Relational Algebra and SQL

Slides by:

Joe Hellerstein

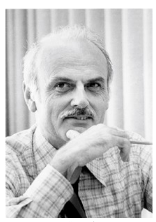
hellerstein@berkeley.edu

A bit of computing history

- Pre-1969: databases were more like data *structures*
  - i.e. hackery

The Relational Model

- A mathematical abstraction of a database, independent of data structures
  - Indeed, allows you to modify data structures at will without affecting program correctness!
  - So-called *Data Independence*



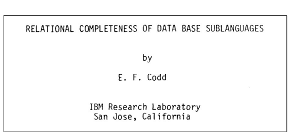
Edgar F. "Ted" Codd
(1923 - 2003)
Turing Award 1981

Codd's Main Contributions

1. *Relational model*: Mathematical relations with typed attributes, primary keys and consistency constraints, *independent* of physical properties like sort order or indexes. (1969)
2. A *Relational Algebra* of simple operations on relations (1972)
   - In the spirit of abstract algebra (groups, rings, fields, etc)
   - Inspiration for functional libraries like Pandas
3. A *Relational Calculus* of truth expressions over relations (1972)
   - Inspiration for declarative languages like SQL, Datalog, as well as visual languages

Codd's Theorem (1972)

- The relational calculus and the relational algebra have *equivalent expressive power*.

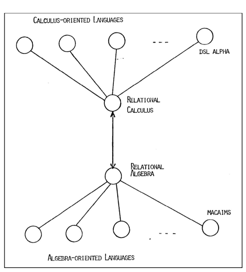


RELATIONAL COMPLETENESS OF DATA BASE SUBLANGUAGES

by

E. F. Codd

IBM Research Laboratory
San Jose, California



Historical Perspective

- **1969**: Codd's Relational Model paper
- 1974: IBM System R and Berkeley Ingres research projects begin
- **1979**: Oracle released first commercial SQL system for DEC Vax minicomputer
- 1981: Ted Codd receives Turing Award
- 1983: IBM DB2 released for MVS mainframe
- 1984-87: Teradata, Informix SQL and Sybase released
- 1988: Berkeley Postgres project begins
- 1989: Microsoft SQL Server released (derived from Sybase)
- **1992**: First meaningful SQL standard
- 1995: PostgreSQL released ("Postgres 95"), MySQL released
- 2000: Sqlite released
- 2004: Google MapReduce paper
- 2010: Apache Hive (SQL on Hadoop) released
- **2012**: Pandas library popularized

Relational Terminology

- *Database*: Set of Relations
- *Relation* (*Table*):
  - *Schema (metadata)*
    - A unique name for the relation
    - A list of *k* distinct Attribute names, each associated with a type.
    - Optional constraints (*key constraints*)
  - *Instance (data)*
    - Set of *k*-tuples satisfying the schema
- *Attribute* (*Column, Field*)
- *Tuple* (*Row, Record*)

The schema of a database is the set of schemas of its relations.

Boat Club Schema

sailors(sid integer, sname text, rating integer, age float)

boats(bid integer, bname text, color text)

reserves(sid integer, bid integer, day date)

Boat Club
Example Instances

Boats

bid	bname	color
101	Interlake	blue
102	Interlake	red
104	Marine	red
103	Clipper	green

Note: primary keys underlined

R1

sid	bid	day
22	101	10/10/16
58	103	11/12/96

S1

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S2

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Why learn Relational Algebra

- Intuitive for programmers
  - Imperative: apply this, then apply that
  - Set-oriented: no need for for-loops, low-level reasoning
- Basis of functional libraries like Pandas
  - Pandas (over-?) complicates things
  - Nice to have a clean foundation
- Simple optimization rules
  - Will help you think about writing efficient data-centric programs
- Common currency
  - Most data folk know the relational algebra operators

Relational Algebra Preliminaries

- Algebra of operators on *relational instances*

$$\pi_{S.name}(\sigma_{R.bid=100 \wedge S.rating>5}(R \bowtie_{R.sid=S.sid} S))$$

  - **Closed**: result is also a relational instance
    - Enables rich composition!
  - **Typed**: input schema and operator determines output
    - Why is this important?
- Pure relational algebra has ***set* semantics**
  - **No duplicate** tuples in a relation instance
  - vs. SQL, which has *multiset* (bag) semantics

Relational Algebra Operators

Unary Operators: *operate on **single** relation instance*

- **Projection (π):** *Retains only desired columns (vertical)*
- **Selection (σ):** *Selects a subset of rows (horizontal)*
- **Renaming (ρ):** *Rename attributes and relations.*

Binary Operators: *operate on **pairs** of relation instances*

- **Union ($\cup$):** *Tuples in r1 or in r2.*
- **Intersection ($\cap$):** *Tuples in r1 and in r2.*
- **Set-difference (—):** *Tuples in r1, but not in r2.*
- **Cross-product ($\times$):** *Allows us to combine two relations.*
- **Joins ($\bowtie_\theta$, $\bowtie$):** *Combine relations that satisfy predicates*

Projection (π)

Selects a subset of columns (vertical)

$$\pi_{\underbrace{sname,age}_{\text{List of Attributes}}}(S2)$$

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

sname	age
yuppy	35.0
lubber	55.5
guppy	35.0
rusty	35.0

- Schema determined by schema of attribute list
  - Names and types correspond to input attributes

Projection (π)

Selects a subset of columns (vertical)

$$\pi_{age}(S2)$$

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Multiset

age
35.0
55.5
35.0
35.0

Set

age
35.0
55.5

- Set semantics → results in fewer rows
  - SQL systems don't automatically remove duplicates
  - Why?

Selection(σ)

Selects a subset of rows (horizontal)

$$\sigma_{\underbrace{rating>8}_{\text{Selection Condition (Boolean Expression)}}}(S2)$$

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
~~31~~	~~lubber~~	~~8~~	~~55.5~~
~~44~~	~~guppy~~	~~5~~	~~35.0~~
58	rusty	10	35.0

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

- Output schema same as input
- Duplicate Elimination?

Composing Select and Project

- Names of sailors with rating > 8

$$\pi_{sname}(\sigma_{rating>8}(S2))$$

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

$\sigma_{rating>8}$

sid	sname	rating	age
28	yuppy	9	35.0
58	rusty	10	35.0

π_{name}

sname
yuppy
rusty

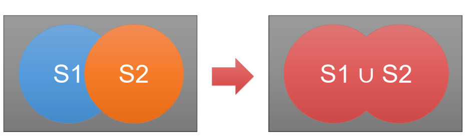
- What about:

$$\sigma_{rating>8}(\pi_{sname}(S2))$$

Invalid types. Input to $\sigma_{rating>8}$ does not contain rating.

Union (U)

S1 U S2



Two input relations, must be *compatible*:
- Same number of fields.
- Fields in the same position have same **type**

Union (U)

Relational *Instance* **S1**

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

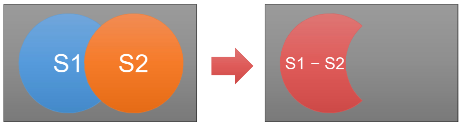
S1 U S2

sid	sname	rating	age
22	dustin	7	45
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

Duplicate elimination?

Set Difference (−)

S1 − S2



Same as with union, both input relations must be *compatible*.

Set Difference (−)

Relational *Instance* **S1**

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

S1 − S2

sid	sname	rating	age
22	dustin	7	45

Symmetric?

S2 − S1

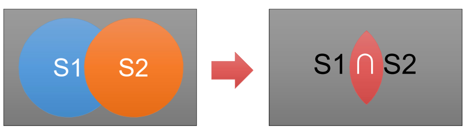
sid	sname	rating	age
28	yuppy	9	35.0
44	guppy	5	35.0

Duplicate elimination?

- Not required

Intersection (⋂)

S1 ⋂ S2



Same as with union, both input relations must be *compatible*.

Intersection (⋂)

Relational *Instance* **S1**

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Relational *Instance* **S2**

sid	sname	rating	age
28	yuppy	9	35.0
31	lubber	8	55.5
44	guppy	5	35.0
58	rusty	10	35.0

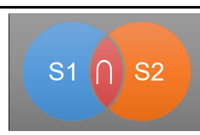
S1 ⋂ S2

sid	sname	rating	age
31	lubber	8	55.5
58	rusty	10	35.0

Is intersection essential?

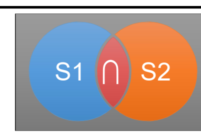
- Implement it with earlier ops. ?

Intersection (⋂)

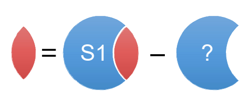


$S1 \cap S2 = S1 - ?$

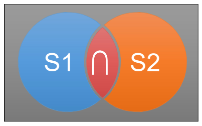
Intersection (⋂)



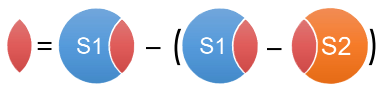
$S1 \cap S2 = S1 - ?$



Intersection (⋂)



S1 ⋂ S2 = S1 – (S1 – S2)



Cross-Product (×)

R1 × S1: *Each row of **R1** paired with each row of **S1***

R1:

sid	bid	day
22	101	10/10/96
58	103	11/12/96

×

S1:

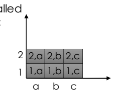
sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

=

R1 × S1

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

Sometimes also called **Cartesian Product:**



How many rows in the result? | R1 | * | R2 |

Schema compatibility? No requirements.

One field per field in original schemas.

What about duplicate names? Renaming operator

Renaming (ρ = *"rho"*)

Renames relations and their attributes:

$$\rho(\underbrace{\text{Temp1}}_{\text{Output Relation Name}}(\underbrace{1 \rightarrow \text{sid1}, 4 \rightarrow \text{sid2}}_{\text{Renaming List position} \rightarrow \text{New Name}}), \underbrace{\text{R1} \times \text{S1}}_{\text{Input Relation}})$$

R1 × S1

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

Temp1

sid1	bid	day	sid2	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
22	101	10/10/96	31	lubber	8	55.5
22	101	10/10/96	58	rusty	10	35.0
58	103	11/12/96	22	dustin	7	45.0
58	103	11/12/96	31	lubber	8	55.5
58	103	11/12/96	58	rusty	10	35.0

- Relational algebra can also be defined *positionally*, without names. $\pi_{f5}(\sigma_{f6>f8}(S2))$
- Difficult to read …

Compound Operator: Join

- Joins are compound operators (like intersection):
  - **Cross product** followed by **selection** and possibly **projection** (for natural join)
- Hierarchy of common kinds:
  - **Theta Join ($\bowtie_\theta$):** *join on logical expression θ*
    - **Equi-Join:** *theta join with conjunction equalities*
      - **Natural Join ($\bowtie$):** *equi-join on all matching column names*
- Note: we should use a join, not a cross-product, if we can! Easier to read, clarifies opportunities for using efficient join algorithms.

Theta Join ($\bowtie_\theta$)

$$\mathbf{R \bowtie_\theta S = \sigma_\theta(R \times S)}$$

Example: *Pair each sailor with older sailors.*

S1 $\bowtie_{\text{age} < \text{age}}$ S1

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

Theta Join ($\bowtie_\theta$)

$$\mathbf{R \bowtie_\theta S = \sigma_\theta(R \times S)}$$

Example: *Pair each sailor with older sailors.*

S1 $\bowtie_{\text{age} < \text{age}}$ S1

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1 × S1

S1				S1			
sid	sname	rating	age	sid	sname	rating	age
~~22~~	~~dustin~~	~~7~~	~~45.0~~	~~22~~	~~dustin~~	~~7~~	~~45.0~~
22	dustin	7	45.0	31	lubber	8	55.5
22	dustin	7	45.0	58	rusty	10	35.0
~~31~~	~~lubber~~	~~8~~	~~55.5~~	~~22~~	~~dustin~~	~~7~~	~~45.0~~
~~31~~	~~lubber~~	~~8~~	~~55.5~~	~~31~~	~~lubber~~	~~8~~	~~55.5~~
31	lubber	8	55.5	58	rusty	10	35.0
~~58~~	~~rusty~~	~~10~~	~~35.0~~	~~22~~	~~dustin~~	~~7~~	~~45.0~~
~~58~~	~~rusty~~	~~10~~	~~35.0~~	~~31~~	~~lubber~~	~~8~~	~~55.5~~
~~58~~	~~rusty~~	~~10~~	~~35.0~~	~~58~~	~~rusty~~	~~10~~	~~35.0~~

Theta Join ($\bowtie_\theta$)

$$R \bowtie_\theta S = \sigma_\theta(R \times S)$$

Example: *Pair each sailor with older sailors.*

$$S1 \bowtie_{age < age} S1$$

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

S1				S1			
sid	sname	rating	age	sid	sname	rating	age
22	dustin	7	45.0	31	lubber	8	55.5
22	dustin	7	45.0	58	rusty	10	35.0
31	lubber	8	55.5	58	rusty	10	35.0

- *Result schema* same as that of cross-product.
- Special Case:
  - **Equi-Join:** *theta join with conjunction equalities*
    - *Special special case* ***Natural Join*** *…*

Natural Join ($\bowtie$)

*Special case of **equi-join** in which equalities are specified for all matching attributes, and duplicate attributes are projected away*

$$R \bowtie S = \pi_{\text{unique attr.}}\ \sigma_{\text{eq. matching attr.}}(R \times S)$$

- Compute $R \times S$
- Select rows where attributes appearing in both relations have equal values
- Project onto the set of all unique attributes.

Natural Join ($\bowtie$)

$$R \bowtie S = \pi_{\text{unique attr.}}\ \sigma_{\text{eq. matching attr.}}(R \times S)$$

Example:

R1:

sid	bid	day
22	101	10/10/96
58	103	11/12/96

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

$R1 \bowtie S1$

sid	bid	day	sid	sname	rating	age
22	101	10/10/96	22	dustin	7	45.0
~~22~~	~~101~~	~~10/10/96~~	~~31~~	~~lubber~~	~~8~~	~~55.5~~
~~22~~	~~101~~	~~10/10/96~~	~~58~~	~~rusty~~	~~10~~	~~35.0~~
~~58~~	~~103~~	~~11/12/96~~	~~22~~	~~dustin~~	~~7~~	~~45.0~~
~~58~~	~~103~~	~~11/12/96~~	~~31~~	~~lubber~~	~~8~~	~~55.5~~
58	103	11/12/96	58	rusty	10	35.0

Natural Join ($\bowtie$)

$$R \bowtie S = \pi_{\text{unique attr.}}\ \sigma_{\text{eq. matching attr.}}(R \times S)$$

Example:

R1:

sid	bid	day
22	101	10/10/96
58	103	11/12/96

S1:

sid	sname	rating	age
22	dustin	7	45.0
31	lubber	8	55.5
58	rusty	10	35.0

$R1 \bowtie S1$

sid	bid	day	sname	rating	age
22	101	10/10/96	dustin	7	45.0
58	103	11/12/96	rusty	10	35.0

Commonly used for foreign key joins (as above).

Exercise:

Find names of sailors who've reserved boat #103

Boats(bid,bname,color)
Sailors(sid, sname, rating, age)
Reserves(sid, bid, day)

- Solution 1:

$$\pi_{sname}(\sigma_{bid=103}(\text{Sailors} \bowtie \text{Reserves}))$$

- Solution 2:

$$\pi_{sname}(\text{Sailors} \bowtie \sigma_{bid=103}(\text{Reserves}))$$

Exercise:

Find names of sailors who've reserved a red boat

Boats(bid,bname,color)
Sailors(sid, sname, rating, age)
Reserves(sid, bid, day)

- Solution 1:

$$\pi_{sname}(\sigma_{color='red'}(\text{Boats}) \bowtie \text{Res} \bowtie \text{Sailors})$$

- More "efficient" Solution 2:

$$\pi_{sname}(\pi_{sid}(\pi_{bid}(\sigma_{color='red'}(\text{Boats})) \bowtie \text{Res}) \bowtie \text{Sailors})$$

In general many possible equivalent expressions: **algebra**…

Relational Algebra Rules

- **Selections:**
  - $\sigma_{c1 \wedge \ldots \wedge cn}(R) \equiv \sigma_{c1}(\ldots(\sigma_{cn}(R))\ldots)$ *(cascade)*
  - $\sigma_{c1}(\sigma_{c2}(R)) \equiv \sigma_{c2}(\sigma_{c1}(R))$ *(commute)*
- **Projections:**
  - $\pi_{a1}(R) \equiv \pi_{a1}(\ldots(\pi_{a1, \ldots, an-1}(R))\ldots)$ *(cascade)*
- **Cartesian Product**
  - $R \times (S \times T) \equiv (R \times S) \times T$ (associative)
  - $R \times S \equiv S \times R$ (commutative)
  - *Applies for joins as well but* be careful with join predicates …

Caution with Join Ordering

Boats(bid,bname,color)
Sailors(sid, sname, rating, age)
Reserves(sid, bid, day)

- Consider the following:

$$\text{Boats} \bowtie_{bid} \text{Reserves} \bowtie_{sid} \text{Sailors}$$

- Commute and Associate:

$$\text{Boats} \bowtie_{bid} (\text{Sailors} \bowtie_{sid} \text{Reserves})$$

- Incompatible join predicate:

$$(\text{Boats} \bowtie_{bid} \text{Sailors}) \bowtie_{sid} \text{Reserves}$$

Caution with Join Ordering

Boats(bid,bname,color)
Sailors(sid, sname, rating, age)
Reserves(sid, bid, day)

- Consider the following:

$$\text{Boats} \bowtie_{bid} \text{Reserves} \bowtie_{sid} \text{Sailors}$$

- Commute and Associate:

$$\text{Boats} \bowtie_{bid} (\text{Sailors} \bowtie_{sid} \text{Reserves})$$

- Incompatible join predicate:

$$(\text{Boats} \times \text{Sailors}) \bowtie_{sid,bid} \text{Reserves}$$

More Relational Algebra Rules

Commuting of selection operators

- $\sigma_c(R \times S) \equiv \sigma_c(R) \times S$ (c only has fields in R)
- $\sigma_c(R \bowtie S) \equiv \sigma_c(R) \bowtie S$ (c only has fields in R)

Commuting of projection operators

- $\pi_a(R \times S) \equiv \pi_{a1}(R) \times \pi_{a2}(S)$
  - a_1 is subset of a that mentions R and a_2 is subset of a that mentions S
  - Similar result holds for joins

A Standard Extension

- Group By / Aggregation Operator (γ):

$$\gamma_{age,\ AVG(rating)}(Sailors)$$

  - *With selection (HAVING clause):*

$$\gamma_{age,\ AVG(rating),\ COUNT(*)>2}(Sailors)$$

Recall Codd also had a Relational Calculus

- A *declarative* logic language
  - Find all tuples such that the following properties hold …
  - Says "what" the output should be, not "how" to get it.
- SQL is based on the relational calculus
  - Even though, under the hood, database engines translate to algebra expressions!

SQL Language

- Two sublanguages:
  - DDL – Data Definition Language
    - Define and modify schema
  - DML – Data Manipulation Language
    - Queries can be written intuitively.
- Relational Database Management System (RDBMS) responsible for efficient evaluation.
  - Choose and run algorithms for declarative queries

We will learn SQL interactively

- Frontend: psql command line, Jupyter Notebook
- Backend: PostgreSQL