1 Title

Multiplication Via Recursion

2 Abstract

Multiplication via recursion breaks down complex multiplication into simpler steps through two phases: first decomposing multiplication into repeated addition, then applying addition via recursion to compute the final result. This approach systematically reduces complex calculations to elementary operations.

3 Recursive Definition

For non-negative integers a and b:

$$a \times b = \begin{cases} 0 & \text{if } b = 0 \quad \text{(Base Case)} \\ a + (a \times (b-1)) & \text{if } b > 0 \quad \text{(Recursive Case)} \end{cases}$$

4 Complete Calculation Procedure

The complete calculation of $a \times b$ follows these sequential steps:

4.1 Phase 1: Multiplication Decomposition

4.1.1 Start with the initial problem: $a \times b$

4.1.2 Apply the recursive formula: $a \times b = a + (a \times (b-1))$

4.1.3 Continue decomposing until reaching the base case:

$$\begin{aligned} a \times b &= a + (a \times (b-1)) \\ &= a + (a + (a \times (b-2))) \\ &= a + a + (a \times (b-3)) \\ &\vdots \\ &= \underbrace{a + a + \cdots + a}_{b \text{ times}} + (a \times 0) \\ &= \underbrace{a + a + \cdots + a}_{b \text{ times}} + 0 \\ &= \underbrace{a + a + \cdots + a}_{b \text{ times}} \end{aligned}$$

4.2 Phase 2: Addition Calculation

After decomposing multiplication into repeated addition, we must calculate each addition operation separately. This requires multiple applications of the addition via recursion process:

4.2.1 Identify all addition operations

The result of Phase 1 is an expression with b instances of a being added together:

$$\underbrace{a + a + \cdots + a}_{b \text{ times}}$$

4.2.2 Apply addition via recursion to each pair of terms

For each addition operation in the sequence, we must apply the addition via recursion pattern separately:

$$\begin{aligned}
\text{First addition:} \quad a + a &= (a + (a-1)) + 1 \\
&= ((a + (a-2)) + 1) + 1 \\
&\vdots \\
&= (\cdots((a+0)+1) + \cdots + 1)
\end{aligned}$$

4.2.3 Process additions sequentially

After calculating the first addition, we must continue with the next addition operation:

$$\begin{aligned}
\text{Second addition:} \quad (a + a) + a &= ((a + a) + (a-1)) + 1 \\
&= (((a + a) + (a-2)) + 1) + 1 \\
&\vdots
\end{aligned}$$

4.2.4 Continue until all additions are resolved

This process continues for all $b - 1$ addition operations in the expression:

$$\begin{aligned}
\text{Third addition:} \quad ((a + a) + a) + a &= \dots \\
&\vdots
\end{aligned}$$

4.2.5 Combine all results

After applying addition via recursion to each addition operation, we obtain the final result:

$$\underbrace{a + a + \cdots + a}_{b \text{ times}} = a \times b$$

5 Summary

Multiplication via recursion follows a clear two-phase process:

1. Phase 1: Decompose multiplication into repeated addition, resulting in b instances of a being added together

2. Phase 2: Calculate each addition operation separately using addition via recursion, requiring $b - 1$ distinct applications of the addition process

This approach demonstrates how complex operations (multiplication) can be systematically reduced to simpler operations (multiple separate additions) and ultimately to the most basic operation (adding 1). The key insight is that each addition operation in the sequence must be calculated individually using the addition via recursion process.