1 Question:

Prove that the product of any two odd integers is always odd.

2 Answer:

Assume, for contradiction, that the product of two odd integers is even. That is, assume:

$$a \cdot b = 2k, \quad \text{for some integer } k. \tag{1}$$

Since a and b are odd, we write:

$$a = 2m + 1, \quad b = 2n + 1. \tag{2}$$

Multiplying both values:

$$a \cdot b = (2m + 1)(2n + 1) = 4mn + 2m + 2n + 1 = 2(2mn + m + n) + 1. \tag{3}$$

Since $2(2mn + m + n) + 1$ is odd, this contradicts our assumption that $a \cdot b$ is even. Therefore, $a \cdot b$ must be odd.