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Machine Learning and Vision Lab
Pre-Report 8

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Contents

Pyramid in Optical Flow

The core idea behind using a pyramid in optical flow is to adopt a coarse-to-fine strategy. The algorithm starts by estimating motion at a lower resolution (coarser level) and progressively refines the estimate at higher resolutions (finer levels). This approach is motivated by the observation that large motions can be captured at lower resolutions, while finer details are addressed at higher resolutions. One common enhancement to optical flow algorithms involves the use of a pyramid structure. A pyramid, in the context of optical flow, refers to a multi-resolution representation of the input images. It is created by successively down-sampling the original image to create a series of lower-resolution images, forming a pyramid-like structure. This pyramid is used in optical flow computations for several reasons:

1. **Scale Sensitivity:** Optical flow techniques with a pyramid are scale-sensitive, detecting motion at different scales. This allows handling both large and small motions effectively.

2. **Robustness to Noise:** Lower-resolution images in the pyramid are less affected by noise, enhancing robustness. The algorithm focuses on salient features across resolutions, reducing sensitivity to noise.

3. **Computational Efficiency:** Working with a pyramid reduces the overall workload. Higher-level images require fewer computations, improving computational efficiency, crucial in real-time applications.

4. **Handling Large Displacements:** A pyramid helps in dealing with large displacements effectively. Lower resolutions capture coarse motion, allowing progressive refinement at higher resolutions.

5. **Improved Convergence:** The pyramid aids in reliable convergence. Starting with a broad estimation and refining progressively often leads to better convergence and more accurate results.

Optimization Proof

Consider the minimization problem:

$$\text{minimize} \sum_{i=-1}^{1} \sum_{j=-1}^{1} (f_x(i,j)u + f_y(i,j)v + f_t(i,j))^2$$

Now, take the partial derivatives with respect to u and v and set them equal to zero to find the critical points.

1. **Partial Derivative with Respect to u:**

$$\frac{\partial}{\partial u} = 2 \sum_{i=-1}^{1} \sum_{j=-1}^{1} (f_x(i,j)u + f_y(i,j)v + f_t(i,j)) \cdot f_x(i,j) = 0$$

   Expressing in matrix form:

$$A^T(Au - b) = 0$$

2. **Partial Derivative with Respect to v:**

$$\frac{\partial}{\partial v} = 2 \sum_{i=-1}^{1} \sum_{j=-1}^{1} (f_x(i,j)u + f_y(i,j)v + f_t(i,j)) \cdot f_y(i,j) = 0$$

   In matrix form:

$$A^T(Av - b) = 0$$

The solution to the linear system is given by:

$$\begin{bmatrix} u \\ v \end{bmatrix} = (A^T A)^{-1} A^T b$$