

1 Question:

Prove that the sum of two even integers is always even.

2 Answer:

Assume, for contradiction, that the sum of two even integers is odd. That is, assume:

$$a + b = 2k + 1, \quad \text{for some integer } k. \quad (1)$$

Since a and b are even, they can be written as:

$$a = 2m, \quad b = 2n. \quad (2)$$

Adding these values:

$$a + b = 2m + 2n = 2(m + n). \quad (3)$$

Since $2(m + n)$ is even, this contradicts our assumption that $a + b$ is odd. Therefore, the sum of two even numbers must be even.