

1 Question:

Prove that the product of any two odd integers is always odd.

2 Answer:

Let a and b be two odd integers. By definition, an odd integer can be written as:

$$a = 2m + 1, \quad b = 2n + 1, \quad \text{where } m, n \text{ are integers.} \quad (1)$$

The product of a and b is:

$$a \cdot b = (2m + 1)(2n + 1). \quad (2)$$

Expanding the product:

$$a \cdot b = 4mn + 2m + 2n + 1. \quad (3)$$

Factoring out 2 from the first three terms:

$$a \cdot b = 2(2mn + m + n) + 1. \quad (4)$$

Since $2mn + m + n$ is an integer, $a \cdot b$ is of the form $2k + 1$, which is odd.