LIF neuron model and its variants

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1. Introduction

The primary goal of this project is to conduct a comprehensive analysis by manipulating key parameters of the LIF model and its variants (ELIF & AELIF). Through this, we aim to gain insights into the impact of parameters on spiking dynamics, firing patters, and the overall network behavior.

2. Simple LIF Model

Parameter	Value	Description
u_{rest}	-80	resting potential
$threshold$	-55	threshold potential
u_{reset}	-90	reset potential
R	1	membrane resistance
τ	10	membrane time constant
$Time\ Resolution$	0.1	dt in euler's method
$Iterations$	600	number of iterations

Table 1. *Base parameters for the LIF model.*

The provided table displays the foundational parameters for the LIF model. In this section, all graphs pertaining to this model utilize these parameters. The sole manipulation involves adjusting the input current to observe its effects on the neuron's firing pattern. The specific values for the input current can be verified within the corresponding plots.

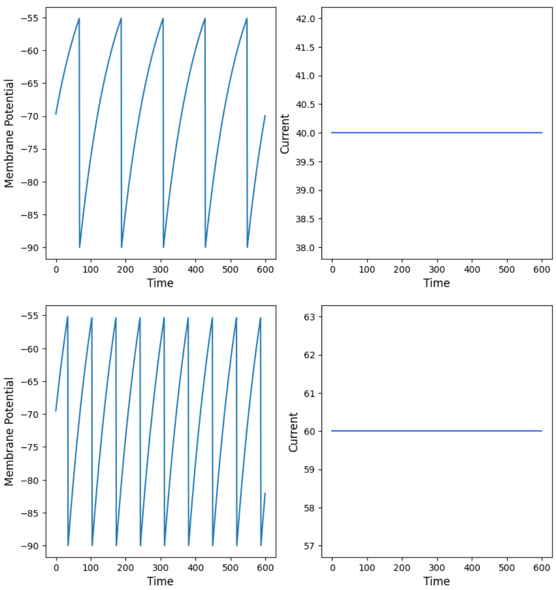

Figure 1. *LIF model with constant current.*

Raising the current causes the neuron to fire more frequently. If the current is insufficient, the neuron remains inactive. However, by increasing either the duration or the input current, the neuron reaches its threshold and transitions into a repetitive firing state.

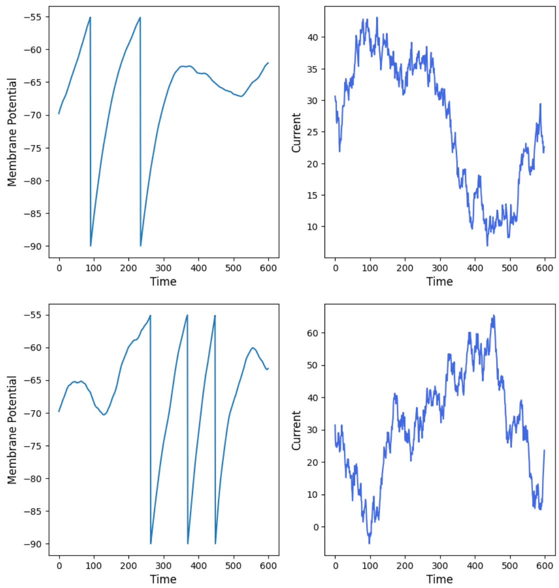

Figure 2. *LIF model with noisy input current. Negative input current can be interpreted as some inhibitory neuron's activity.*

The simulation of noisy input current is represented by the equation:
$I+ = Normal(\mu = 0, \sigma = 1.5)$. This type of input current induces a more irregular firing pattern in the neuron when the noise amplitude is elevated.

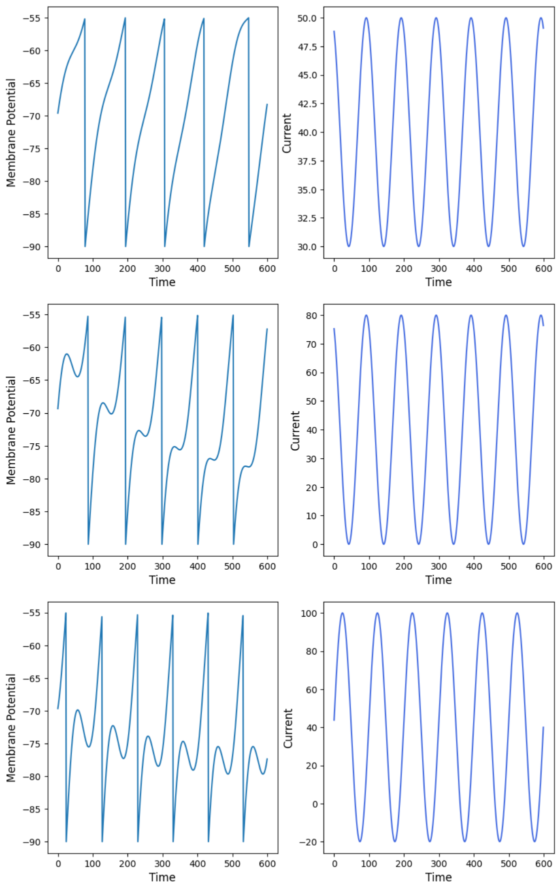

Figure 3. *LIF model with sinusoidal input current with different parameters. Negative input current can be interpreted as some inhibitory neuron's activity.*

The sinusoidal input current is modeled by the equation: $I += A \times sin(2\pi f i \times dt + \phi)$ where i represents the iteration number and ϕ is the phase shift. The neuron's behavior is clearly evident in the accompanying plots.

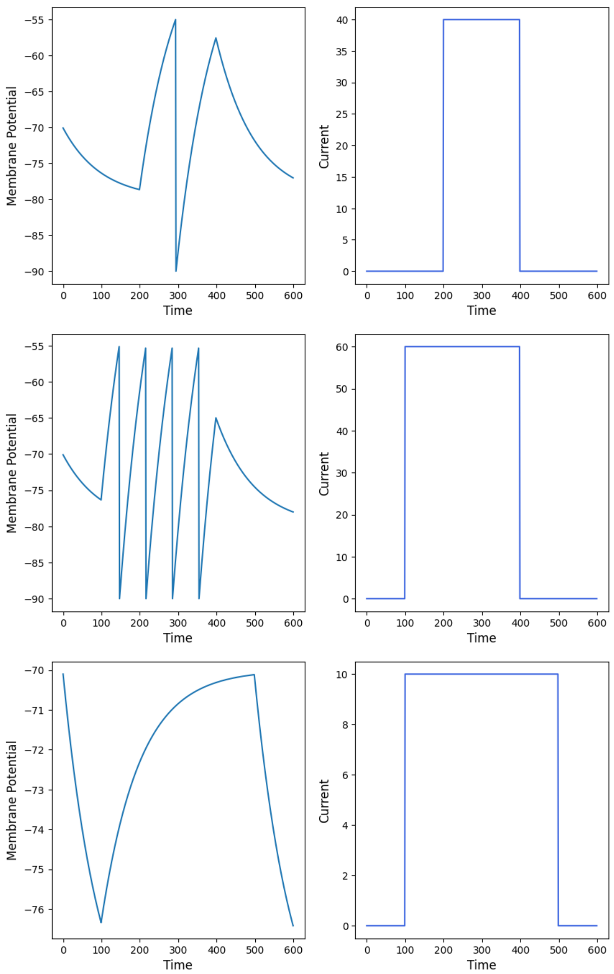

Figure 4. *LIF model with step input current.*

Resetting the current to its initial value (0) causes the potential to converge towards rest potential.

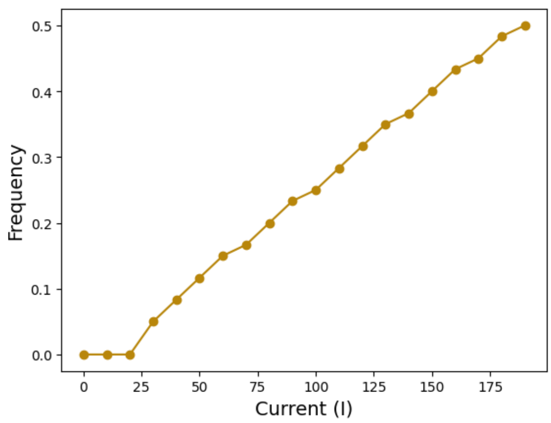

Figure 5. *FI curve for LIF model with constant input current I for different values of I.*

Observing the data, it's apparent that the firing rate experiences a linear increase with the input current once it surpasses a certain threshold, indicated by a zero value for F. In this context, for $I = 0, 10, 20$, the firing rate remains zero. Further insights into this behavior are provided in the explanation of Figure 1. The simulation encompasses current values ranging from $I = 0$ to $I = 190$.

For each step of a simulation, we added some noise to the

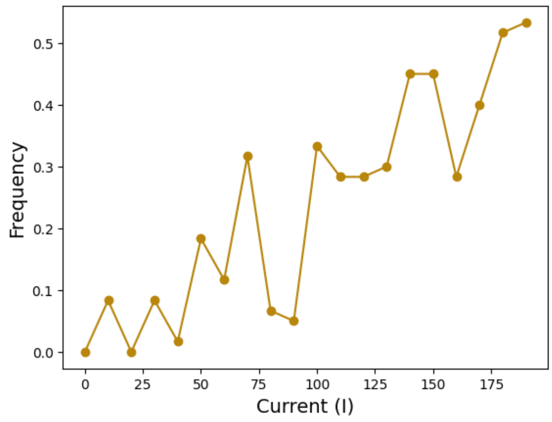

Figure 6. *FI curve for LIF model with noisy input current I. (strong noise!)*

current using a normal distribution with mean 0 and standard deviation 1.5 . The curve looks somewhat random, but increasing since the initial value of the current is increasing for simulations.

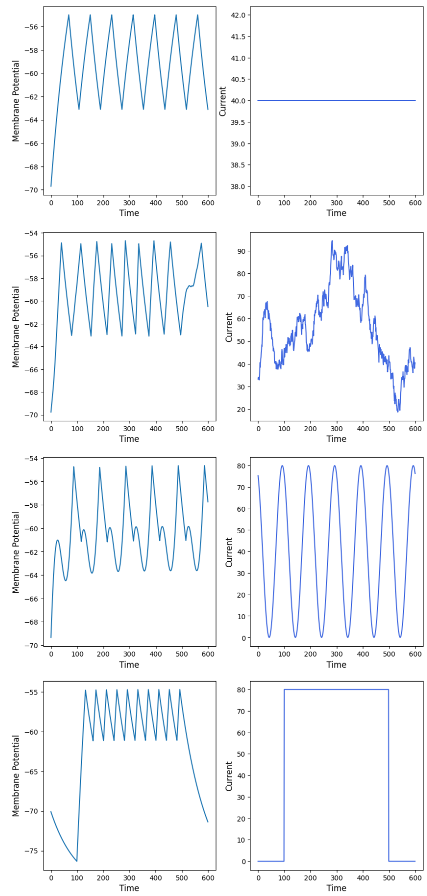

Figure 7. *Simulating refractory period by blocking the input current for a period of time after the neuron fires.*

These plots illustrate the neuron's response when the input current is blocked following a firing event. Notably, the potential does not automatically reset after a spike; however, it can be reset while simultaneously blocking the input current.

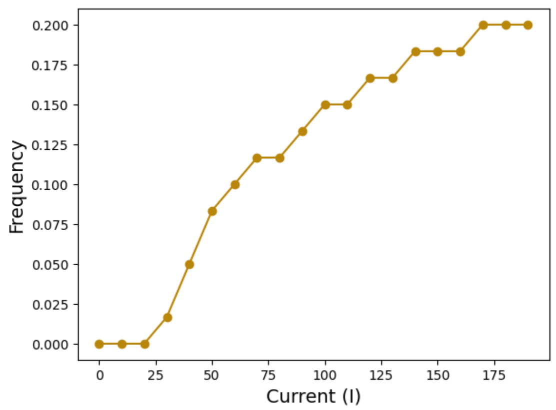

Figure 8. *FI curve for LIF model with refractory (by blocking the input current). The simulation is done for the current values of* $I = 0, 10, 20, ..., 190$.

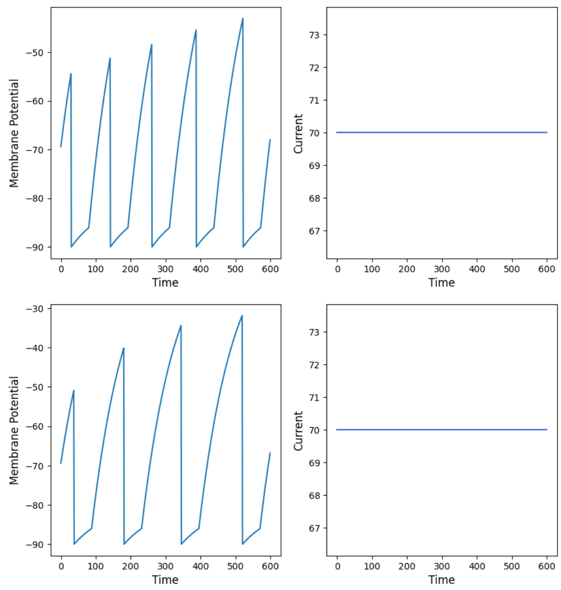

Figure 9. *Simulating refractory period by adapting the threshold potential after the neuron fires.*

The differential equation for the adaptive threshold is
$\tau_{adapt}\frac{d}{dt}\vartheta(t) = -[\vartheta(t) - \vartheta_0] + \theta \sum_f \delta(t - t^{(f)})$
where τ_{adapt} is the time constant of adaptation. After each spike the threshold ϑ is increased by an amount θ, while during a quiescent period the threshold approaches its stationary value ϑ_0.
As expected, it takes more time for the neuron to reach ϑ_0 when τ_{adapt} is larger.

3. ELIF Model

Parameter	Value	Description
u_{rest}	-80	resting potential
$threshold$	-55	threshold potential
u_{reset}	-90	reset potential
R	1	membrane resistance
τ	10	membrane time constant
Δ_T	1	sharpness of the exponential
$Time\ Resolution$	0.1	dt in euler's method
$Iterations$	600	number of iterations

Table 2. *Base parameters for the ELIF model.*

The provided table outlines the fundamental parameters for the ELIF model. All graphs in this section pertaining to this model utilize these parameters. The sole manipulation involves adjusting the input current to observe its effects on the neuron's firing pattern, with specific input current values available in the corresponding plots.

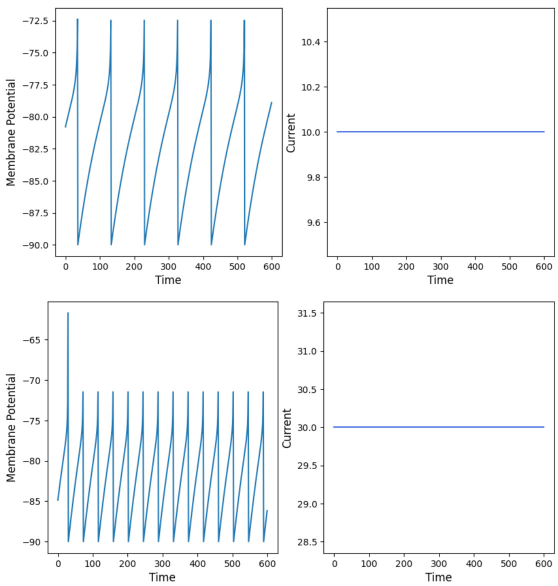

Figure 10. *ELIF model with constant current.*

As anticipated, elevating the current results in the neuron firing more frequently. However, the threshold potential in the plots does not appear to stabilize at the set value of -55. This deviation is attributed to the model's implementation and the sharpness of the exponential behavior, evident from the plots.

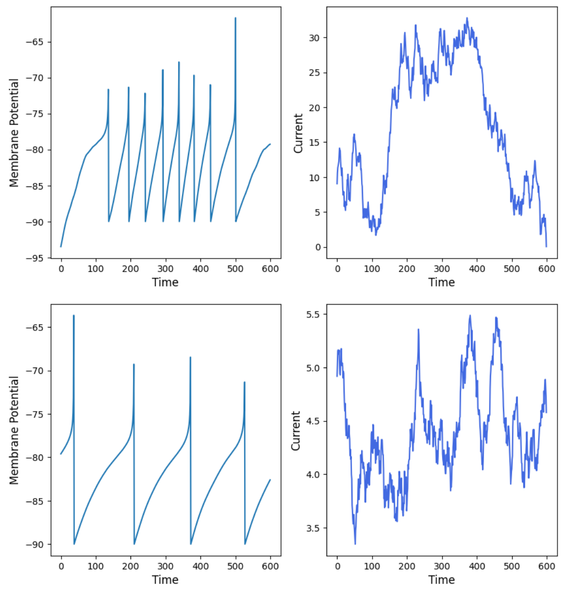

Figure 11. *ELIF model with noisy input current.*

The equation simulating the noisy input current is expressed as:
$I+ = Normal(\mu = 0, \sigma = 1.5)$. This kind of input current doesn't seem to make the Contrary to the LIF model, this type of input current doesn't appear to induce a more irregular firing pattern in the neuron. The rationale behind this lies in the nature of the exponential term within the model's differential equation.

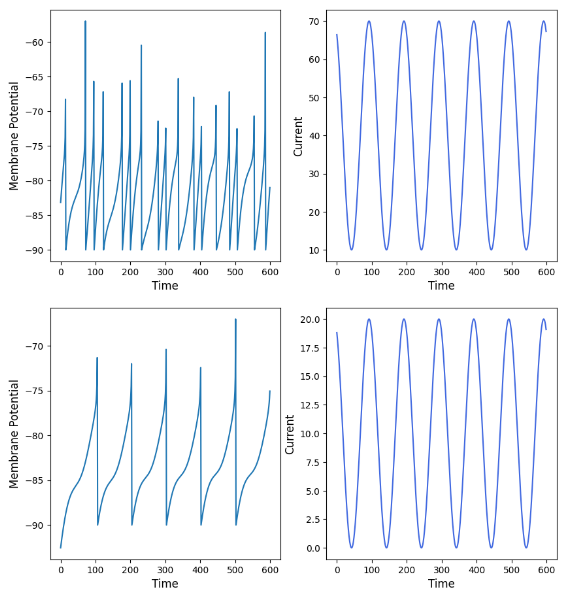

Figure 12. *ELIF model with sinusoidal input current with different parameters.*

The equation simulating the sinusoidal input current is:
$I += A \times sin(2\pi fi \times dt + \phi)$ This mirrors the approach used in the preceding section. The neuron's behavior is evident from the accompanying plots.

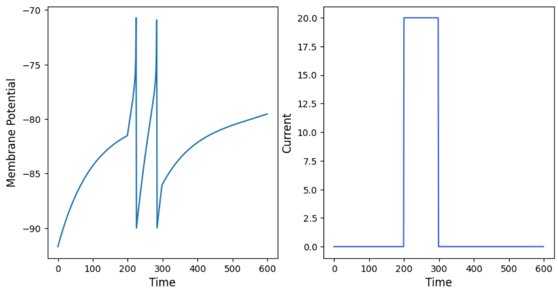

Figure 13. *ELIF model with step input current.*

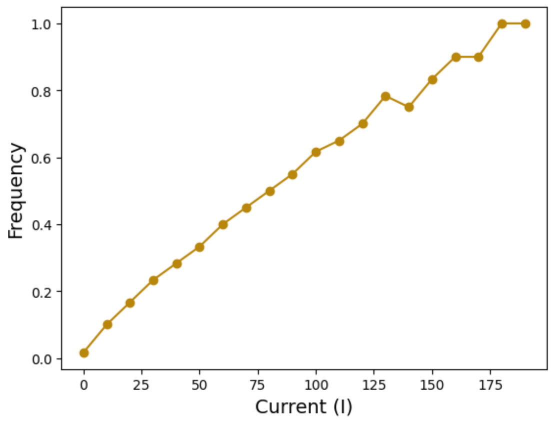

Figure 14. *FI curve for ELIF model with constant input current I for different values of I.*

As observed, the firing rate demonstrates a linear increase with the input current, akin to the LIF model. The simulation encompasses current values ranging from $I = 0$ to $I = 190$, mirroring the approach used in the LIF model.

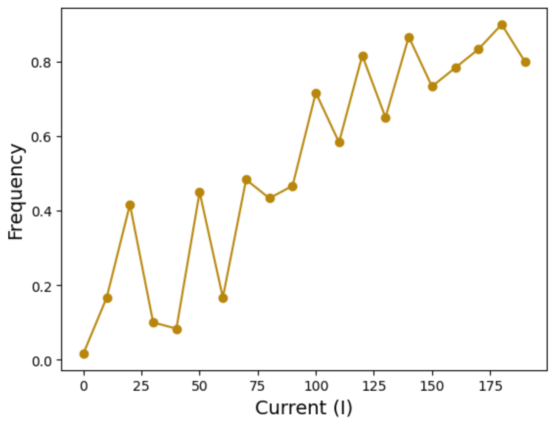

Figure 15. *FI curve for ELIF model with noisy input current I.*

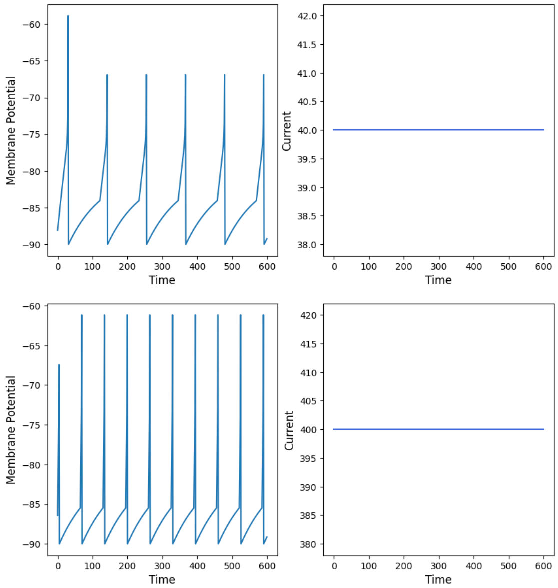

Figure 16. *Simulating refractory period by blocking the input current for a period of time after the neuron fires.*

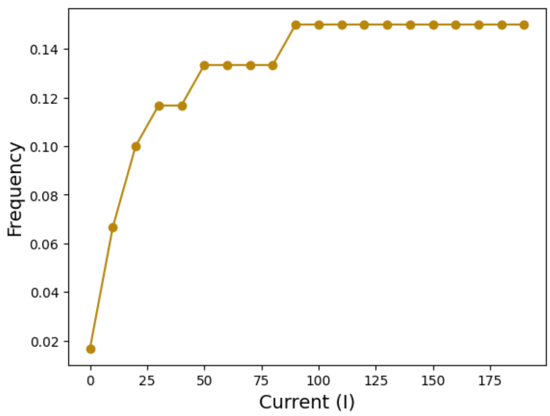

Figure 17. *FI curve for ELIF model with refractory (by blocking the input current). The simulation is done for the current values of $I = 0, 10, 20, ..., 190$.*

4. AELIF Model

Parameter	Value	Description
u_{rest}	-80	resting potential
threshold	-55	threshold potential
u_{reset}	-90	reset potential
R	1	membrane resistance
τ	10	membrane time constant
Δ_T	1	sharpness of the exponential
θ_{rh}	-75	rheobase threshold
a	1	
b	100	
τ_w	100	time constant of adaptation
Time Resolution	0.1	dt in euler's method
Iterations	600	number of iterations

Table 3. *Base parameters for the AELIF model.*

The provided table outlines the fundamental parameters for the AELIF model. All graphs in this section pertaining to this model utilize these parameters. The sole manipulation involves adjusting the input current to observe its effects on the neuron's firing pattern, with specific input current values available in the corresponding plots.

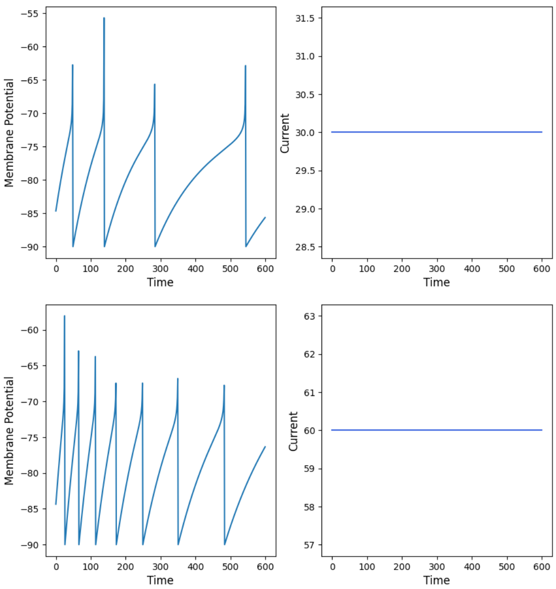

Figure 18. *AELIF model with constant current.*

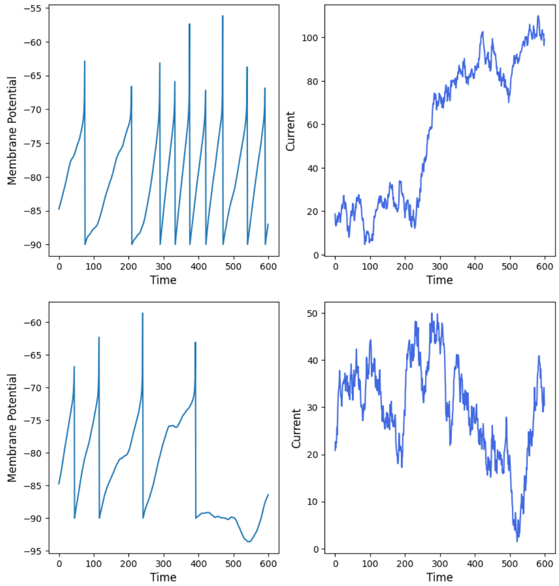

Figure 19. *AELIF model with noisy input current.*

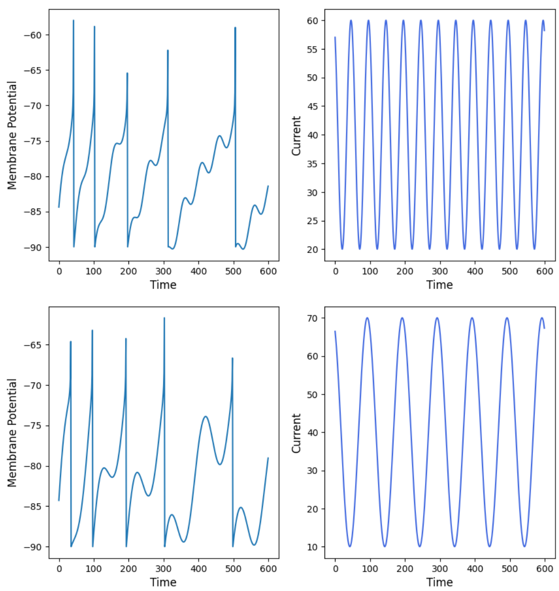

Figure 20. *AELIF model with sinusoidal input current with different parameters.*

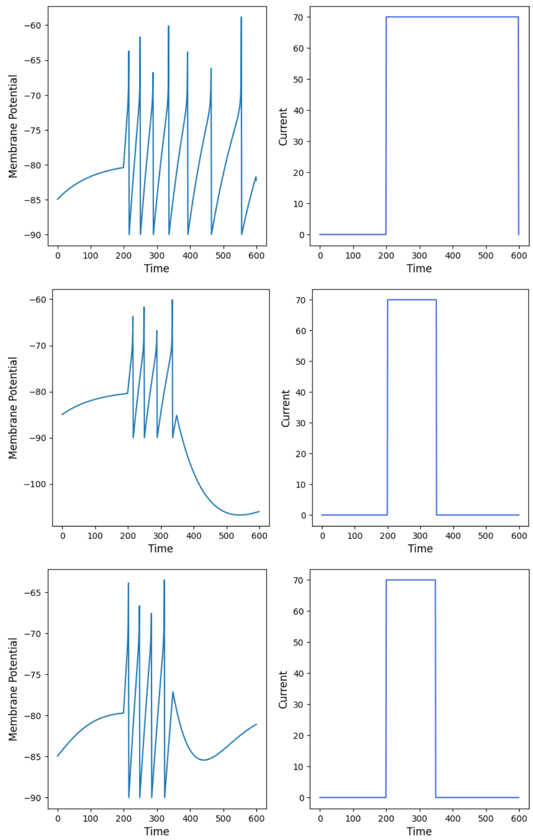

Figure 21. *AELIF model with step input current.*

The second plot appears peculiar, and this anomaly can be attributed to the substantial value of τ_w. In the third plot, we adjusted the value of τ_w to 10, resulting in a different representation.

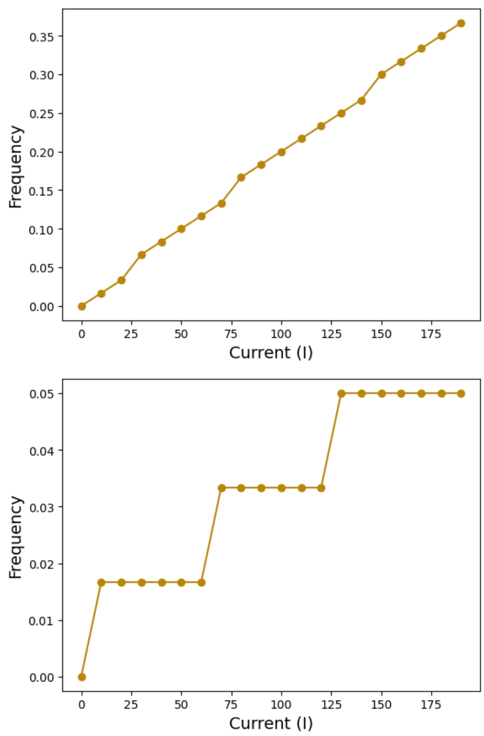

Figure 22. *FI curve for AELIF model with constant input current I for different values of I.*

Note that the AELIF model's firing rate is not periodic because of the adaptivity. In order to define the gain function (FI curve), we count the number of spikes from $t = 0$ to the end of the simulation and divide it by the total time.
The firing rate exhibits a linear increase with the input current, similar to the patterns observed in the LIF and ELIF models. However, this trend is not universal. Notably, adjusting the value of b to an extreme value like 1000(!) produces a curve resembling the one depicted in the second plot.

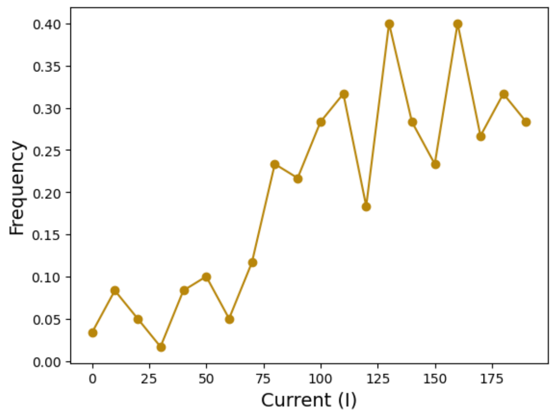

Figure 23. *FI curve for AELIF model with noisy input current I.*

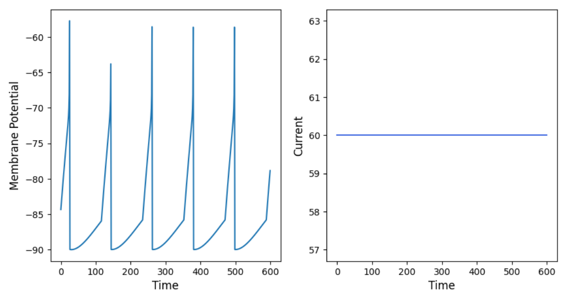

Figure 24. *Simulating refractory period by blocking the input current for a period of time after the neuron fires.*

5. Manipulating Parameters (LIF)

In this section, we'll adjust the parameters of the LIF model to examine their impact on the neuron's firing pattern. The targeted parameters for manipulation include the membrane resistance R, the membrane time constant τ, and refractoriness parameters. (We use only constant input current for convenient in this section.)

Take a look at the table in the second section (LIF Model). We aim to make modifications to this table.

Parameter	Value
u_{rest}	-80
$threshold$	-55
u_{reset}	-90
R	1
τ	10
$Time\ Resolution$	0.1
$Iterations$	600

Table 4

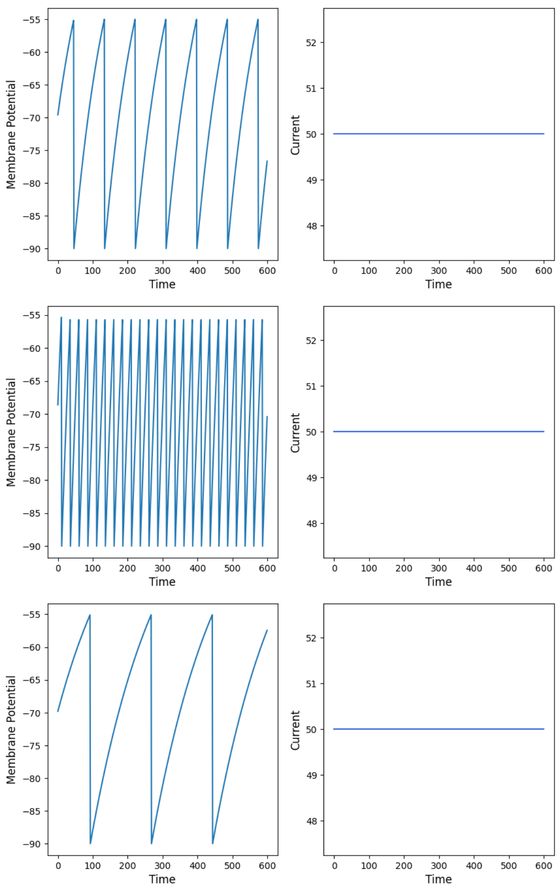

Figure 25. *$R = 3$ in the second plot. In the third one $tau = 20$.*

In the second plot, by increasing the resistance R, the term RI becomes more significant. As a result, the voltage across the membrane increases more rapidly in response to the fixed input current. This means that the neuron reaches the firing threshold more quickly. Similarly in the third plot where we increased τ to 20, the neuron reaches the firing threshold more slowly.

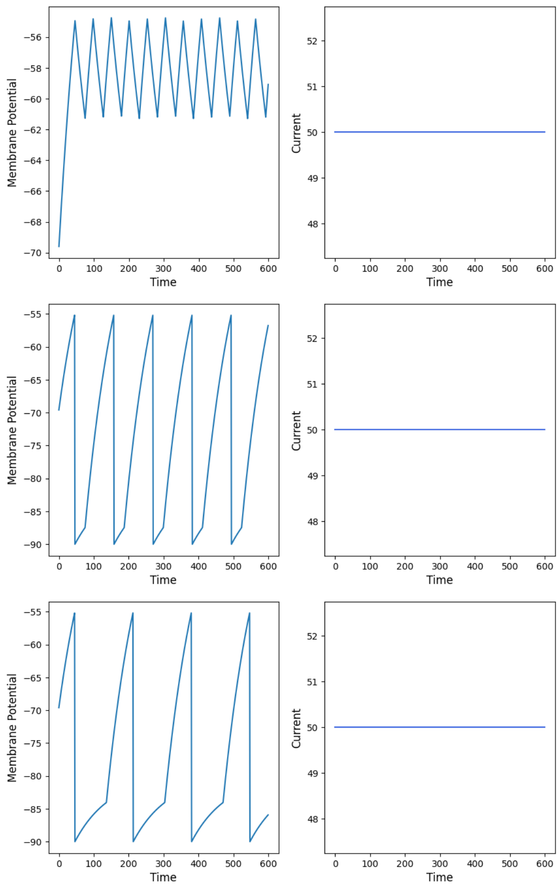

Figure 26. *1st plot: $T = 30$ iterations.*
2nd plost: $T = 30$ iterations.
3rd plot: $T = 90$ iterations.

As observed, a refractory period is added to the model. The main difference between the first and second plots is that we make the neuron's potential go back to u_{reset} after it fires. When we do this and stop the electric flow, the neuron goes back to its resting state, just like we expected. If the refractory time is too large, the membrane potential would be fixed at the rest potential after it reaches there. And the reason obviously is that the input current is blocked for a long time.

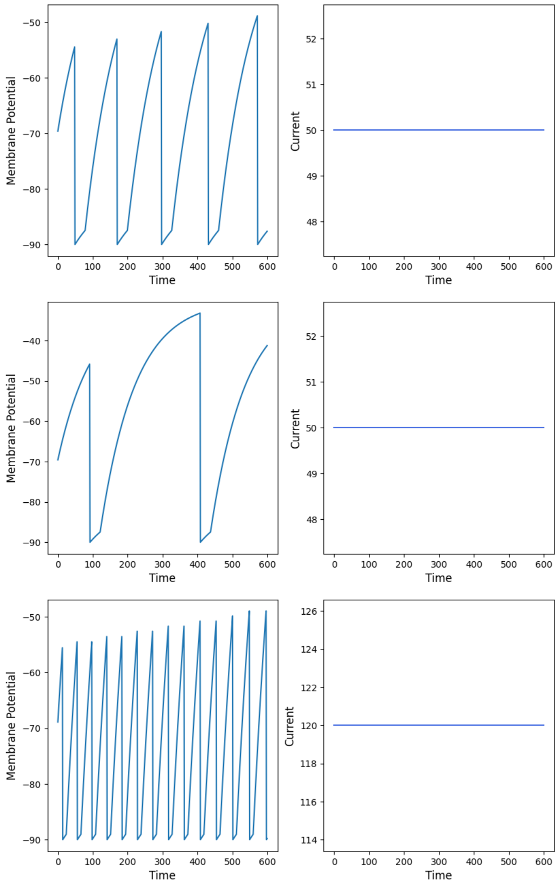

Figure 27. *1st plot: $\theta = 10$, θ_0 = -30, τ_a = 200.*
2nd plost: $\theta = 10$, θ_0 = -30, τ_a = 20.
3rd plot: $\theta = 10$, θ_0 = -30, τ_a = 200, I = 120.

In this figure, the neuron has an adaptiveness behavior. The threshold potential is increased by an amount θ after each spike, until it reaches θ_0. The role of each parameter in the equation provided in the first section for threshold adaptiveness is clear.

6. Manipulating Parameters (ELIF)

In this section, we'll adjust the parameters of the ELIF model to examine their impact on the neuron's firing pattern. The targeted parameters for manipulation include the membrane time constant τ, the sharpness of the exponential Δ_T, and the refractoriness parameters. (We use only constant input current for convenient in this section.)

We manipulate the mentioned parameters using the base parameters in the third section.

Parameter	Value
u_{rest}	-80
$threshold$	-55
u_{reset}	-90
R	1
τ	10
Δ_T	1
$Time\ Resolution$	0.1
$Iterations$	600

Table 5

We ommit to add any more description since the behavior of the models are clear to us now. Thus we only provide the plots in this section and the next one with the value of parameters in the caption of each figure.

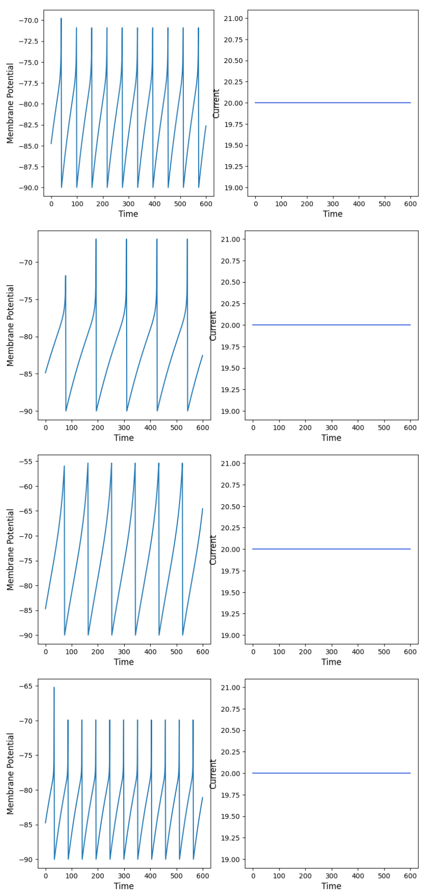

Figure 28. *2nd plot: $\tau = 20$.*
3rd plot: $\Delta_T = 10$.
4th plot: $\Delta_T = 0.1$.

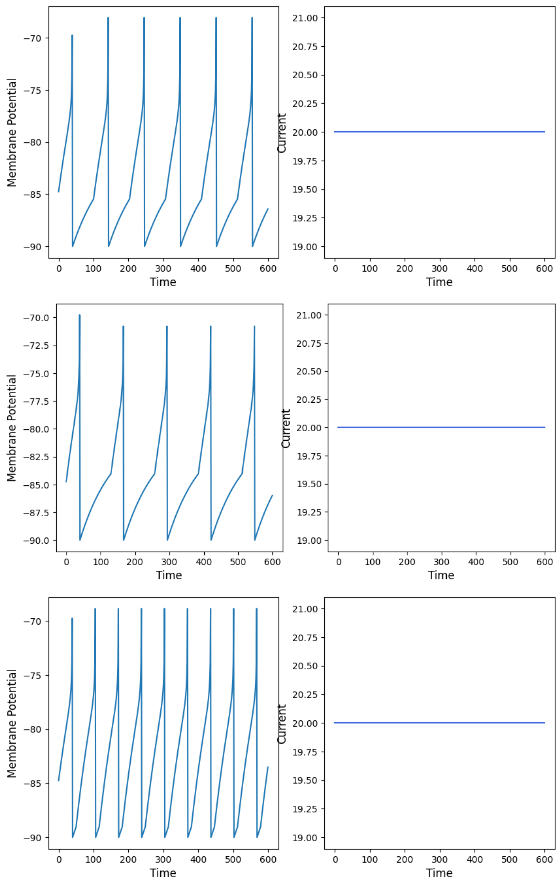

Figure 29. *Blocking the input current for different values of T*

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7. Manipulating Parameters (AELIF)

In this section, we'll adjust the parameters of the AELIF model to examine their impact on the neuron's firing pattern. The targeted parameters for manipulation include the membrane time constant τ, the sharpness of the exponential Δ_T, time constant of adaptation τ_w, and a and b parameters which are used in the adaptation term. (We use only constant input current for convenient in this section.)

We manipulate the mentioned parameters using the base parameters in the fourth section.

Parameter	Value
u_{rest}	-80
$threshold$	-55
u_{reset}	-90
R	1
τ	10
Δ_T	1
θ_{rh}	-75
a	1
b	100
τ_w	100
$Time\ Resolution$	0.1
$Iterations$	600

Table 6

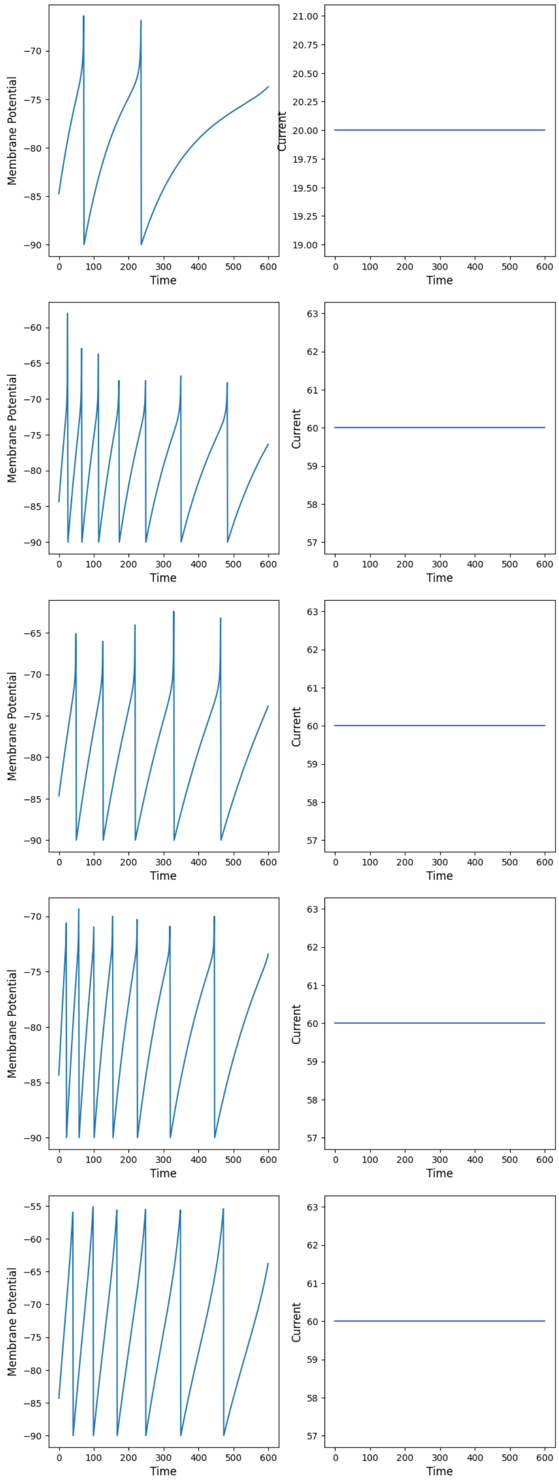


Figure 30. *3rd plot: $\tau = 20$.*
4th plot: $\Delta_T = 0.5$
5th plot: $\Delta_T = 10$

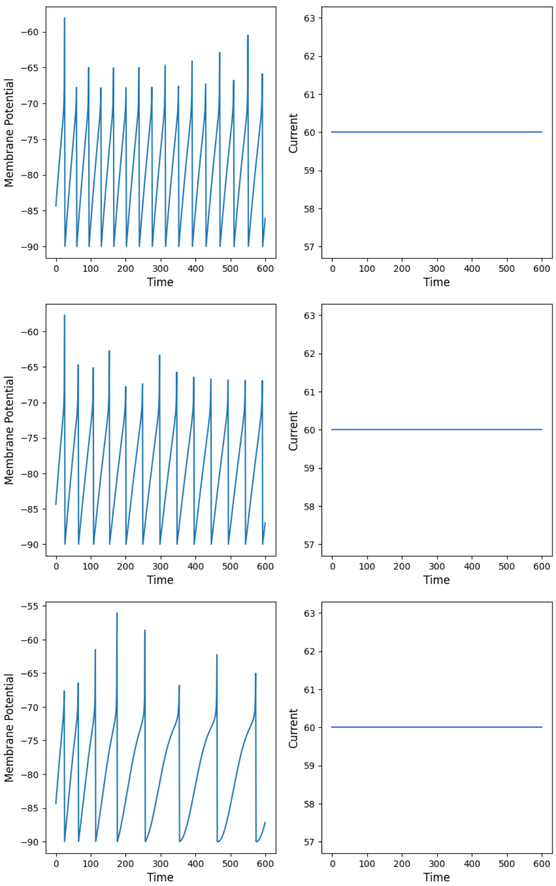
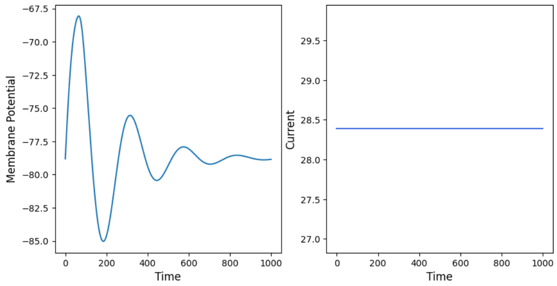


Figure 31. *1st plot: $b = 10$*
2nd plot: $\tau_w = 10$
3rd plot: $a = 100$

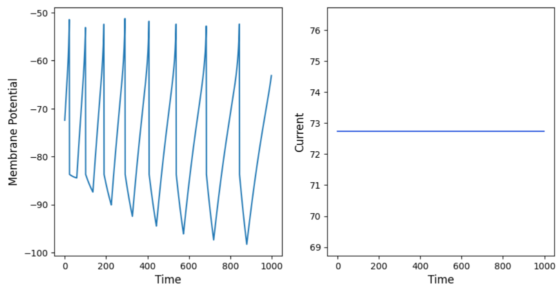
8. Additional samples on the AELIF model

Since the AELIF model is the most complex and accurate one with right parameters, we provide some more samples on this model to have a better understanding of its behavior.



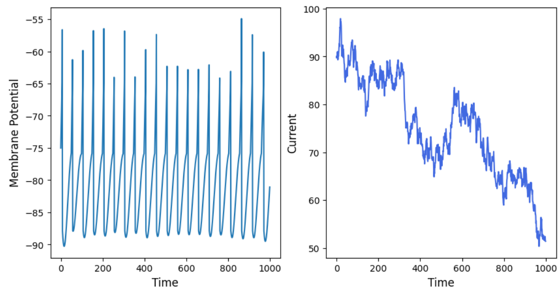
Parameter	Value
u_{rest}	-79
$threshold$	-54
u_{reset}	-88
R	1.2
τ	10
Δ_T	1
θ_{rh}	-70
a	96
b	133
τ_w	185
T_{block}	2.3
$Time\ Resolution$	0.1
$Iterations$	1000

Figure 32



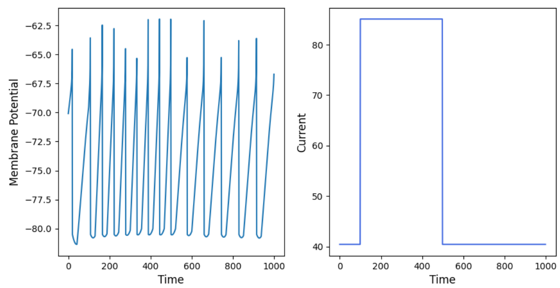
Parameter	Value
u_{rest}	-72
$threshold$	-50
u_{reset}	-83
R	1.34
τ	13.7
Δ_T	4
θ_{rh}	-70
a	6.3
b	114
τ_w	187
T_{block}	3.5
$Time\ Resolution$	0.1
$Iterations$	1000

Figure 33



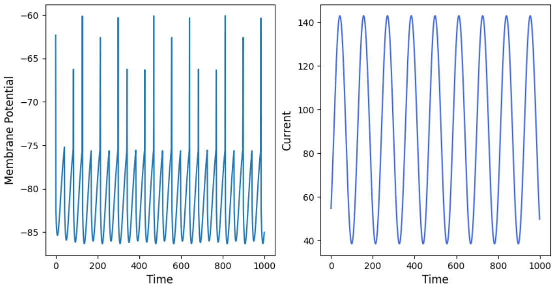
Parameter	Value
u_{rest}	-79
$threshold$	-50
u_{reset}	-87
R	0.5
τ	2.5
Δ_T	0.99
θ_{rh}	-70
a	186
b	183
τ_w	69
T_{block}	4.2
$Time\ Resolution$	0.1
$Iterations$	1000

Figure 34



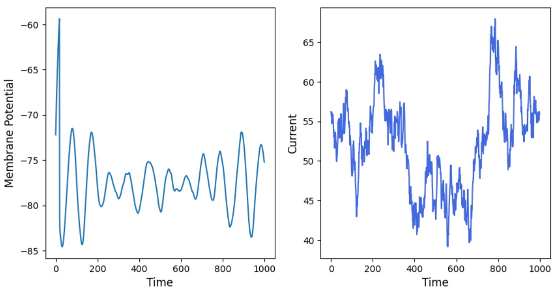
Parameter	Value
u_{rest}	-74
$threshold$	-59
u_{reset}	-80
R	0.9
τ	19.8
Δ_T	056
θ_{rh}	-70
a	167
b	104
τ_w	134
T_{block}	2.29
$Time\ Resolution$	0.1
$Iterations$	1000

Figure 35



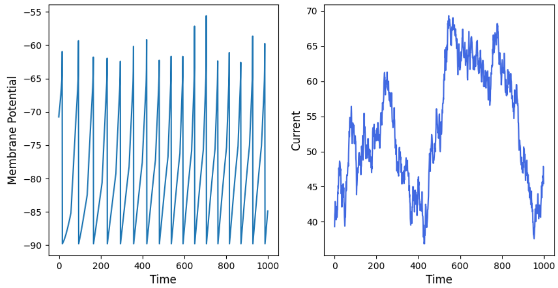
Parameter	Value
u_{rest}	-72
$threshold$	-58
u_{reset}	-87
R	1.3
τ	0.82
Δ_T	2.5
θ_{rh}	-70
a	61
b	122
τ_w	198
T_{block}	4.1
$Time\ Resolution$	0.1
$Iterations$	1000

Figure 36



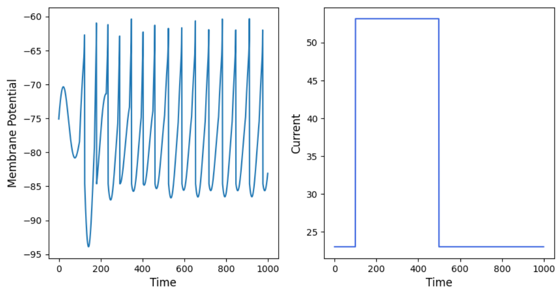
Parameter	Value
u_{rest}	-78
$threshold$	-58
u_{reset}	-81
R	1.7
τ	8.7
Δ_T	3.7
θ_{rh}	-70
a	154
b	188
τ_w	72
T_{block}	0.15
$Time\ Resolution$	0.1
$Iterations$	1000

Figure 37



Parameter	Value
u_{rest}	-76
$threshold$	-53
u_{reset}	-89
R	0.96
τ	10
Δ_T	0.99
θ_{rh}	-70
a	64
b	36
τ_w	196
T_{block}	4.1
$Time\ Resolution$	0.1
$Iterations$	1000

Figure 38



Parameter	Value
u_{rest}	-77
$threshold$	-59
u_{reset}	-84
R	1.1
τ	6
Δ_T	1.3
θ_{rh}	-70
a	106
b	169
τ_w	65
T_{block}	4.2
$Time\ Resolution$	0.1
$Iterations$	1000

Figure 39