1 Question:

Let $A = [a_{ij}]$ and $B = [b_{ij}]$ be upper triangular matrices. Prove that AB is upper triangular with diagonal $a_{11}b_{11}, a_{22}b_{22}, \ldots, a_{nn}b_{nn}$.

2 Answer:

Let $AB = [c_{ij}]$. Then $c_{ij} = \sum_{k=1}^{n} a_{ik}b_{kj}$ and $c_{ii} = \sum_{k=1}^{n} a_{ik}b_{ki}$. Suppose $i > j$. Then, for any k, either $i > k$ or $k > j$, so that either $a_{ik} = 0$ or $b_{kj} = 0$. Thus, $c_{ij} = 0$, and AB is upper triangular. Suppose $i = j$. Then, for $k < i$, we have $a_{ik} = 0$; and, for $k > i$, we have $b_{ki} = 0$. Hence, $c_{ii} = a_{ii}b_{ii}$, as claimed. [This proves one part of Theorem 2.5(i); the statements for $A + B$ and kA are left as exercises.]