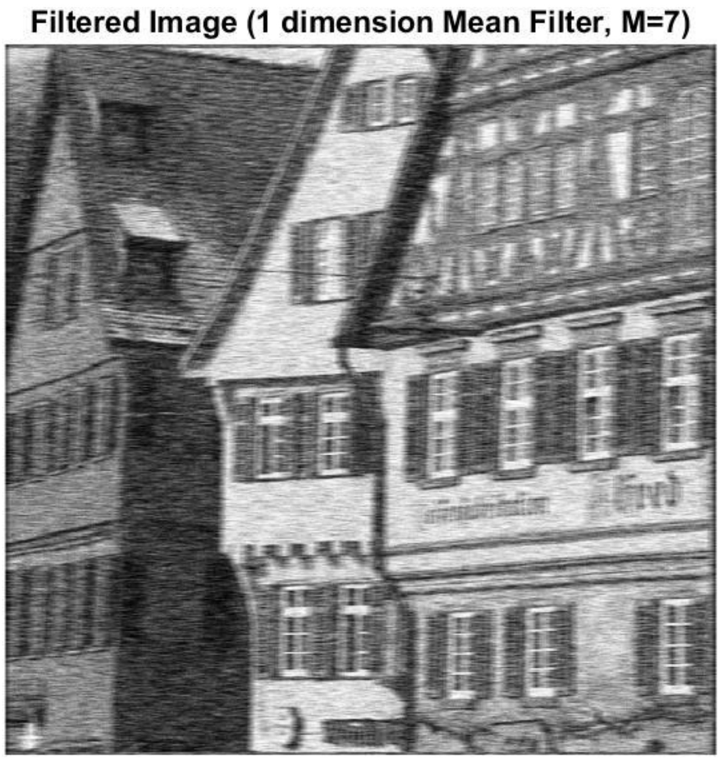
1.1)

Image

1.2)

Noisy Image (Variance = 0.2)

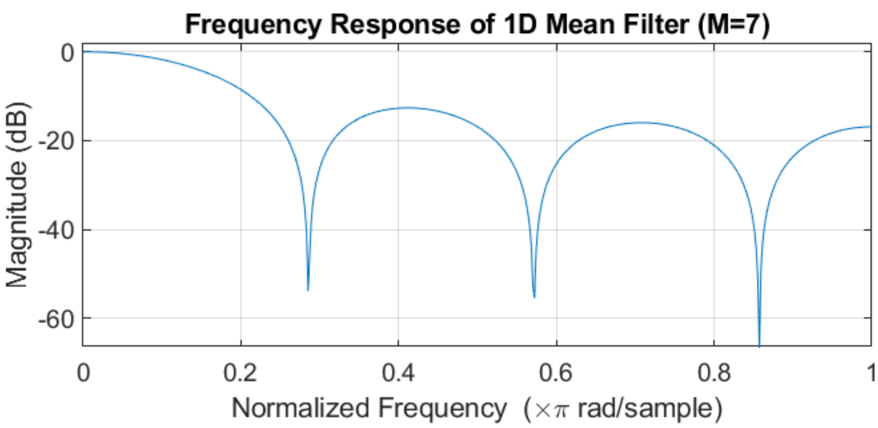
1.3)



As can be observed, in uniform regions such as walls that were speckled with noise, the image has become smoother and more uniform. However, at the edges and boundaries, it has also become more blurred. Therefore, it appears that the image has been denoised, but some of its details have been lost as well.

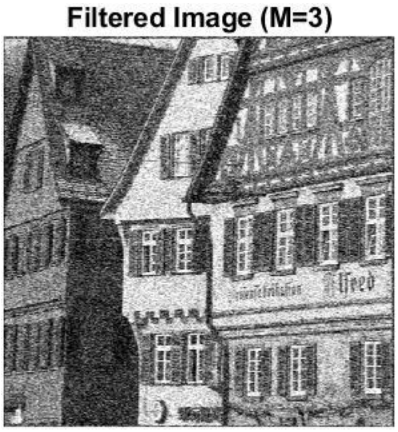
The filter we used is non-causal. This is because if we consider the pixel being calculated as the present time, the three pixels to its left (on the same row with lower column numbers) represent the past, and the three pixels to its right represent the future. Consequently, the output depends not only on the past and present but also on the future.

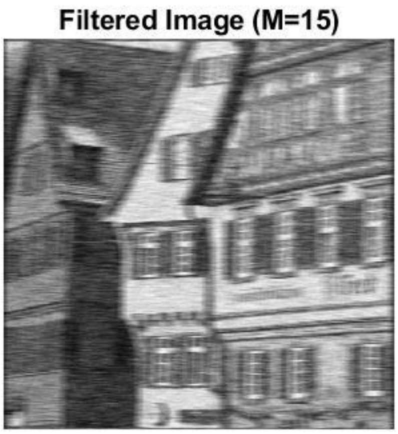
1.4)

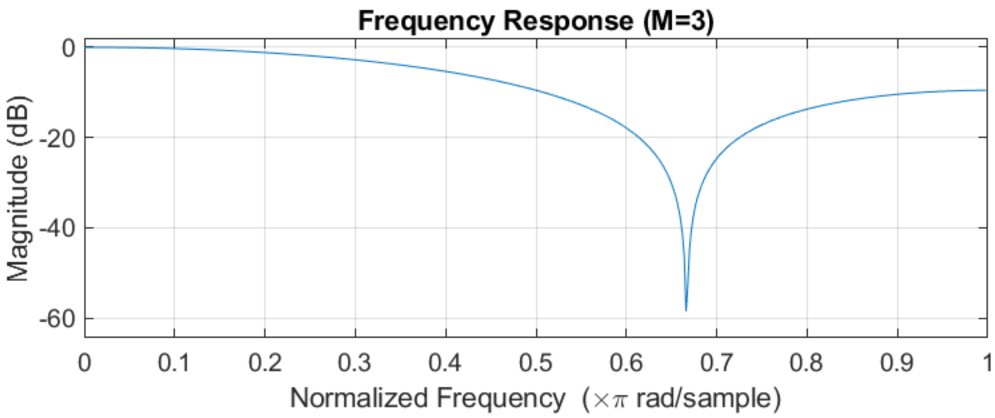


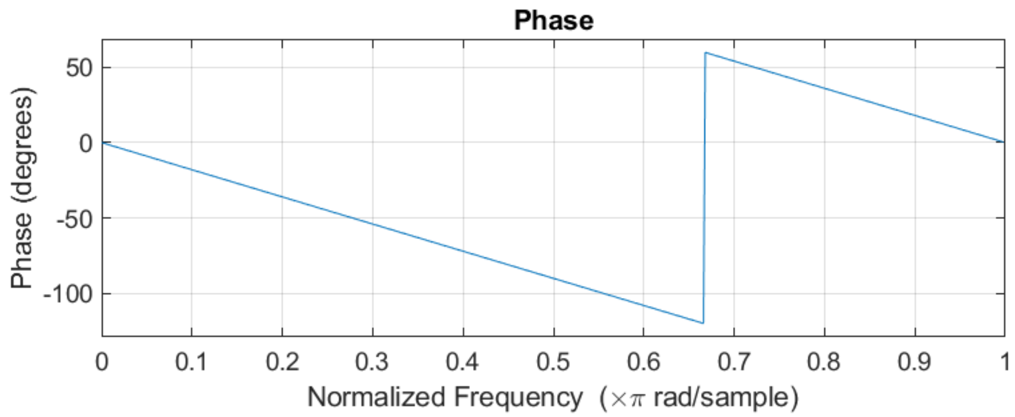
According to the frequency response plot, frequencies from 0 to 0.2 lie in the passband, while frequencies from 0.2 to 1 fall within the stopband. Image edges and boundaries in the frequency domain are generally dominated by high-frequency components; therefore, most of them lie in the stopband. This means that noise in these regions is strongly attenuated; however, at the same time, some image details are also suppressed and become blurred. On the other hand, smoother regions, such as walls, lie in the passband, where only a small amount of noise is removed.

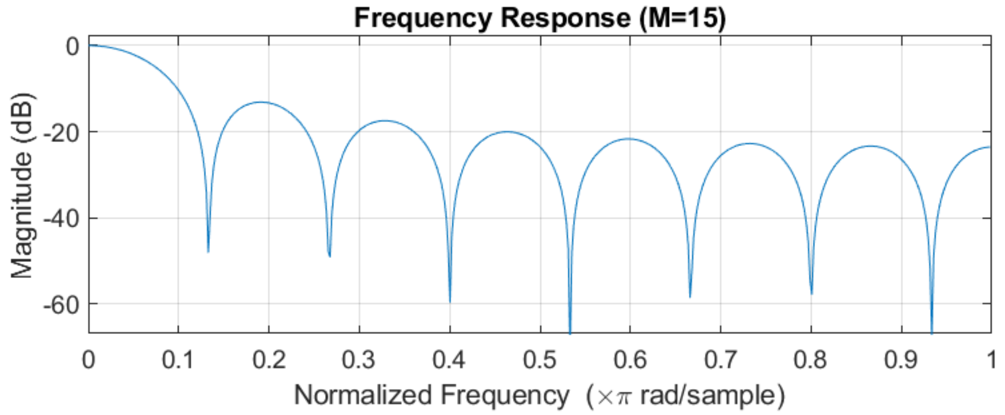
1.5 & 1.6)

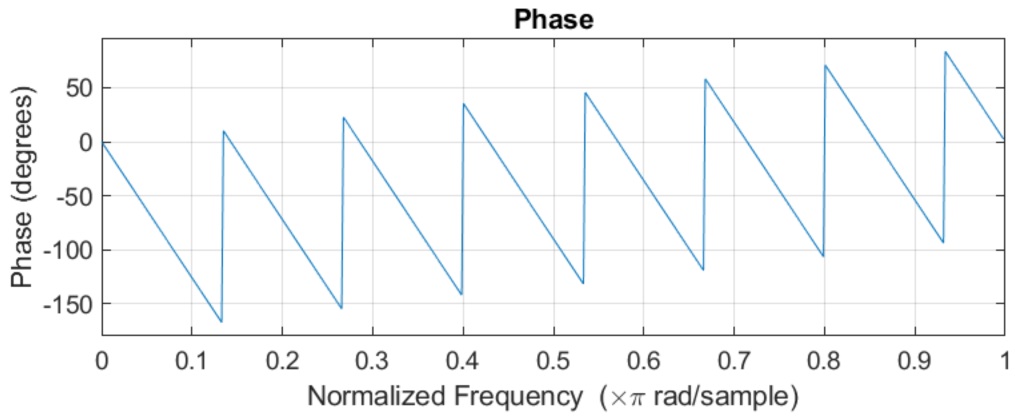












As the value of M increases, the width of the passband decreases.
For M=3: passband ranges from 0 to approximately 0.5.
For M=7: passband ranges from 0 to approximately 0.2.
For M=15: passband ranges from 0 to approximately 0.1.

This implies that as M increases, fewer edge and boundary components, as well as fewer uniform regions such as walls, fall within the passband. Consequently, a larger portion of the image—ranging from edges to smooth regions, especially edges—is affected by the filter's stopband, where both noise and image details are strongly attenuated. This is undesirable, since the objective is to remove noise only, not the image information itself.

In other words, although increasing M leads to stronger denoising over larger regions of the image, an excessive increase results in the removal of image information and fine details over wider areas, particularly around edges.

Therefore, it can be stated that there exists a trade-off between increasing denoising (by increasing M) and preserving image information (to recover the original image).

Accordingly, as the noise level increases, M should also be increased to enhance denoising. However, due to the aforementioned trade-off, M should not be increased excessively, since a significant portion of image information and details—especially at edges—would be removed along with the noise.

2.1)

Filtered Image (2D, 7x7)

Filtered Image (2D, 9x9)

In the first experiment, based on both the mathematical formulation and the observed images, the noise—particularly along the horizontal direction—is reduced. However, in the second filter, the noise is reduced in both the horizontal and vertical directions, resulting in overall better denoising performance. On the other hand, this also leads to increased overall image blurring, especially around edges.

In contrast, in the first experiment, the blurring is more pronounced along the horizontal direction, which causes the image to appear slightly stretched in that direction.

Moreover, according to the mathematical expressions of these two filters, the one-dimensional filter requires fewer computations compared to the two-dimensional filter.

2.2)

Here, similar to the previous one-dimensional filter, there exists a trade-off between increasing denoising (increasing M) and preserving image information and details. In this filter, due to averaging in both horizontal and vertical directions, increasing MMM has a more pronounced and noticeable effect on both noise removal and the attenuation of image details, especially around edges.

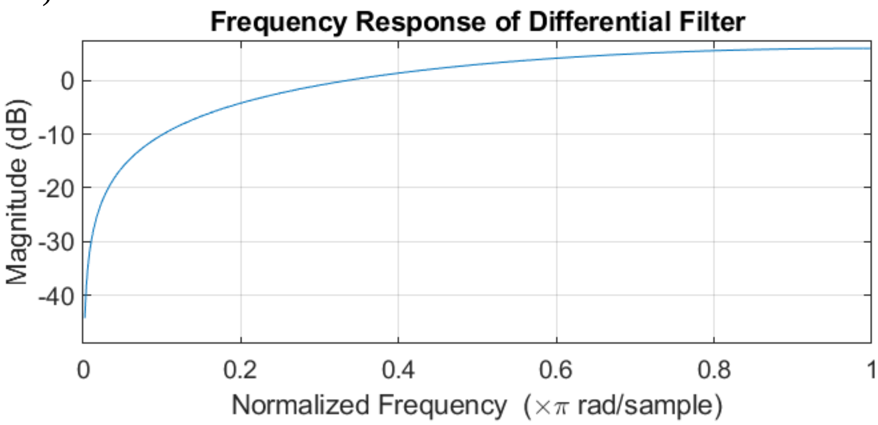
Therefore, as the noise level increases, M should also be increased to enhance denoising. However, considering the trade-off mentioned above, M should not be increased excessively, as a significant portion of image information and details—particularly at the edges—would be removed along with the noise.

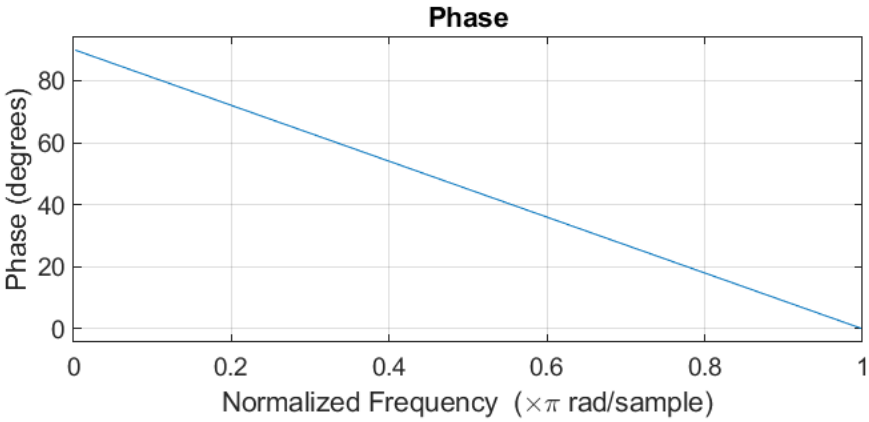
3.1)

Differential Filtered Image

As can be seen in the image, the differential filter is used to detect rapid changes in the image (edges), which is why the edges are highlighted. In other words, the smaller the changes (in this filter, measured horizontally with a one-pixel distance), the darker the resulting pixel appears, whereas larger changes produce a whiter pixel.

3.2)





According to the frequency response plot, lower frequencies—which correspond to smoother and more homogeneous regions of the image—lie in the stopband (as shown in the figure, from 0 to approximately 0.1) and are therefore attenuated. Higher frequencies, which correspond to regions with more rapid intensity changes in the image (i.e., edges), lie in the passband (as shown in the figure, from approximately 0.1 to 1). This is reasonable, since this filter is used to emphasize edges.