In the Name of God

Introduction to Machine Learning (25737-2)

Problem Set 03

Spring Semester 1401-02

Department of Electrical Engineering

Sharif University of Technology

Instructor: Dr. S. Amini

Due on Khordad 5th, 1401 at 23:59
Late submisstion is allowed untill
Khordad 12th, 1401 at 23:59

(∗) starred problems are optional and have a bonus mark!

1 Design Simple Neural Network

Design a neural network with one hidden-layer that implements the following function:

$$(A \vee \bar{B}) \oplus (\bar{C} \vee \bar{D})$$

Draw the network and determine all its weigths.

2 Vector Derivative

Consider following functions:

$$\mathbf{f}_1(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}) = \begin{bmatrix} \frac{1}{\pi}\sin(\pi x_2) \\ e^{x_1-1}x_2^2 \\ x_1 x_2 \end{bmatrix}, \quad \mathbf{f}_2(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}) = \begin{bmatrix} x_1+x_2+x_3 \\ x_1^2+x_2^2+x_3^2 \end{bmatrix}$$

$$\mathbf{f}(\mathbf{x}) = (\mathbf{f}_2 \circ \mathbf{f}_1)(\mathbf{x})$$

Determine $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}$ at point $\mathbf{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

*Note that your solution must follow the methods mentioned in the course slides.

3 One Convolutional Layer

Consider we have one convolutional layer with following equation:

$$\begin{bmatrix} z_1 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} k_1 & \dots & k_d & & & \\ & k_1 & \dots & k_d & & \\ & & & \ddots & & \\ & & & k_1 & \dots & k_d \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} + \begin{bmatrix} b \\ \vdots \\ b \end{bmatrix}$$

and also we know:

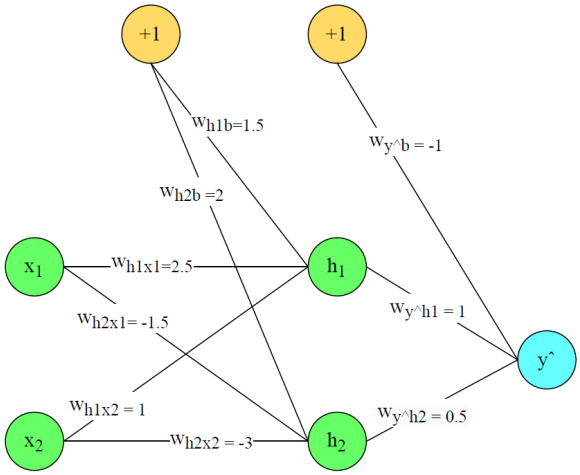
$$\frac{\partial L}{\partial z_i} = \alpha_i$$

Calculate $\frac{\partial L}{\partial k_j}$ and $\frac{\partial L}{\partial b}$ in terms of x_i and z_i.

4 Backpropagation Algorithm

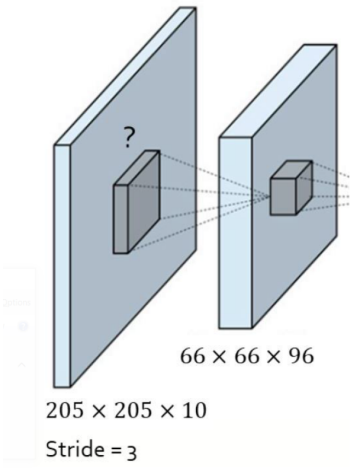
The following image shows a two-layer neural network with two nodes in the hidden layer and one node in the output. x_1 and x_2 are the two inputs to the network. Each node has a bias with value of 1.
Assume that the value of the learning rate is 0.1 and the activation function is sigmoid in both hidden-layer and output-layer.

1. Calculate the value at nodes $\hat{y}, h_1, h_2$ for input $\{x_1 = 0, x_2 = 1\}$.
2. Execute one step of backpropagation algorithm for the previous input in part (a) and output $y = 1$.
3. Calculate the updated weights for the hidden-layer and output-layer (a total of 9 weights) by executing one step of the gradient descent algorithm.



5 Model Parameters

Consider the following two-layer convolutional network.



1. Based on the input and output dimensions shown in the figure, determine the size of the kernel used for this operation.

2. Determine the number of trainable parameters in this layer.
3. Calculate the number of multiplication operations required to obtain the output.