Report 1.0.1

Simulation of Humanoid using MATLAB Teach Toolbox

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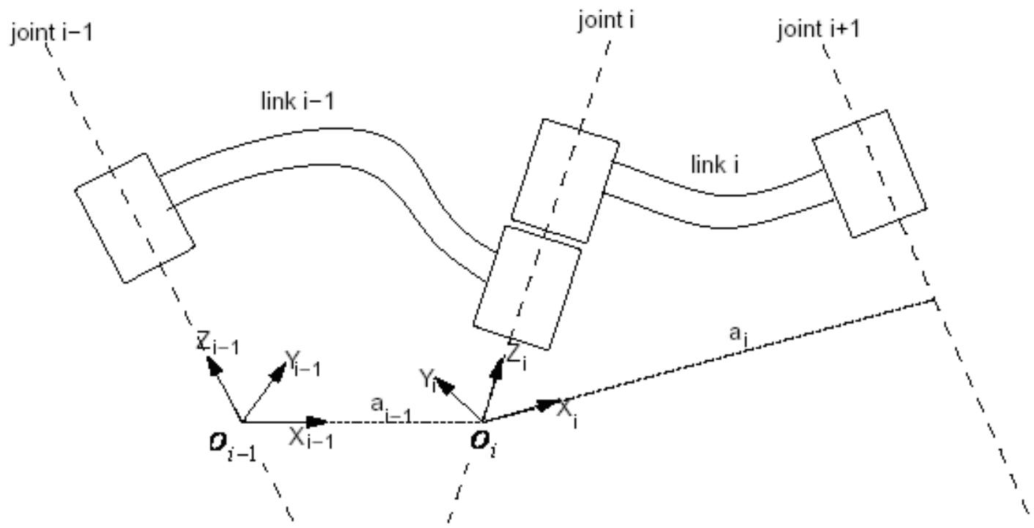
Contents

Introduction

DH Parameters

Denavit-Hartenberg (DH) comes in variety of modified and standard. The link and joint parameters in the modified convention as shown in figure below are as follows:

- Twist angle, α_{i-1} is the angle between z_{i-1} to z_i measured about x_{i-1}
- Link length, a_{i-1} is the distance from z_{i-1} to z_i measured along x_{i-1}
- Offset length, d_i is the distance from x_{i-1} to x_i measured along z_i
- Joint angle, θ_i is the angle between x_{i-1} to x_i measured about z_i



The D-H parameters are determined as per table below.

Link, i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				

The frame transformation ${}^{i-1}T_i$ describing the finite motion from link i -1 to link i may then be expressed as the following sequence of elementary transformations, starting from link (i – 1):

1. A rotation α_{i-1} about x_{i-1}.
2. A translation a_{i-1} along the x_{i-1} axis
3. A rotation θ_i about $\boldsymbol{z_i}$;
4. A translation d_i along the same axis $\boldsymbol{z_i}$;

The homogeneous transformation ${}^{i-1}T_i$ is represented as a product of four basic transformations as follows:

$${}^{i-1}T_i = R(x_{i-1,}\alpha_{i-1})T(x_{i-1,}a_{i-1})R(z_i,\theta_i)T(z_{i,}d_i)$$

$$=$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_{i-1} & -S\alpha_{i-1} & 0 \\ 0 & S\alpha_{i-1} & C\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -d_i S\alpha_{i-1} \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & d_i C\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

An alternative representation of ${}^{base}T_{end-effector}$ can be written as

$${}^{base}T_{end-effector} = {}^{b}T_e = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where r_{kj}'s represent the rotational elements of transformation matrix (k and j = 1, 2 and 3). p_x, p_y and p_z denote the elements of the position vector. For a six jointed manipulator, the position and orientation of the end-effector with respect to the base is given by

$${}^{0}T_6 = {}^{0}T_1(q_1)\,{}^{1}T_2(q_2)\,{}^{2}T_3(q_3)\,{}^{3}T_4(q_4)\,{}^{4}T_5(q_{51})\,{}^{5}T_6(q_6)$$

where q_i is the joint variable (revolute or prismatic joint) for joint i, (i = 1, 2, …6).

D-H parameter	Classical convention	Modified convention
Joint axis	z_{i-1} is for joint i	z_i is for joint i
Link length (a_i)	The distance from o_i to the intersection of the z_{i-1} and x_i axes along the x_i axis	The distance from z_i to z_{i+1} measured along x_i
Twist angle(α_i)	The angle from the z_{i-1} axis to the z_i axis about the x_i axis	The angle between z_i to z_{i+1} measured about x_i
Offset length (d_i)	The distance from the origin of the (i-1) frame to the intersection of the z_{i-1} axis with the x_i axis along the z_{i-1} axis	The distance from x_{i-1} to x_i measured along z_i
Joint angle (θ_i)	The angle between the x_{i-1} and x_i axes about the z_{i-1} axis	The angle between x_{i-1} to x_i measured about z_i

Algorithm for modified D-H Convention:

Step - 1: Assigning of base frame: the base frame {0} is assigned to link 0. The base frame {0} is arbitrary. For simplicity chose z_0 along z_1 axis when the first joint variable is zero. Using this convention, we have $a_0 = 0$ and $\alpha_0 = 0$. This also ensures that $d_1 = 0$ if the joint is revolute and $\theta_1 = 0$ if the joint is prismatic.

Step - 2: Identify links. The link frames are named by number according to the link to which they are attached (i.e. frame {i} is attached rigidly to link *i*). For example, the frame {2} is attached to link 2.

Identify joints. The z-axis of frame {i}, called z_i , is coincident with the joint axis *i*. The link *i* has two joint axes, z_i and z_{i+1} . The z_i axis is assigned to joint *i* and z_{i+1} is assigned to joint (*i*+1).

For *i* = 1,….., *n* perform steps 3 to 6.

Step – 3: Identify the common normal between z_i and z_{i+1} axes, or point of intersection. The origin of frame {i} is located where the common normal (a_i) meets the z_i axis.

Step – 4: Assign the z_i axis pointing along the ith joint axis.

Step – 5: Assign x_i axis pointing along the common normal (a_i) in the direction from z_i axis to z_{i+1} axis. In the case of a_i = 0, x_i is normal to the plane of z_i and z_{i+1} axes.

- As seen from figure 3.7, the joints may not necessarily be parallel or intersecting. As a result, the *z*-axes are skew lines. There is always one line mutually perpendicular to any two skew lines, called the common normal, which has the shortest distance between them. We always assign the *x*-axis of the local reference frames in the direction of the common normal. Thus, if a_i represents the common normal between z_i and z_{i+1}, the direction x_i is along a_i.
- If two joint *z*-axes are parallel, there are an infinite number of common normals present. We will pick the common normal that is collinear with the common normal of the previous joint.
- If the *z*-axes of two successive joints are intersecting, there is no common normal between them (or it has zero length). We will assign the *x*-axis along a line perpendicular to the plane formed by the two axes.

Step – 6: The y_i axis is selected to complete right-hand coordinate system.

Step – 7: Assigning of end-effector frame: If the joint n is revolute, the direction of x_n is chosen along the direction of x_{n-1} when $\theta_n = 0$ and the origin of frame {n} is chosen so that $d_n = 0$. If the joint n is prismatic, the direction of x_n is chosen so that $\theta_n = 0$ and the origin of frame {n} is chosen at the intersection of x_{n-1} with z_n so that $d_n = 0$.

Step – 8: The link parameters are determined as mentioned in table 4.

Link, i	a_{i-1}	a_{i-1}	d_i	θ_i
1				
2				

- a_{i-1} = the distance from z_{i-1} to z_i measured along x_{i-1}
- α_{i-1} = the angle between z_{i-1} to z_i measured about x_{i-1}
- d_i is the distance from x_{i-1} to x_i measured along z_i
- θ_i is the angle between x_{i-1} to x_i measured about z_i

Step - 9: Form ${}^0T_n = {}^0T_1\,{}^1T_2\,{}^2T_3 \ldots {}^{n-1}T_n$. This gives the position and orientation of the end-effector frame expressed in the base coordinates.

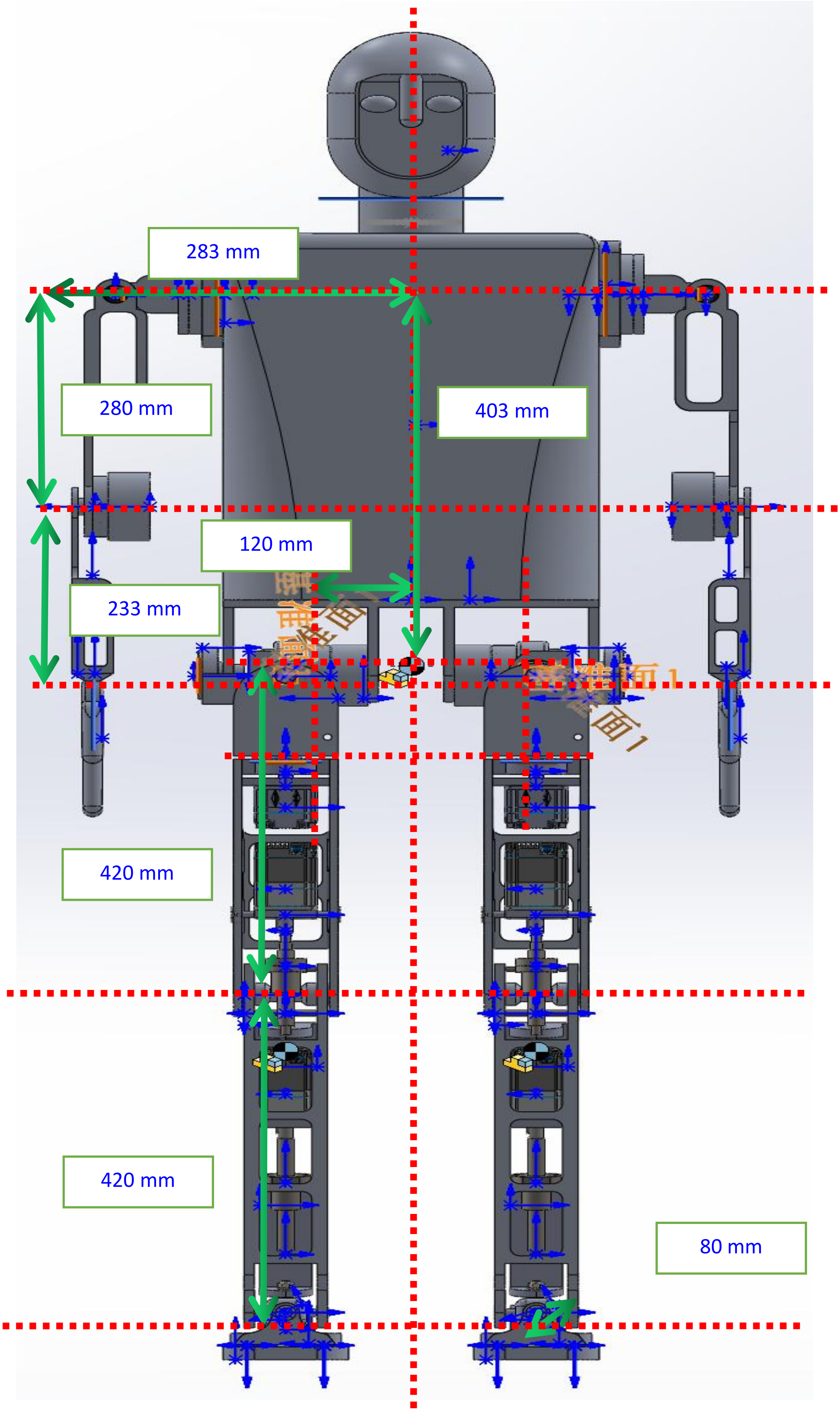
Two additional parameters are Sigma and Offset.

Link [THETA, D ,A ,ALPHA ,SIGMA ,OFFSET]

OFFSET is a constant displacement between the user joint angle vector and the true kinematic solution. SIGMA=0 for a revolute and 1 for a prismatic joint.

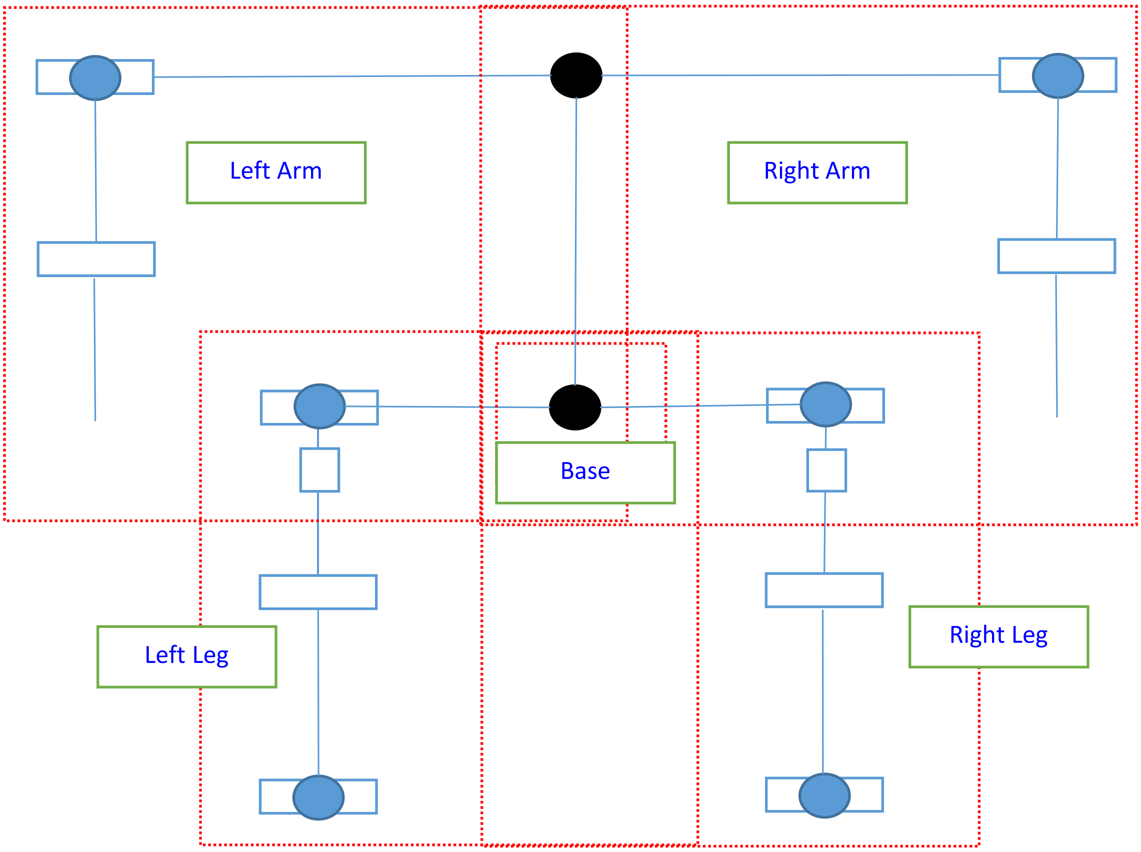
Structure

Solid Works Model



Free-Body Diagram

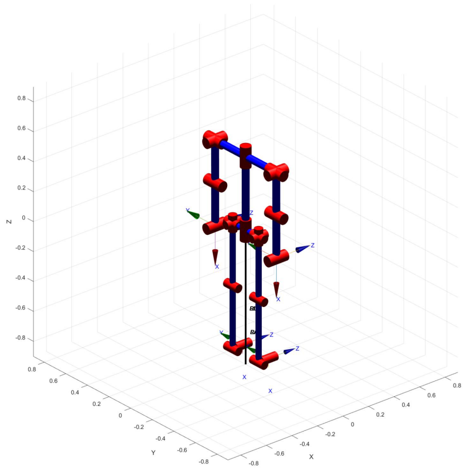
Complete free-body diagram of the humanoid looks like below.



However, we cannot get the DH parameters for the whole robot at once. Therefore, we split it to 5 parts: Left Arm, Right Arm, Base, Left Leg, and Right Leg. Then we can start from the base and apply the DH convention to get the DH parameters for each part.

Results

Final Model in MATLAB Teach Toolbox



Denavit-Hartenberg Parameters

Platform

platform:: 1 axis, R, modDH, slowRNE

j	theta	d	a	alpha	offset
1	q	0	0	0	0

Left Arm

LA:: 6 axis, RRRRRR, modDH, slowRNE

j	theta	d	a	alpha	offset
1	q1	0	0	0	0
2	q2	0.49077	0	0	0
3	q3	-0.283	0	1.57	-1.57
4	q4	0	0	1.57	0
5	q5	0	0.287	1.57	0
6	q6	0	0.287	1.57	0

Right Arm

RA:: 6 axis, RRRRRR, modDH, slowRNE

j	theta	d	a	alpha	offset
1	q1	0	0	0	0
2	q2	0.49077	0	0	0
3	q3	0.283	0	1.57	-1.57
4	q4	0	0	1.57	0
5	q5	0	0.287	1.57	0
6	q6	0	0.287	1.57	0

Left Leg

LL:: 8 axis, RRRRRRRR, modDH, slowRNE

j	theta	d	a	alpha	offset
1	q1	0	0	0	0
2	q2	-0.12	0	1.57	0
3	q3	0	0	1.57	-1.57
4	q4	0	0	-1.57	-1.57
5	q5	0	0.42	1.57	0
6	q6	0	0.42	0	0
7	q7	0.0772	0	-1.57	0
8	q8	0	0	0	0

Right Leg

RL:: 8 axis, RRRRRRRR, modDH, slowRNE

j	theta	d	a	alpha	offset
1	q1	0	0	0	0
2	q2	0.12	0	1.57	0
3	q3	0	0	1.57	-1.57
4	q4	0	0	-1.57	-1.57
5	q5	0	0.42	1.57	0
6	q6	0	0.42	0	0
7	q7	0.0772	0	-1.57	0
8	q8	0	0	0	0

Jacobian Matrices

Left Arm

```
Jacob_LA =

   -0.0002   -0.0002         0         0         0         0
    0.0011    0.0011   -0.0014   -0.5740    0.0002         0
   -0.2825   -0.2825    0.5740   -0.0009   -0.2870         0
   -1.0000   -1.0000         0         0         0         0
   -0.0008   -0.0008   -1.0000    0.0016    1.0000         0
    0.0008    0.0008   -0.0024   -1.0000    0.0008    1.0000
```

Right Arm

```
Jacob_RA =

    0.0002    0.0002         0         0         0         0
   -0.0002   -0.0002   -0.0014   -0.5740    0.0002         0
    0.2835    0.2835    0.5740   -0.0009   -0.2870         0
   -1.0000   -1.0000         0         0         0         0
   -0.0008   -0.0008   -1.0000    0.0016    1.0000         0
    0.0008    0.0008   -0.0024   -1.0000    0.0008    1.0000
```

Left Leg

```
Jacob_LL =

    0.0000   -0.0772   -0.0001    0.0000   -0.0772   -0.0772         0         0
    0.0764    0.0007   -0.0765    0.8400    0.0003         0         0         0
   -0.1193    0.8400    0.0007         0    0.4200         0         0         0
   -1.0000   -0.0000    1.0000         0         0         0         0         0
   -0.0008   -1.0000   -0.0008         0   -1.0000   -1.0000         0         0
   -0.0008    0.0008    0.0008    1.0000    0.0008    0.0008    1.0000    1.0000
```

Right Leg

```
Jacob_RL =

   -0.0002   -0.0772   -0.0001    0.0000   -0.0772   -0.0772         0         0
    0.0766    0.0007   -0.0765    0.8400    0.0003         0         0         0
    0.1207    0.8400    0.0007         0    0.4200         0         0         0
   -1.0000   -0.0000    1.0000         0         0         0         0         0
   -0.0008   -1.0000   -0.0008         0   -1.0000   -1.0000         0         0
   -0.0008    0.0008    0.0008    1.0000    0.0008    0.0008    1.0000    1.0000
```