1 Title

Addition Via Recursion

2 Abstract

Recursion is an effective way to add numbers. Using recursion, a more complicated addition is reduced to simpler additions. Hence, through a recursive procedure, it turns into several steps of addition by 1, where $a+1$ denotes the next number.

3 Procedure

For non-negative integers a and b:

$$a+b=\begin{cases} a & \text{if } b=0 \quad \text{(Base Case)} \\ (a+(b-1))+1 & \text{if } b>0 \quad \text{(Recursive Case)} \end{cases}$$

The recursive case systematically reduces any addition problem to successive simpler cases through these steps:

1. **Initial Problem**: $a+b$ where $b>0$
2. **Recursive Decomposition**:

   $$a+b=(a+\underbrace{(b-1)}_{\text{Simpler term}})+1$$

   This creates:

   - A simpler subproblem: $a+(b-1)$
   - A pending operation: $+1$
3. **Iterative Reduction**: Repeat until reaching base case:

   $$\begin{aligned} & a+b \\ & \Downarrow \\ & (a+(b-1))+1 \\ & \Downarrow \\ & ((a+(b-2))+1)+1 \\ & \Downarrow \\ & \quad \vdots \\ & \Downarrow \\ & (\cdots((a+0)+1)+\cdots+1) \end{aligned}$$

4. **Base Case Resolution**: When $b - n = 0$:

$$\underbrace{(\cdots((a+0)}_{\text{Base case}}\underbrace{+1)+\cdots+1)}_{b\text{ times}}$$

5. **Result Construction**:

$$a + \underbrace{1 + 1 + \cdots + 1}_{b\text{ times}} = a + b$$

4 Examples

4.1 Example 1

4.1.1 Question

Calculate this $3 + 2$:

4.1.2 Answer

$$\begin{aligned}
3 + 2 &= (3 + 1) + 1 && \text{(First decomposition)} \\
&= ((3 + 0) + 1) + 1 && \text{(Second decomposition)} \\
&= (3 + 1) + 1 && \text{(Base case applied)} \\
&= 4 + 1 && \text{(First increment)} \\
&= 5 && \text{(Final result)}
\end{aligned}$$

4.2 Example 2

4.2.1 Question

Calculate the result of $a + b$:

4.2.2 Answer

$$\begin{aligned}
a + b &= (a + (b - 1)) + 1 \\
&= ((a + (b - 2)) + 1) + 1 \\
&\vdots \\
&= (\cdots((a + 0) + 1) + \cdots + 1) \\
&= a + \underbrace{1 + 1 + \cdots + 1}_{b\text{ times}} \\
&= a + b
\end{aligned}$$