1 Question:

Prove Theorem 2.3(iv): $(AB)^T = B^T A^T$.

2 Answer:

Let $A = [a_{ik}]$ and $B = [b_{kj}]$. Then the ij-entry of AB is

$$a_{i1}b_{1j} + a_{i2}b_{2j} + \cdots + a_{im}b_{mj}$$

This is the ji-entry (reverse order) of $(AB)^T$. Now column j of B becomes row j of B^T, and row i of A becomes column i of A^T. Thus, the ij-entry of $B^T A^T$ is

$$[b_{1j}, b_{2j}, \ldots, b_{mj}]\,[a_{i1}, a_{i2}, \ldots, a_{im}]^T = b_{1j}a_{i1} + b_{2j}a_{i2} + \cdots + b_{mj}a_{im}$$

Thus, $(AB)^T = B^T A^T$ on because the corresponding entries are equal.