Sharif University of Technology
Electrical Engineering Department

Machine Learning and Vision Lab
Pre-Report 3

Amir Hossein Yari
99102507

October 29, 2023

Contents

LDA Proof

$$\arg\max \frac{u_1^T S_B u_1}{u_1^T S_W u_1}$$

$$L(u_1) = \frac{u_1^T S_B u_1}{u_1^T S_W u_1}$$

$$\frac{\partial L}{\partial u_1} = 0 \Rightarrow \frac{2S_B u_1 (u_1^T S_W u_1) - 2S_W u_1 (u_1^T S_B u_1)}{(u_1^T S_W u_1)^2} = 0$$

$$\Rightarrow \frac{S_B u_1 (u_1^T S_W u_1)}{u_1^T S_W u_1} - \frac{S_W u_1 (u_1^T S_B u_1)}{u_1^T S_W u_1} = 0 \Rightarrow S_B u_1 = \lambda S_W u_1 \Rightarrow S_W^{-1} S_B u_1 = \lambda u_1$$

So u_1 is eigenvector of $S_W^{-1} S_B$ and the eigenvalue is $\frac{u_1^T S_B u_1}{u_1^T S_W u_1}$.

Data Classification

$$\mu_{class1} = \begin{bmatrix}3 & 3.6\end{bmatrix} \quad \mu_{class2} = \begin{bmatrix}3.3 & 2\end{bmatrix}$$

$$S_{W_1} = \sum_{i=1}^{5}(x^i - \begin{bmatrix}3 & 3.6\end{bmatrix})(x^i - \begin{bmatrix}3 & 3.6\end{bmatrix})^T$$

$$= \begin{bmatrix}-2 & -1.6\end{bmatrix}^T \begin{bmatrix}-2 & -1.6\end{bmatrix} + \begin{bmatrix}-1 & -0.6\end{bmatrix}^T \begin{bmatrix}-1 & -0.6\end{bmatrix} + \begin{bmatrix}0 & -0.6\end{bmatrix}^T \begin{bmatrix}0 & -0.6\end{bmatrix}$$

$$+ \begin{bmatrix}1 & 1.4\end{bmatrix}^T \begin{bmatrix}1 & 1.4\end{bmatrix} + \begin{bmatrix}2 & 1.4\end{bmatrix}^T \begin{bmatrix}2 & 1.4\end{bmatrix} = \begin{bmatrix}10 & 8 \\ 8 & 7.2\end{bmatrix}$$

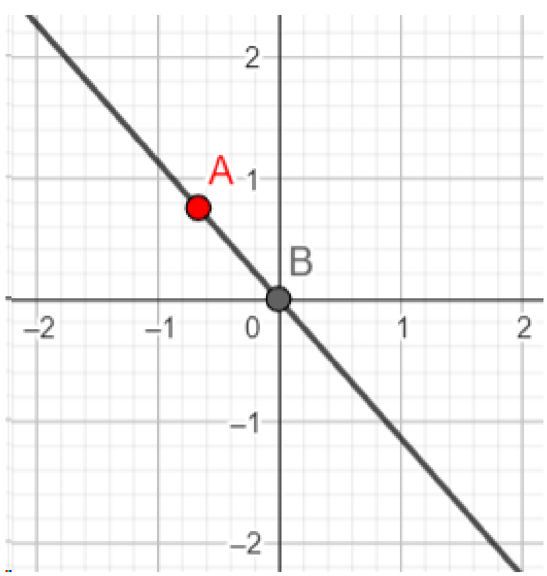
$$S_{W_2} = \sum_{i=1}^{6}(x^i - \begin{bmatrix}3.3 & 2\end{bmatrix})(x^i - \begin{bmatrix}3.3 & 2\end{bmatrix})^T = \begin{bmatrix}17.3 & 16 \\ 16 & 16\end{bmatrix}$$

$$S_W = S_{W_1} + S_{W_2} = \begin{bmatrix}27.3 & 24 \\ 24 & 23.2\end{bmatrix} \Rightarrow S_W^{-1} = \begin{bmatrix}0.39 & -0.41 \\ -0.41 & 0.47\end{bmatrix}$$

$$S_B = 5 \times (\begin{bmatrix}3 \\ 3.6\end{bmatrix} - \begin{bmatrix}3.2 \\ 2.7\end{bmatrix})(\begin{bmatrix}3 & 3.6\end{bmatrix} - \begin{bmatrix}3.2 & 2.7\end{bmatrix}) + 6 \times (\begin{bmatrix}3.3 \\ 2\end{bmatrix} - \begin{bmatrix}3.2 \\ 2.7\end{bmatrix})(\begin{bmatrix}3.3 & 2\end{bmatrix} - \begin{bmatrix}3.2 & 2.7\end{bmatrix})$$

$$= \begin{bmatrix}0.26 & -1.32 \\ -1.32 & 6.99\end{bmatrix}$$

$$S_W^{-1}S_B = \begin{bmatrix}0.64 & -3.38 \\ -0.73 & 3.83\end{bmatrix} \Rightarrow \text{eigenvector} = \begin{bmatrix}-0.66 \\ 0.75\end{bmatrix}$$



Now define projected data and mean od them.

$$P_1 = \begin{bmatrix}-0.66 & 0.75\end{bmatrix} \begin{bmatrix}1 & 2 & 3 & 4 & 5\\ 2 & 3 & 3 & 5 & 5\end{bmatrix} = \begin{bmatrix}0.84 & 0.93 & 0.27 & 1.11 & 0.45\end{bmatrix}$$

$$P_2 = \begin{bmatrix}-0.66 & 0.75\end{bmatrix} \begin{bmatrix}1 & 2 & 3 & 3 & 5 & 6\\ 0 & 1 & 1 & 2 & 3 & 5\end{bmatrix} = \begin{bmatrix}-0.66 & -0.57 & -1.23 & -0.48 & -1.05 & -0.21\end{bmatrix}$$

$$\mu_1 = \begin{bmatrix}-0.66 & 0.75\end{bmatrix} \begin{bmatrix}3\\ 3.6\end{bmatrix} = \begin{bmatrix}0.72\end{bmatrix}$$

$$\mu_2 = \begin{bmatrix}-0.66 & 0.75\end{bmatrix} \begin{bmatrix}3.3\\ 2\end{bmatrix} = \begin{bmatrix}-0.678\end{bmatrix}$$

for each data calculate euclidean norm of distance between project of each data and projected mean of class and minimum norm define the class of data. So classification done correctly ✓