Proof: 8 Divides the Difference of Squares of Two Odd Numbers

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1 Theorem

Let a and b be odd integers. Then:

$$8 \mid (a^2 - b^2)$$

2 Proof

We prove this by expressing odd numbers in their general form and simplifying.

2.1 Step 1: Representation of Odd Numbers

Any odd integer can be written as:

$$a = 2k + 1, \quad b = 2m + 1 \quad \text{where } k, m \in \mathbb{Z}.$$

2.2 Step 2: Compute $a^2 - b^2$

Using the difference of squares:

$$a^2 - b^2 = (a - b)(a + b).$$

Substitute a and b:

$$a^2 - b^2 = (2k + 1 - 2m - 1)(2k + 1 + 2m + 1) = 2(k - m) \cdot 2(k + m + 1).$$

Simplify:

$$a^2 - b^2 = 4(k - m)(k + m + 1).$$

2.3 Step 3: Divisibility by 8

We show that $4(k-m)(k+m+1)$ is divisible by 8:

- **Case 1**: If $k-m$ is even, then $(k-m)=2n$ for some $n \in \mathbb{Z}$. Thus:

$$a^2 - b^2 = 4(2n)(k+m+1) = 8n(k+m+1).$$

- **Case 2**: If $k-m$ is odd, then $k+m+1$ must be even (since the sum of an odd and even term is even). Let $k+m+1=2n$. Thus:

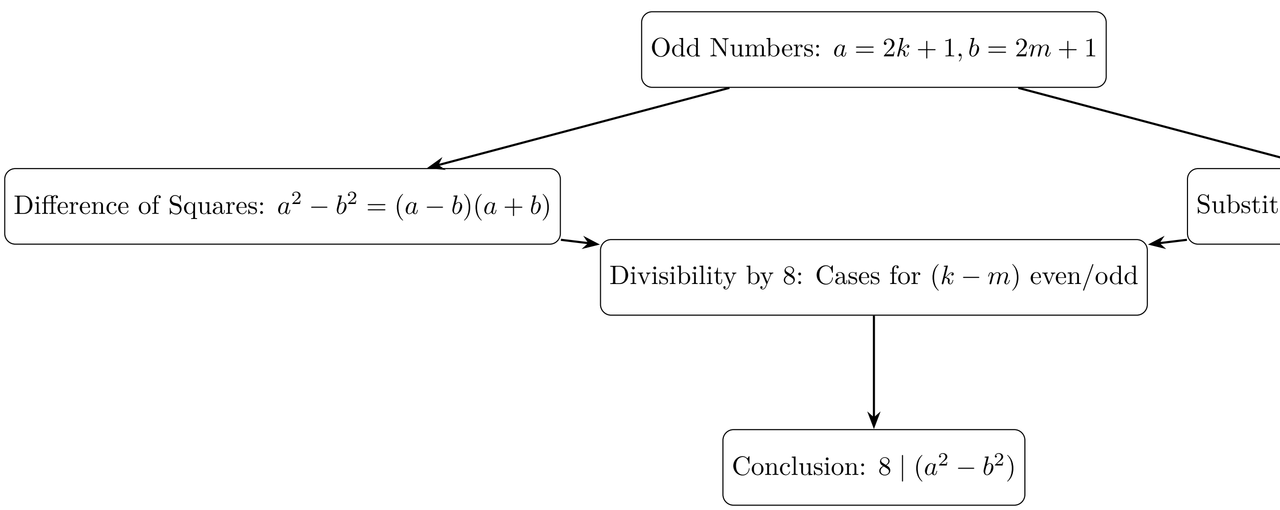
$$a^2 - b^2 = 4(k-m)(2n) = 8n(k-m).$$

In both cases, $a^2 - b^2$ is divisible by 8.

2.4 Conclusion

For any two odd integers a and b, 8 divides $a^2 - b^2$.

3 DAG Diagram of the Proof



The DAG illustrates:

- The **definition of odd numbers** leads to algebraic manipulation.
- The **difference of squares** and **substitution** steps converge.
- **Divisibility by 8** is shown by case analysis.
- The **conclusion** follows from the cases.