# Natural Language Processing MEMMs and CRFs

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- The goal of sequence labeling is to assign tags to words, or more generally, to assign discrete labels to discrete elements in a sequence [Eisenstein, 2018].
- Well known examples of this problem are: part-of-speech tagging (POS) and Named Entity Recognition (NER).
- Maximum-entropy Markov models (MEMMs) make use of log-linear multi-class models for sequence labeling tasks [McCallum et al., 2000].
- In the early NLP literature, logistic regression was often called maximum entropy classification [Eisenstein, 2018].
- Hence, MEMMs will look very similar to the multi-class softmax models seen in the lecture about linear models.
- In contrast to HMMs, here we rely on parameterized functions.

The goal of MEMMs is model the following conditional distribution:

$$P(s_1, s_2 \ldots, s_m | x_1, \ldots, x_m)$$

- Where each x<sub>j</sub> for j = 1...m is the j-th input symbol (for example the j-th word in a sentence), and each s<sub>i</sub> for j = 1...m is the j-th tag.<sup>1</sup>
- We would expect P(DET,NOUN,VERB|the,dog,barks) to be higher than P(VERB,VERB,VERB|the,dog,barks) in a model trained from a POS-tagging training dataset.

<sup>&</sup>lt;sup>1</sup>These slides are based on lecture notes of Michael Collins http://www.cs.columbia.edu/~mcollins/crf.pdf. The notation and terminology has been adapted to be consistent with the rest of the course.

- We use S to denote the set of possible tags.
- We assume that S is a finite set.
- For example, in part-of-speech tagging of English, S would be the set of all
  possible parts of speech in English (noun, verb, determiner, preposition, etc.).
- Given a sequence of words  $x_1, \ldots, x_m$ , there are  $k^m$  possible part-of-speech sequences  $s_1, \ldots, s_m$ , where k = |S| is the number of possible parts of speech.
- We want to estimate a distribution over these  $k^m$  possible sequences.

In a first step, MEMMs use the following decomposition ( $s_0$  has always a special tag \*):

$$P(s_{1}, s_{2}..., s_{m}|x_{1},..., x_{m}) = \prod_{i=1}^{m} P(s_{i}|s_{1}..., s_{i-1}, x_{1},..., x_{m})$$

$$= \prod_{i=1}^{m} P(s_{i}|s_{i-1}, x_{1},..., x_{m})$$
(1)

- The first equality is exact (it follows by the chain rule of conditional probabilities).
- The second equality follows from an independence assumption, namely that for all i,

$$P(s_i|s_1\ldots,s_{i-1},x_1,\ldots,x_m)=P(s_i|s_{i-1},x_1,\ldots,x_m)$$

- Hence we are making a first order Markov assumption similar to the Markov assumption made in HMMs<sup>2</sup>.
- The tag in the *i*-th position depends only on the tag in the (i-1)-th position.
- Having made these independence assumptions, we then model each term using a multiclass log-linear (softmax) model:

$$P(s_{i}|s_{i-1},x_{1},...,x_{m}) = \frac{\exp(\vec{w} \cdot \vec{\phi}(x_{1},...,x_{m},i,s_{i-1},s_{i}))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(x_{1},...,x_{m},i,s_{i-1},s'))}$$
(2)

<sup>&</sup>lt;sup>2</sup>We actually made a second order Markov assumption in HMMs. MEMMs can also be extended to second order assumptions.

Here  $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)$  is a feature vector where:

- $x_1, \dots, x_m$  is the entire sentence being tagged.
- *i* is the position to be tagged (can take any value from 1 to *m*).
- $s_{i-1}$  is the previous tag value (can take any value in S).
- $s_i$  is the new tag value (can take any value in S).

The scope of the feature vector is **restricted** to the whole input sequence  $x_1, x_m$ , and only the previous and current tag values. This restriction allows efficient training of both MEMMs and CRFs.

# Example of Features used in Part-of-Speech Tagging

- 1.  $\vec{\phi}(x_1, \cdots, x_m, i, s_{i-1}, s_i)_{[1]} = 1$  if  $s_i = \mathsf{ADVERB}$  and word  $x_i$  ends in "-ly"; 0 otherwise. If the weight  $\vec{w}_{[1]}$  associated with this feature is large and positive, then this feature is essentially saying that we prefer labelings where words ending in -ly get labeled as ADVERB.
- 2.  $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)_{[2]} = 1$  if  $i = 1, s_i = VERB$ , and  $x_m = ?$ ; 0 otherwise. If the weight  $\vec{w}_{[2]}$  associated with this feature is large and positive, then labelings that assign VERB to the first word in a question (e.g., "Is this a sentence beginning with a verb?") are preferred.
- 3.  $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)_{[3]} = 1$  if  $s_{i-1} = \text{ADJECTIVE}$  and  $s_i = \text{NOUN}$ ; 0 otherwise. Again, a positive weight for this feature means that adjectives tend to be followed by nouns.
- 4.  $\vec{\phi}(x_1, \cdots, x_m, i, s_{i-1}, s_i)_{[4]} = 1$  if  $s_{i-1}$ = PREPOSITION and  $s_i$ = PREPOSITION. A negative weight  $\vec{w}_{[4]}$  for this function would mean that prepositions don't tend to follow prepositions.

<sup>3</sup>Source: https://blog.echen.me/2012/01/03/ introduction-to-conditional-random-fields/

#### **Feature Templates**

It is possible to define more general feature templates covering unigrams, bigrams, n-grams of words as well as tag values of  $s_{i-1}$  and  $s_i$ .

- 1. A word unigram and tag unigram feature template:
  - $\vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i)_{[index(j,z)]} = 1$  if  $s_i = \mathsf{TAG}_{[j]}$  and  $x_i = \mathsf{WORD}_{[z]}$ ; 0 otherwise  $\forall j, z$ .
  - Notice that j is and index spanning all possible tags in S and z is another index spanning the words in the vocabulary V.
- 2. A word bigram and tag bigram feature template:

$$\vec{\phi}(x_1,\cdots,x_m,i,s_{i-1},s_i)_{[index(j,z,u,v)]}=1$$
 if  $s_{i-1}=\mathsf{TAG}_{[j]}$  and  $s_i=\mathsf{TAG}_{[z]}$  and  $x_{i-1}=\mathsf{WORD}_{[u]}$  and  $x_i=\mathsf{WORD}_{[v]}$ ; 0 otherwise  $\forall j,z,u,v$ .

The function index(j, k, ...) will map each different feature to a unique index in the feature vector.

Notice that the resulting vector will be very high-dimensional and sparse.

## Example

$$P(s_i|s_{i-1}, x_1, \dots, x_m) = \frac{\exp(\vec{w} \cdot \vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s_i))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(x_1, \dots, x_m, i, s_{i-1}, s'))}$$

#### Example:

1 2 3 4 The dog barks loudly DT NN VB ADV

Let's check that  $P(s_4 = \text{ADV}|s_3 = \text{VB}, \text{the,dog,barks,loudly}) > P(s_4 = \text{VB}|s_3 = \text{VB}, \text{the,dog,barks,loudly})$ 

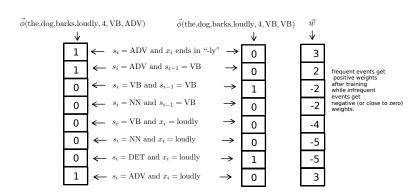
$$P(s_4 = \text{ADV} | s_3 = \text{VB}, \text{the,dog,barks,loudly}) = \frac{\exp(\vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB}, \text{ADV}))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB}, s'))}$$

$$P(s_4 = \text{VB} | s_3 = \text{VB}, \text{the,dog,barks,loudly}) = \frac{\exp(\vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB}, \text{VB}))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB}, s'))}$$

## Example

#### This is the same as checking if:

 $\vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB, ADV}) > \vec{w} \cdot \vec{\phi}(\text{the,dog,barks,loudly}, 4, \text{VB, VB})$ 



$$\begin{split} &\vec{w}\cdot\vec{\phi}(\text{the,dog,barks,loudly,4,VB,ADV}) = 1*3+1*2+1*3 = 6\\ &> \vec{w}\cdot\vec{\phi}(\text{the,dog,barks,loudly,4,VB,VB}) = 1*-2+1*-5 = -7 \end{split}$$

#### MEMMs and Multi-class Softmax

- Notice that the log-linear model from above is very similar to the multi-class softmax model presented in the lecture about linear models.
- A general log-linear model has the following form:

$$P(y|x; \vec{w}) = \frac{\exp(\vec{w} \cdot \vec{\phi}(x, y))}{\sum_{y' \in Y} \exp(\vec{w} \cdot \vec{\phi}(x, y'))}$$

A multi-class softmax model has the following form:

$$\hat{\vec{y}} = \operatorname{softmax}(\vec{x} \cdot W + \vec{b}) 
\hat{\vec{y}}_{[i]} = \frac{e^{(\vec{x} \cdot W + \vec{b})_{[i]}}}{\sum_{j} e^{(\vec{x} \cdot W + \vec{b})_{[j]}}}$$
(3)

#### MEMMs and Multi-class Softmax

- Difference 1: in the log-linear model we have a fixed parameter vector  $\vec{w}$  instead of having multiple vectors (one column of W for each class value).
- Difference 2: the feature vector of the log-linear model  $\vec{\phi}(x,y)$  includes information of the label y, whereas the input vector  $\vec{x}$  of the softmax model is independent of y.
- Log-linear models allow using features that consider the interaction between x and y (e.g., x ends in "ly" and y is an ADVERB).

## Training MEMMs

- Once we've defined the feature vectors  $\vec{\phi}$ , we can train the parameters  $\vec{w}$  of the model in the usual way linear models are trained.
- We set the negative log-likelihood as the loss function and optimize parameters using gradient descent from the training examples.
- This is equivalent as using the cross-entropy loss.
- "Any loss consisting of a negative log-likelihood is a cross-entropy between the empirical distribution defined by the training set and the probability distribution defined by model" [Goodfellow et al., 2016].

- The decoding problem is as follows.
- We are given a new test sequence  $x_1, \ldots, x_m$ .
- Our goal is to compute the most likely state sequence for this test sequence,

$$\arg\max_{s_1,\ldots,s_m} P(s_1,\ldots,s_m|x_1,\ldots,x_m). \tag{4}$$

- There are k<sup>m</sup> possible state sequences, so for any reasonably large sentence length m brute-force search through all the possibilities will not be possible.
- We can use the Viterbi alogrithm in a similar way as used for HMMs.

- The basic data structure in the algorithm will be a dynamic programming table π with entries π[j, s] for j = 1,..., m, and s ∈ S.
- π[j, s] will store the maximum probability for any state sequence ending in state s
  at position j.
- · More formally, our algorithm will compute

$$\pi[j,s] = \max_{s_1,\ldots,s_{j-1}} \left( P(s|s_{j-1},x_1,\ldots,x_m) \prod_{k=1}^{j-1} P(s_k|s_{k-1},x_1,\ldots,x_m) \right)$$

for all j = 1, ..., m, and for all  $s \in S$ .

#### The algorithm is as follows:

• Initialization: for  $s \in S$ 

$$\pi[1,s] = P(s|s_0,x_1,\ldots,x_m)$$

where  $s_0$  is a special "initial" state.

• For  $j \in \{2, ..., m\}$ ,  $s \in \{1, ..., k\}$ 

$$\pi[j, s] = \max_{s' \in S} [\pi[j-1, s'] \times P(s|s', x_1, \dots, x_m)]$$

• Finally, having filled in the  $\pi[j, s]$  values for all j, s, we can calculate

$$\max_{s_1,\ldots,s_m}=\max_s \ \pi[m,s].$$

- The algorithm runs in  $O(mk^2)$  time (i.e., linear in the sequence length m, and quadratic in the number of tags k).
- As in the Viterbi algorithm for HMMs, we can compute the highest-scoring sequence using backpointers in the dynamic programming algorithm.

## Comparison between MEMMs and HMMs

- So what is the motivation for using MEMMs instead of HMMs?
- Note that the Viterbi decoding algorithms for the two models are very similar.
- In MEMMs, the probability associated with each state transition  $s_{i-1}$  to  $s_i$  is

$$P(s_i|s_{i-1},x_1,...,x_m) = \frac{\exp(\vec{w} \cdot \vec{\phi}(x_1,...,x_m,i,s_{i-1},s_i))}{\sum_{s' \in S} \exp(\vec{w} \cdot \vec{\phi}(x_1,...,x_m,i,s_{i-1},s'))}$$

In HMMs, the probability associated with each transition is:

$$P(s_i|s_{i-1},x_1,\ldots,x_m) = P(s_1|s_{i-1})P(x_i|s_i)$$

## Comparison between MEMMs and HMMs

- The key advantage of MEMMs is that the use of feature vectors  $\vec{\phi}$  allows much richer representations than those used in HMMs.
- For example, the transition probability can be sensitive to any word in the input sequence x<sub>1</sub>,..., x<sub>m</sub>.
- In addition, it is very easy to introduce features that are sensitive to spelling features (e.g., prefixes or suffixes) of the current word x<sub>i</sub>, or of the surrounding words.
- These features are useful in many NLP applications, and are difficult to incorporate within HMMs in a clean way.

- We now turn to Conditional Random Fields (CRFs) [Lafferty et al., 2001].
- Notation: for convenience, we'll use  $x_{1:m}$  to refer to an input sequence  $x_1, \ldots, x_m$ , and  $s_{1:m}$  to refer to a sequence of tags  $s_1, \ldots, s_m$ .
- The set of all possible tags is again S.
- The set of all possible tag sequences is  $S^m$ .
- In conditional random fields we'll again build a model of

$$P(s_1,...,s_m|x_1,...,x_m) = P(s_{1:m}|x_{1:m})$$

 A first key idea in CRFs will be to define a feature vector that maps an entire input sequence x<sub>1:m</sub> paired with an entire tag sequence s<sub>1:m</sub> to some d-dimensional feature vector:

$$\vec{\Phi}(x_{1:m}, s_{1:m}) \in \mathcal{R}^d$$

- We'll soon give a concrete definition for  $\vec{\Phi}$ .
- For now just assume that some definition exists.
- We will often refer to  $\vec{\Phi}$  as being a "global" feature vector.
- It is global in the sense that it takes the entire state sequence into account.

In CRFs we build a giant log-linear model:

$$P(s_{1:m}|x_{1:m}; \vec{w}) = \frac{\exp(\vec{w} \cdot \vec{\Phi}(x_{1:m}, s_{1:m}))}{\sum_{s'_{1:m} \in S^m} \exp(\vec{w} \cdot \vec{\Phi}(x_{1:m}, s'_{1:m}))}$$

- This is "just" another log-linear model, but it is is "giant".
- The space of possible values for  $s_{1:m}$  is huge  $S^m$ .
- The normalization constant (denominator in the above expression) involves a sum over all possible tag sequences S<sup>m</sup>.
- These issues might seem to cause severe computational problems.
- Under appropriate assumptions we can train and decode efficiently with this type of model.

• We define the global feature vector  $\vec{\Phi}(x_{1:m}, s_{1:m})$  as follows:

$$\vec{\Phi}(x_{1:m}, s_{1:m}) = \sum_{j=1}^{m} \vec{\phi}(x_{1:m}, j, s_{j-1}, s_j)$$

where  $\vec{\phi}(x_{1:m}, j, s_{j-1}, s_j)$  are the same as the feature vectors used in MEMMs.

- Example:  $\vec{\Phi}(\text{[the,dog,barks]}, \text{DET,NOUN,VERB]}) = \vec{\phi}(\text{[the,dog,barks]}, 1, *, \text{DET}) + \vec{\phi}(\text{[the,dog,barks]}, 2, \text{DET, NOUN}) + \vec{\phi}(\text{[the,dog,barks]}, 3, \text{NOUN, VERB})$
- Essentially, we are adding up many sparse vectors.

## Example

$x_i$ =will $\land y_i = NN$				
$y_{i-1}$ =START $\wedge y_i = NN$				
$x_i$ =will $\land y_i = MD$				
$y_{i-1}$ =START $\wedge y_i = MD$				
$x_i=to \land y_i = TO$				
$y_{i-1}=NN \wedge y_i = TO$				
$y_{i-1}=MD \land y_i = TO$				
$x_i$ =fight $^ y_i = VB$				
V: 1=TO ∧ V: = VB				

will φ(x, 1, y <sub>1</sub> , y <sub>0</sub> )	tο φ(x, 2, y <sub>2</sub> , y <sub>1</sub> )	fight <sub>(x, 3, y3, y2)</sub>	Φ(x, NN TO VB)
1	0	0	1
1	0	0	1
0	0	0	0
0	0	0	0
0	1	0	1
0	1	0	1
0	0	0	0
0	0	1	1
0	0	1	1

#### <sup>4</sup>source:

http://people.ischool.berkeley.edu/~dbamman/
nlpF18/slides/12\_neural\_sequence\_labeling.pdf

• We are assuming that for any dimension of  $\vec{\Phi}_{[k]}, k=1,\ldots,d$ , the k'th global feature is:

$$\vec{\Phi}(x_{1:m}, s_{1:m})_{[k]} = \sum_{j=1}^{m} \vec{\phi}(x_{1:m}, j, s_{j-1}, s_j)_{[k]}$$

- Thus  $\vec{\Phi}(x_{1:m}, s_{1:m})_{[k]}$  is calculated by summing the "local" feature vector  $\vec{\phi}(x_{1:m}, j, s_{j-1}, s_j)_{[k]}$  over the m different tag transitions in  $s_1, \ldots, s_m$ .
- We would expect each local vector to encode relevant information about the tag transition by turning on some vector dimensions (setting the value to one).
- We now turn to two critical practical issues in CRFs: first, decoding, and second, parameter estimation (training).

## **Decoding with CRFs**

- The decoding problem in CRFs is as follows.
- For a given input sequence x<sub>1:m</sub> = x<sub>1</sub>, x<sub>2</sub>,..., x<sub>m</sub>, we would like to find the most likely underlying state sequence under the model, that is,

$$arg \max_{s_{1:m} \in S^{m}} P(s_{1:m}|x_{1:m}; \vec{w}) = arg \max_{s_{1:m} \in S^{m}} \frac{\exp(\vec{w} \cdot \vec{\Phi}(x_{1:m}, s_{1:m}))}{\sum_{s'_{1:m} \in S^{m}} \exp(\vec{w} \cdot \vec{\Phi}(x_{1:m}, s'_{1:m}))}$$

$$= arg \max_{s_{1:m} \in S^{m}} \exp(\vec{w} \cdot \vec{\Phi}(x_{1:m}, s_{1:m}))$$

$$= arg \max_{s_{1:m} \in S^{m}} \vec{w} \cdot \vec{\Phi}(x_{1:m}, s_{1:m})$$

$$= arg \max_{s_{1:m} \in S^{m}} \vec{w} \cdot \sum_{j=1}^{m} \vec{\phi}(x_{1:m}, j, s_{j-1}, s_{j})$$

$$= arg \max_{s_{1:m} \in S^{m}} \sum_{j=1}^{m} \vec{w} \cdot \vec{\phi}(x_{1:m}, j, s_{j-1}, s_{j})$$

## **Decoding with CRFs**

 We have shown that finding the most likely sequence under the model is equivalent to finding the sequence that maximizes:

$$arg \max_{s_{1:m} \in S^m} \sum_{j=1}^m \vec{w} \cdot \vec{\phi}(x_{1:m}, j, s_{j-1}, s_j)$$

- This problem has a clear intuition. Each transition from tag s<sub>j-1</sub> to tag s<sub>j</sub> has an associated score: w̄ · φ̄(x<sub>1:m</sub>, j, s<sub>i-1</sub>, s<sub>j</sub>)
- This score could be positive or negative.
- Intuitively, this score will be relatively high if the state transition is plausible, relatively low if this transition is implausible.
- The decoding problem is to find an entire sequence of states such that the sum
  of transition scores is maximized.
- We can again solve this problem using a variant of the Viterbi algorithm, in a very similar way to the decoding algorithm for HMMs or MEMMs.

# Parameter Estimation in CRFs (training)

- For parameter estimation, we assume we have a set of n labeled examples,  $\{(x_{1:m}^i, s_{1:m}^i)\}_{i=1}^n$ . Each  $x_{1:m}^i$  is an input sequence  $x_1^i, \ldots, x_m^i$  each  $s_{1:m}^i$  is a tag sequence  $s_1^i, \ldots, s_m^i$ .
- We again set the negative log-likelihood (or cross-entropy) as the loss function L
  as optimize parameters using gradient descent.
- The main challenge here is that gradient calculations  $\frac{\partial L}{\partial \vec{w}_{[k]}}$  involve summing over  $S^m$  (a very large set containing all possible tag sequences).
- This sum can be computed efficiently using the Forward-backward algorithm<sup>5</sup>.
- This is another dynamic programming algorithm that is closely related to the Viterbi algorithm.

<sup>5</sup>http://www.cs.columbia.edu/~mcollins/fb.pdf

#### **CRFs and MEMMs**

- CRFs and MEMMS are discriminative sequence labeling models: they model the conditional probability directly via a parameterized log-linear multi-class function (softmax).
- HMMs, on the other hand, are generative models.
- In MEMM the normalization (denominator of the softmax) is local: it happens at each tag step (the sum runs over all possible tag values S).
- In CRFs the normalization is global: the sum runs over all possible tag sequences S<sup>m</sup>.
- Training a MEMM is quite easy: just train a multi-class log-linear model for for a given word to the label. This classifier is used at each word step to predict the whole sequence.
- Training CRF is more complex. The objective is to maximize the log probability of the most likely sequence.

## CRFs and MEMMs: the label bias problem

- MEMMs end up making up decision at each time step independently.
- This leads to a problem called label bias: in some tag space configurations, MEMMs essentially completely ignore important aspects of the context.
- Example: The right POS labeling of sentence "will to fight" (la voluntad de pelear) is "NN TO VB". 6
- Here NN stands for "noun", TO stands for "infinitive to", and VB stands for "verb base form".
- Modals (MD) show up much more frequently at the start of the sentence than nouns do (e.g., questions).
- Hence, tag "MD" will receive a higher score than tag "NN" when  $x_0$ ="will" :  $P(s_1 = MD|s_0 = *, x_1 = \text{`will''}, ...) > P(s_1 = NN|s_{i-1} = *, x_1 = \text{`will''}).$
- But we know that MD + TO is very rare: "... can to eat", "... would to eat".

#### CRFs and MEMMs: the label bias problem

- The word "to" is relatively deterministic (almost always has tag TO) so it doesn't
  matter what tag precedes it.
- Because of the local normalization of MEMMs,  $P(s_i = TO | s_{i-1}, x_1, \dots, x_i = \text{"to"}, \dots, x_n)$  will always be 1 when  $x_i = \text{"to"}$  regardless of the value of  $s_{i-1}$  (MD or NN).
- That means our prediction for "to" can't help us disambiguate "will".
- We lose the information that MD + TO sequences rarely happen.
- As a consequence: a MEMMS would likely label the first word to "MD".
- CRF overcomes this issue by doing a global normalization: it considers the score
  of the whole sequence before normalizing to make it a probability distribution.

#### Links

- http://people.ischool.berkeley.edu/~dbamman/nlpF18/slides/ 11\_memm\_crf.pdf
- http://people.ischool.berkeley.edu/~dbamman/nlpF18/slides/ 12\_neural\_sequence\_labeling.pdf
- https://www.depends-on-the-definition.com/ sequence-tagging-lstm-crf/
- https://www.quora.com/
   What-are-the-pros-and-cons-of-these-three-sequence-models-MaxEnt
- https:
  //people.cs.umass.edu/~mccallum/papers/crf-tutorial.pdf
- http://www.davidsbatista.net/blog/2017/11/13/Conditional\_ Random\_Fields

Questions?

Thanks for your Attention!

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