Adding Multiple Numbers

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1 Introduction

All addition ultimately reduces to pairwise operations. This course shows how we systematically break down sums with multiple numbers into sequences of binary additions using parentheses.

2 Core Principle

Addition is binary: We can only add two numbers at any given moment. For multiple numbers:

- We need $(n-1)$ additions for n numbers
- Parentheses specify operation order
- Different groupings yield same result (associativity)

3 General Case Pattern

For numbers $a_1 + a_2 + \cdots + a_n$:

$$\begin{aligned} \text{Step 1:} \quad & (a_1 + a_2) \\ \text{Step 2:} \quad & (\text{Result}_1 + a_3) \\ & \vdots \\ \text{Step } n-1\text{:} \quad & (\text{Result}_{n-2} + a_n) \end{aligned}$$

4 Basic Examples

4.1 Example

$$\begin{aligned} & 2 + 3 + 4 \\ & = (2 + 3) + 4 \\ & = 5 + 4 \\ & = 9 \end{aligned}$$

Alternative grouping:

$$\begin{aligned} & 2 + (3 + 4) \\ & = 2 + 7 \\ & = 9 \end{aligned}$$

4.2 Example

$$\begin{aligned} & 1 + 4 + 2 + 5 \\ & = ((1 + 4) + 2) + 5 \\ & = (5 + 2) + 5 \\ & = 7 + 5 \\ & = 12 \end{aligned}$$

Alternative sequence:

$$\begin{aligned} & 1 + (4 + (2 + 5)) \\ & = 1 + (4 + 7) \\ & = 1 + 11 \\ & = 12 \end{aligned}$$

4.3 Example

Add: $3 + 1 + 4 + 2 + 5$

$$\begin{aligned} & ((((3 + 1) + 4) + 2) + 5) \\ & = ((4 + 4) + 2) + 5 \\ & = (8 + 2) + 5 \\ & = 10 + 5 \\ & = 15 \end{aligned}$$