1 Theorem

Let a, b be two odd integers. The form of an odd integer is the following:

a is odd if and only if $a = 2k + 1$ for some $k \in \mathbb{Z}$.

We need to show that, $a + b$ is an even integer, if a and b are odd integers.

2 Proof

Let a and b be two odd integers. Then, with no loss of generality, $a = 2m + 1$ and $b = 2n + 1$, for m, n in $\mathbb{Z}$. Then, the sum is the following:

$$2m + 2n + 2 = 2(m + n + 1).$$

However, $2(m + n + 1)$ is a form of an even integer because the integers are closed under addition, or

$$m, n \in \mathbb{Z} \rightarrow m + n \in \mathbb{Z}.$$