Introduction to Subtraction via Recursion

1 Definition

Subtraction of two non-negative integers a and b, denoted $a - b$, can be defined recursively using the predecessor operation (decrementing by 1). This definition assumes $a \geq b$, as negative results are not covered here:

- **Base Case**: If $b = 0$, then $a - b = a$.
- **Recursive Case**: If $b > 0$, then $a - b = (a - 1) - (b - 1)$.

This reduces subtraction by b to recursively subtracting 1 from both a and b until $b = 0$.

2 Examples

2.1 Example 1: $5 - 0$

Applying the base case directly:

$$5 - 0 = 5$$

2.2 Example 2: $5 - 3$

Breaking down the recursion step-by-step:

$$\begin{aligned}
5 - 3 &= (5 - 1) - (3 - 1) \\
&= 4 - 2 \\
&= (4 - 1) - (2 - 1) \\
&= 3 - 1 \\
&= (3 - 1) - (1 - 1) \\
&= 2 - 0 \\
&= 2
\end{aligned}$$

2.3 Example 3: $6-6$

Demonstrating equal values:

$$\begin{aligned}
6-6 &= (6-1)-(6-1) \\
&= 5-5 \\
&= (5-1)-(5-1) \\
&= 4-4 \\
&\vdots \quad \text{(repeating until base case)} \\
&= 0-0 \\
&= 0
\end{aligned}$$

2.4 Example 4: $4-2$

Recursive steps:

$$\begin{aligned}
4-2 &= (4-1)-(2-1) \\
&= 3-1 \\
&= (3-1)-(1-1) \\
&= 2-0 \\
&= 2
\end{aligned}$$

2.5 Example 5: $3-1$

Simpler case:

$$\begin{aligned}
3-1 &= (3-1)-(1-1) \\
&= 2-0 \\
&= 2
\end{aligned}$$

3 Important Notes

- This definition assumes $a \geq b$. If $a < b$, the recursion would attempt to decrement a below 0, which is undefined here.
- Subtraction is not commutative: $a-b \neq b-a$ (unless $a=b$).

4 Conclusion

This recursive framework reduces subtraction to repeated decrementing of both a and b, terminating when $b=0$. It highlights how recursion simplifies operations by breaking them into incremental steps. However, it is restricted to non-negative results, emphasizing the importance of $a \geq b$.