

1 Theorem

Proof by Induction: Sum of the First n Natural Numbers

The sum of the first n natural numbers is:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

2 Proof by Enumeration

Consider the sum of the first n numbers:

$$1 + 2 + \cdots + n. \tag{1}$$

Now, let us write the sum from n to 1:

$$n + n - 1 + \cdots + 1. \tag{2}$$

The sum of both sequences is the same; the only difference is the order of the addition. Next, add the first element of the first sequence to the first element of the second sequence, which is $(1 + n)$. Do the same for each of the two elements of both sequences. The result is the following:

$$(1 + n), (2 + (n - 1)), \dots, (n + 1). \tag{3}$$