Proof: The Sum of Two Even Integers is Even

Problem

Prove that the sum of two even integers is always even.

Solution 1: Direct Proof Using Definition

Let a and b be two even integers. By definition, an even integer can be written as:

$$a = 2m, \quad b = 2n, \quad \text{where } m, n \text{ are integers.} \tag{1}$$

The sum of a and b is:

$$a + b = 2m + 2n. \tag{2}$$

Factoring out 2:

$$a + b = 2(m + n). \tag{3}$$

Since $m + n$ is an integer, we conclude that $a + b$ is even.

Solution 2: Proof by Contradiction

Assume, for contradiction, that the sum of two even integers is odd. That is, assume:

$$a + b = 2k + 1, \quad \text{for some integer } k. \tag{4}$$

Since a and b are even, they can be written as:

$$a = 2m, \quad b = 2n. \tag{5}$$

Adding these values:

$$a + b = 2m + 2n = 2(m + n). \tag{6}$$

Since $2(m + n)$ is even, this contradicts our assumption that $a + b$ is odd. Therefore, the sum of two even numbers must be even.

Solution 3: Proof Using Parity

An integer is even if it has remainder 0 when divided by 2. That is,

$$a \equiv 0 \pmod{2}, \quad b \equiv 0 \pmod{2}. \tag{7}$$

Adding both congruences:

$$a + b \equiv 0 + 0 \equiv 0 \pmod{2}. \tag{8}$$

Since $a + b \equiv 0 \pmod{2}$, it follows that $a + b$ is even.