

Steps in an Informal Mathematical Proof

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Classification of Steps in an Informal Mathematical Proof

Informal proofs often do not follow a rigid logical format but instead unfold through a sequence of rhetorical and epistemic moves. This move is encapsulated in the term explanatory chain. Each step follows from the previous one as a mathematical result. A good proof does not include an explanatory jump. Below is a classification of common steps that appear in informal proofs. The goal of creating graphs is to represent the flow of informal reasoning.

1. Framing the Problem (Orientation)

- Rephrasing or interpreting the theorem informally.
- Introducing notation and assumptions.
- Describing intuitions or visualizing the problem.

2. Assembling Tools

- Citing known results, definitions, or heuristics.
- Stating useful lemmas or propositions.
- Drawing analogies with familiar problems.

3. Identifying the Key Insight or Strategy

- Highlighting the central idea or trick.
- Suggesting a change of perspective or transformation.
- Describing the planned method (e.g., contradiction, induction).

4. Developing the Argument

- Proceeding step-by-step from assumptions to conclusion.
- Using logical implications, definitions, and intermediate results.
- Including intuitive commentary or explanatory remarks.

5. Intermediate Constructions or Contradictions

- Constructing objects (e.g., numbers, sets) with desired properties.
- Dividing into cases, if necessary.
- Deriving contradictions if proof by contradiction is employed.

6. Classification of the Steps

- A major mathematical fact is called a theorem.
- A minor mathematical fact is called a lemma.
- An easy conclusion of a theorem is a corollary.
- A true mathematical proposition is called a fact.

7. Concluding the Proof

- Summarizing how the result was obtained.
- Reiterating the initial goal and confirming its achievement.
- Optionally reflecting on the method or result.

8. Optional: Explanatory or Pedagogical Enhancements

- Motivational asides and remarks on intuition.
- Use of diagrams or informal sketches.
- Analogical reasoning or connections to related problems.