Problem Set 0

Please write your solutions in the LATEX and Python templates provided. Aim for concise solutions; convoluted and obtuse descriptions might receive low marks, even when they are correct.

This assignment is meant to be an evaluation of your **individual** understanding coming into the course and should be completed **without collaboration** or outside help. You **may** ask for logistical help concerning LATEX formatting and/or code submission.

Problem 0-1. Let $A = \left\{ i + \binom{5}{i} \ \ i \in \mathbb{Z} \text{ and } 0 \leq i \leq 4 \right\}$ and $B = \{3i \mid i \in \{1, 2, 4, 5\}\}$.

Evaluate: **(a)** $A \cap B$ **(b)** $|A \cup B|$ **(c)** $|A - B|$

Problem 0-2. Let X be the random variable representing the number of heads seen after flipping a fair coin three times. Let Y be the random variable representing the outcome of rolling two fair six-sided dice and multiplying their values. Please compute the following expected values.

Evaluate: **(a)** $\mathrm{E}\,[X]$ **(b)** $\mathrm{E}\,[Y]$ **(c)** $\mathrm{E}\,[X + Y]$

Problem 0-3. Let $A = 600/6$ and $B = 60 \bmod 42$. Are these statements True or False?

Evaluate: **(a)** $A \equiv B \pmod 2$ **(b)** $A \equiv B \pmod 3$ **(c)** $A \equiv B \pmod 4$

Problem 0-4. Prove **by induction** that $\sum_{i=1}^{n} i^3 = \left[\frac{n(n+1)}{2} \right]^2$, for any integer $n \geq 1$.

Problem 0-5. Prove **by induction** that every connected undirected graph $G = (V, E)$ for which $|E| = |V| - 1$ is acyclic.

Problem 0-6. An **increasing subarray** of an integer array is any consecutive sequence of array integers whose values strictly increase. Write Python function `count_long_subarrays(A)` which accepts Python Tuple `A = `$(a_0, a_1, \ldots, a_{n-1})$ of $n > 0$ positive integers, and returns the number of longest increasing subarrays of `A`, i.e., the number of increasing subarrays with length at least as large as every other increasing subarray. For example, if `A = (1,3,4,2,7,5,6,9,8)`, your program should return `2` since the maximum length of any increasing subarray of `A` is three and there are two increasing subarrays with that length: specifically, subarrays `(1,3,4)` and `(5,6,9)`. You can download a code template containing some test cases from the website.