

1 Question:

Prove that the sum of two even integers is always even.

2 Answer:

An integer is even if it has remainder 0 when divided by 2. That is,

$$a \equiv 0 \pmod{2}, \quad b \equiv 0 \pmod{2}. \tag{1}$$

Adding both congruences:

$$a + b \equiv 0 + 0 \equiv 0 \pmod{2}. \tag{2}$$

Since $a + b \equiv 0 \pmod{2}$, it follows that $a + b$ is even.