

## 1 Question:

Prove that  $\sqrt{2}$  is an irrational number.

## 2 Answer:

Suppose  $\sqrt{2}$  is rational, so  $\sqrt{2} = \frac{p}{q}$  in lowest terms. Then:

$$p^2 = 2q^2. \tag{1}$$

Since  $p^2$  is even,  $p$  is even, so we write  $p = 2k$ . Substituting:

$$(2k)^2 = 2q^2 \Rightarrow 4k^2 = 2q^2 \Rightarrow q^2 = 2k^2. \tag{2}$$

Thus,  $q^2$  is even, meaning  $q$  is also even. This creates an infinite descent of smaller and smaller even numbers, contradicting the assumption that  $p$  and  $q$  are in lowest terms. Hence,  $\sqrt{2}$  is irrational.