1 Question:

Prove that $\sqrt{2}$ is an irrational number.

2 Answer:

Suppose $\sqrt{2}$ were rational, meaning $\sqrt{2} = \frac{p}{q}$ for some integers p and q with no common factors. Squaring both sides gives:

$$p^2 = 2q^2. \tag{1}$$

In prime factorization, p^2 has an even number of factors, and $2q^2$ has an odd number of factors of 2. This contradiction shows that p^2 cannot equal $2q^2$, proving that $\sqrt{2}$ is irrational.