agent in the end. The hospital's goal is to minimize the cost of vaccines by minimizing total injections which cost $200 each round. Each round should not include more than 3 antigens. Moreover, antigens 2,5, and 6 cannot be combined in a safe manner, and there is a limit on the amount of each antigen in a single injection.

Total dosage requirements

antigens	1	2	3	4	5	6
dosage	3	2	3	3	1	2

Dosage limit in an injection

antigens	1	2	3	4	5	6
dosage	2	1	2	3	1	12

Vaccine delivery model

Data and parameters

- Maximum number of vaccinations needed: $T = 14$,
- Set of antigens: $J := \{1, 2, \ldots, 6\}$,
- Injection cost: \$200,
- Dose limit in an injection: $D_j, \qquad \forall j \in J$,
- Required antigens after all vaccinations: $R_j, \qquad \forall j \in J$

Vaccine delivery model

Decision variables

The goal is to minimize the cost of vaccination which is equivalent to minimizing the number of vaccinations needed to deliver the required antigens. Therefore, a decision variable is defined to identify how many vaccines are needed

$$v_i = \begin{cases} 1 & \text{if vaccine } i \text{ is produced} \\ 0 & \text{otherwise,} \end{cases}, \qquad \forall i = 1, \ldots, T.$$

Vaccine delivery model

Decision variables (continued)

We still need to track which antigen was mixed into a particular vaccine injection, therefore,

$$x_{ji} : \text{Units of antigen } j \text{ mixed into vaccine } i$$
$$\forall i = 1, \ldots, T, \text{ and } j \in J,$$

However, to satisfy impose compatibility conditions, a binary variable can really help us,

$$y_{ji} = \begin{cases} 1 & \text{if antigen } j \text{ is mixed into vaccine } i, \\ 0 & \text{otherwise,} \end{cases}$$

Vaccine delivery model

Objective function

The goal is to minimize the cost of vaccine delivery which is satisfied if the number of total visits to the immunization center is minimized.

$$\min \sum_{i=1}^{T} 200 v_i,$$

Constraints

- There is a limit on the amount of each antigen in any injection

$$x_{ji} \leq D_j, \qquad \forall i = 1, \ldots, T, \text{ and } j \in J,$$

Vaccine delivery model

Constraints (continued)

- A certain amount of antigens should be delivered after all injections

$$\sum_{i=1}^{T} x_{ji} = R_j, \qquad \forall j \in J,$$

- No more than 3 antigens could be in any vaccine

$$\sum_{j} x_{ji} \leq \boxed{3v_i}, \qquad \forall i = 1, \ldots, T,$$

v_i is a binary variable. When $v_i = 1$, at most 3 antigens could be blended to create vaccine i. If $v_i = 0$, we do not need to blend any antigens. This constraints implicitly describes the relationship between x_{ji} and v_i variables as well.

Vaccine delivery model

Constraints (continued)

- Prevent non-compatible antigens to be mixed together in a single vaccine

$$x_{ji} \leq M y_{ji}, \qquad \forall j \in \{2,5,6\}, \text{ and } i = 1, \ldots, T,$$
$$\sum_{j \in \{2,5,6\}} y_{ji} \leq 1, \qquad \forall i = 1, \ldots, T,$$

where $M = 3$ because at most 3 antigens are mixed into any injection.

Vaccine delivery model: Formal formulation

Model

$$
\begin{array}{rll}
\max & \sum_{i=1}^{T} 200 v_i, & \\
s.t.\ 1: & x_{ji} \leq D_j, & \forall i = 1, \ldots, T, \text{ and } j \in J, \\
2: & \sum_{i=1}^{T} x_{ji} = R_j, & \forall j \in J, \\
3: & \sum_j x_{ji} \leq 3 v_i, & \forall i = 1, \ldots, T, \\
4: & x_{ji} \leq 3 y_{ji}, & \forall j \in \{2,5,6\}, \text{ and } i = 1, \ldots, T \\
5: & \sum_{j \in \{2,5,6\}} x_{ji} \leq 1, & \forall i = 1, \ldots, T, \\
6: & v_i \text{ and } y_{ji} \in \{0,1\}, & \forall i = 1, \ldots, T, \text{ and } j \in J, \\
7: & x_{ji} \geq 0, \text{Integer} & \forall i = 1, \ldots, T, \text{ and } j \in J.
\end{array}
$$

Introduction

- Infectious diseases are the worlds largest killer of children and young adults
- Infectious diseases can cause death directly. They also cause death indirectly by increasing the chance that an individual will contract other diseases such as cancer.

The challenge

Although infectious diseases pose a serious threat to public health, resources for controlling infectious diseases are limited. Decision makers must determine how to allocate limited epidemic control budgets among competing programs and populations so as to achieve the greatest health benefit given the available prevention resources.

Introduction

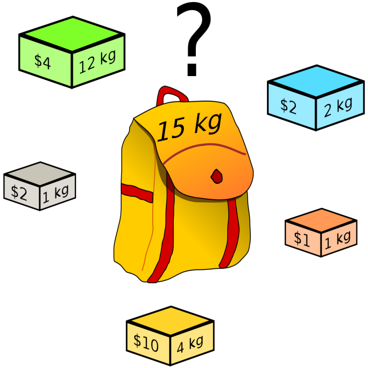
Epidemic programs

- Prevention: Immunization, immigration restriction, environmental abatement programs (insecticides)
- Control: Quarantine, screening and diagnosis
- Treatment: May reduce the spread of an infectious disease

Facing limited resources, decision makers must choose which program to initiate, which populations to target, and for how long to continue each program.

The knapsack problem

United States is faced with an Ebola outbreak. The center for disease control has a number of different options to respond. These options are divided into two categories: 1-Treatment and 2-Quarantine. CDC determined that the 3 programs available in the quarantine category must be carried out completely or they would have no effect. However, the 3 treatment programs may be implemented proportionally. CDC has also determined the effect of each program, E_i, and has access to a total budget B. Find which options yield the maximum effect in controlling the epidemic.



The knapsack problem

Data and parameters

- Number of programs: $I = 1, \ldots, 6$,
- Set of proportional programs: $I_1 := \{1, 2, 3\}$,
- Set of binary programs: $I_2 := \{4, 5, 6\}$,
- Cost of implementing program i: $c_i \qquad \forall i \in I$
- Total budget: B,
- Benefit of implementing program i: $E_i, \qquad \forall i \in I$.

The knapsack problem

Decision variables

The goal is to maximize the total benefit of running these program and keeping within the specified budget. To that end, we need to know which program is implemented

$$x_i : \text{The implemented proportion of program } i, \qquad \forall i \in I_1,$$

$$y_i = \begin{cases} 1 & \text{if program } i \text{ is implemented,} \\ 0 & \text{otherwise,} \end{cases} \qquad \forall i \in I_2.$$

The knapsack problem

Objective function

The goal is to maximize the total benefit of running these program and keeping within the specified budget.

$$\max \sum_{i \in I_1} E_i x_i + \sum_{i \in I_2} E_i y_i,$$

Constraints

Keep within the specified budget

$$\sum_{i \in I_1} c_i x_i + \sum_{i \in I_2} c_i y_i \leq B,$$

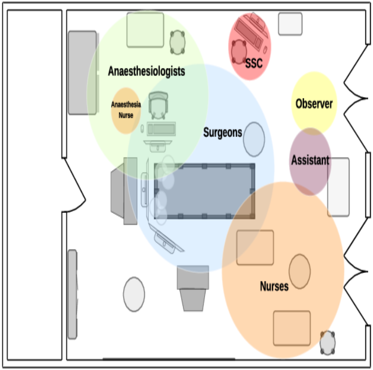
The knapsack problem: Formal formulation

Model

$$\begin{array}{rll} \max & \sum_{i\in I_1} E_i x_i + \sum_{i\in I_2} E_i y_i, & \\ s.t.\ 1: & \sum_{i\in I_1} c_i x_i + \sum_{i\in I_2} c_i y_i \leq B, & \\ 2: & 0 \leq x_i \leq 1, & \forall i \in I_1, \\ 3: & y_i \in \{0,1\}, & \forall i \in I_2. \end{array}$$

The p-median problem

Greenville Health System plans to install infection sensors in all its operation rooms which have a similar layout. The operation rooms are divided into 9 different zones where each zone can be the source of infection with prob. p_j. Assume that different infections spread with a similar and constant speed. The infection travel time between each zone is given. GHS intends to install 3 sensors in each operation room. Find the optimal location of sensors such that any infection can be found quickly.



P-median model: Formal formulation

Parameters

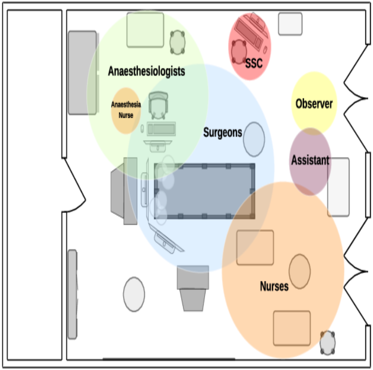
Sensor and infection zones:	$Z := \{1, 2, \dots, z\}$
Probability of being the infection source:	$p_j, \qquad \forall j \in Z,$
Number of sensors:	$N,$
Infection travel time:	$t_{ij} \qquad \forall i \in Z, \forall j \in Z.$

Model

$$
\begin{aligned}
\min \quad & \textstyle\sum_{i \in Z} \sum_{j \in Z} p_j t_{ij} y_{ij}, & \\
s.t.\ 1: \quad & \textstyle\sum_{j \in Z} x_i \leq N, & \\
2: \quad & y_{ij} \leq x_i, & \forall i \in Z, \forall j \in Z, \\
3: \quad & \textstyle\sum_{i \in Z} y_{ij} = 1, & \forall j \in Z, \\
4: \quad & x_i \in \{0, 1\} & \forall j \in Z, \\
5: \quad & y_{ij} \in \{0, 1\} & \forall i \in Z, \forall j \in Z.
\end{aligned}
$$

The p-center (minimax) approach

Some experts have criticized GHS because of the p-median approach in installing all operation room sensors. They point out that while the p-median approach performs better in the long run, it would neglect some areas fo which the probability of being a source is small. If in fact an infection starts from one of those location, the staff would notice it when it is too late. The recommend that GHS prepares for a worst case scenario when infection source is somewhere unexpected. Using the data given before, place the 3 sensors such that the maximum time to detect an infection is minimized.



The p-center (minimax) approach

Decision variables

Two sets of decision variables are defined as before (Facility location and Ambulance service operations).

$$x_j = \begin{cases} 1 & \text{if a sensor is placed in zone } i, \\ 0 & \text{otherwise,} \end{cases} \qquad \forall i \in Z.$$

$$y_{ij} = \begin{cases} 1 & \text{if zone } j \text{ is assigned to sensor } i, \\ 0 & \text{otherwise,} \end{cases} \qquad \forall i \in Z, \forall j \in Z.$$

Another variable is needed to identify the maximum time that it takes to detect an infection.

W : maximum time that it takes to detect an infection by a sensor

The p-center (minimax) approach

Objective function

The objective is to minimize the maximum time that it takes to detect an infection.

$$\min W$$

Constraints

The p-center approach includes all the constraints in the p-median approach. However, another set of constraints are needed to define the relationship between W and the rest of decision variables. Consider an infection starts from zone j. When such an infection is detected?

$$t_{\text{detection}} = \sum_{i \in Z} t_{ij} y_{ij}$$

W is the maximum time, thus

$$W \geq \sum_{i \in Z} t_{ij} y_{ij} \qquad \forall j \in Z,$$

The p-center (minimax) approach: Formal formulation

Parameters

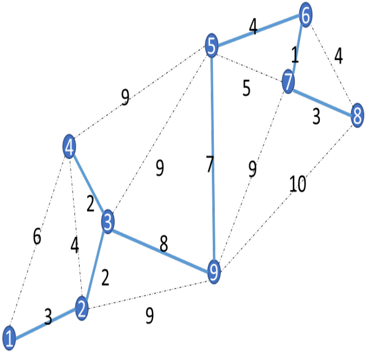
Sensor and infection zones:	$Z := \{1, 2, \ldots, z\}$	
Number of sensors:	N,	
Infection travel time:	t_{ij}	$\forall i \in Z, \forall j \in Z.$

Model

$$
\begin{array}{rll}
\min & W, & \\
s.t.\ 1: & W \geq \sum_{i \in Z} t_{ij} y_{ij} & \forall j \in Z, \\
2: & \sum_{j \in Z} x_i \leq N, & \\
3: & y_{ij} \leq x_i, & \forall i \in Z, \forall j \in Z, \\
4: & \sum_{i \in Z} y_{ij} = 1, & \forall j \in Z, \\
5: & x_i \in \{0, 1\} & \forall j \in Z, \\
6: & y_{ij} \in \{0, 1\} & \forall i \in Z, \forall j \in Z, \\
7: & W \geq 0. &
\end{array}
$$

Minimum spanning tree

CDC is trying to model spreading pattern of the infectious diseases. These patterns can help CDC to set up intervention centers and to allocate resources and personnel more efficiently when faced with an outbreak. Assuming that infectious diseases choose the least resistant path to spread, CDC is trying to figure out what would be the optimal path for an infection to spread from one area to several others.



Minimum spanning tree

Data and parameters

- Set of nodes: $V := \{1, \ldots, 9\}$
- Set of Edges: $E := \{(1,2),(1,4),(2,3),(2,4),(2,9),(3,4),(3,5),(3,9),(4,5),(5,6),(5,7),(5,9),(6,7),(6,8),(7,8),(7,9),(8,9)\}$
- Path resistance score: c_{ij}

Decision variables

The goal is to identify a spanning tree which has the overall minimum resistance score, therefore we have to pick a set of edges

$$x_{ij} := \begin{cases} 1 & \text{if edge i-j is picked,} \\ 0 & \text{otherwise.} \end{cases}$$

Minimum spanning tree

Objective function

To find the minimum spanning tree, the tree should have the least resistance score

$$\min \sum_{i,j \in E} c_{ij} x_{ij},$$

Constraint

- In graph theory, a spanning tree connects every node without creating a cycle. Therefore, the number of edges in a spanning tree is one less than the number of nodes.

$$\sum_{i,j \in E} x_{ij} = |V| - 1 = 9 - 1$$

$|\cdot|$ denotes the dimension of the set V. For example, $|V| = 9$.

Minimum spanning tree

Constraint (continued)

- A set of constraint known as subtour elimination constraints are used to prevent the formulation from allowing edges that create a cycle to be picked,

$$\sum_{i,j \in E, i \in S, j \in S} x_{ij} \leq |S| - 1$$

Note that this constraints create threes in **every subset of the nodes** and thus no cycle would be created since a cycle by definition has equal number of edges and nodes.

Minimum spanning tree: Formal formulation

Data and parameters

- Set of nodes: V
- Set of Edges: E
- Path resistance score: c_{ij}

Model

$$\begin{aligned}
\min \quad & \textstyle\sum_{i,j \in E} c_{ij} x_{ij}, & \\
s.t.\ 1: \quad & \textstyle\sum_{i,j \in E} x_{ij} = |V| - 1, & \\
2: \quad & \textstyle\sum_{i,j \in E, i \in S, j \in S} x_{ij} = |S| - 1, & \forall S \subseteq V \\
3: \quad & x_{ij} \in \{0, 1\} & \forall i, j \in E.
\end{aligned}$$

Class works

- 1: Code the knapsack problem in python using the data files uploaded in Canvas. The total budget is 60.
- 2: Code the p-center problem in python using the data files uploaded in Canvas. The total number of sensors is 5.
- 3: Code the minimum spanning tree problem in python. Use the data given in Slide 26.

Project

- Reformulate the knapsack problem in the *Dynamic Programming* framework. Consider a knapsack with a total capacity of 10 kg, and 5 items which weigh 1, 3, 4, 2, 4 and worth 2, 5, 8, 5, 7 dollars. Maximize your carrying worth.

Sources:

- Brandeau, M.L., Sainfort, F. and Pierskalla, W.P. eds., 2004. Operations research and health care: A handbook of methods and applications (Vol. 70). Springer Science & Business Media.
- Puterman ML. Markov decision processes: Discrete stochastic dynamic programming. John Wiley & Sons. 2014.
- Pinotti AP, Gomes MF, Samay N, Perra N, Vespignani A. The infection tree of global epidemics. Network Science. 2014 Apr. 2(1):132-7.