1 Question:

THEOREM 3.10: A square system $AX = B$ of linear equations has a unique solution if and only if the matrix A is invertible. In such a case, $A^{-1}B$ is the unique solution of the system.

2 Answer:

We only prove here that if A is invertible, then $A^{-1}B$ is a unique solution. If A is invertible, then

$$A\left(A^{-1}B\right) = \left(AA^{-1}\right)B = IB = B$$

and hence, $A^{-1}B$ is a solution. Now suppose v is any solution, so $Av = B$. Then

$$v = Iv = \left(A^{-1}A\right)v = A^{-1}(Av) = A^{-1}B$$

Thus, the solution $A^{-1}B$ is unique.