Introduction to Mathematical Induction with Examples

Math Course

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1 What is Mathematical Induction?

Mathematical induction is a powerful proof technique used in mathematics to prove statements that are asserted to be true for all natural numbers. It is especially useful for proving propositions about:

- Summations and series

- Divisibility properties

- Inequalities

- Combinatorial identities

2 The Principle of Mathematical Induction

Theorem 1 (Principle of Mathematical Induction). *To prove that a proposition $P(n)$ is true for all natural numbers $n \geq n_0$, it suffices to:*

1. ***Base Case:*** *Verify $P(n_0)$ is true*

2. ***Inductive Step:*** *Show that if $P(k)$ is true for some arbitrary $k \geq n_0$ (called the induction hypothesis), then $P(k+1)$ must also be true*

3 The Domino Analogy

Mathematical induction works like falling dominos:

- The base case is like knocking over the first domino

- The inductive step ensures each domino will knock over the next one

- Together, these guarantee that all dominos will fall

4 Key Points to Remember

- Always verify both the base case and inductive step

- The induction hypothesis is crucial - you must assume $P(k)$ is true

- Mathematical induction proves statements for *all* natural numbers beyond the base case

- Choose the appropriate base case (n_0) for your proposition