1 Theorem

Proof by Induction: Sum of the First n Natural Numbers

The sum of the first n natural numbers is:

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

2 Proof by Induction

We proceed by mathematical induction.

2.1 Base Case ($n = 1$)

$$\text{LHS} = 1, \quad \text{RHS} = \frac{1(1+1)}{2} = 1$$

Since LHS = RHS, the base case holds.

2.2 Inductive Hypothesis

Assume the statement holds for some $k \geq 1$:

$$1 + 2 + \cdots + k = \frac{k(k+1)}{2}$$

2.3 Inductive Step ($n = k + 1$)

We must show:

$$1 + 2 + \cdots + k + (k+1) = \frac{(k+1)(k+2)}{2}$$

Starting from the left-hand side:

$$\begin{aligned}
1 + 2 + \cdots + k + (k+1) &= (1 + 2 + \cdots + k) + (k+1) && (1)\\
&= \frac{k(k+1)}{2} + (k+1) \quad \text{(by the inductive hypothesis)} && (2)\\
&= \frac{k(k+1) + 2(k+1)}{2} && (3)\\
&= \frac{(k+1)(k+2)}{2} && (4)
\end{aligned}$$

This matches the right-hand side for $n = k + 1$.

2.4 Conclusion

By the principle of mathematical induction, the formula holds for all $n \geq 1$.