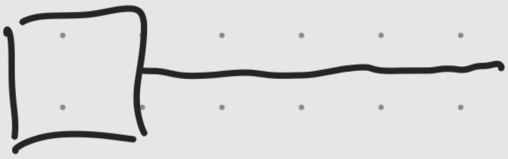
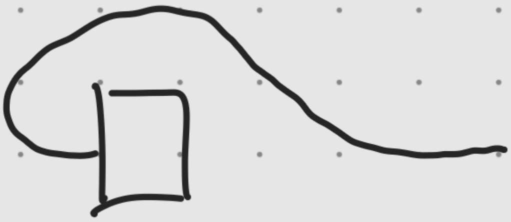
2D: $R = \exp(-e_{12}\pi)^{\lambda}$



$\lambda = 0$

$\lambda = 0{,}25$

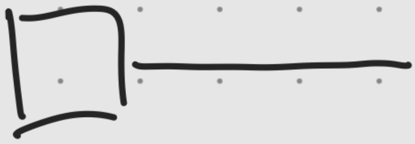


$\lambda = 0{,}5-$

$\lambda = 0{,}5+$

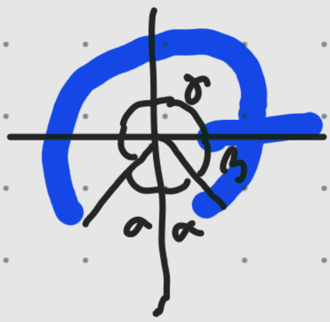
$\lambda = 0{,}75$

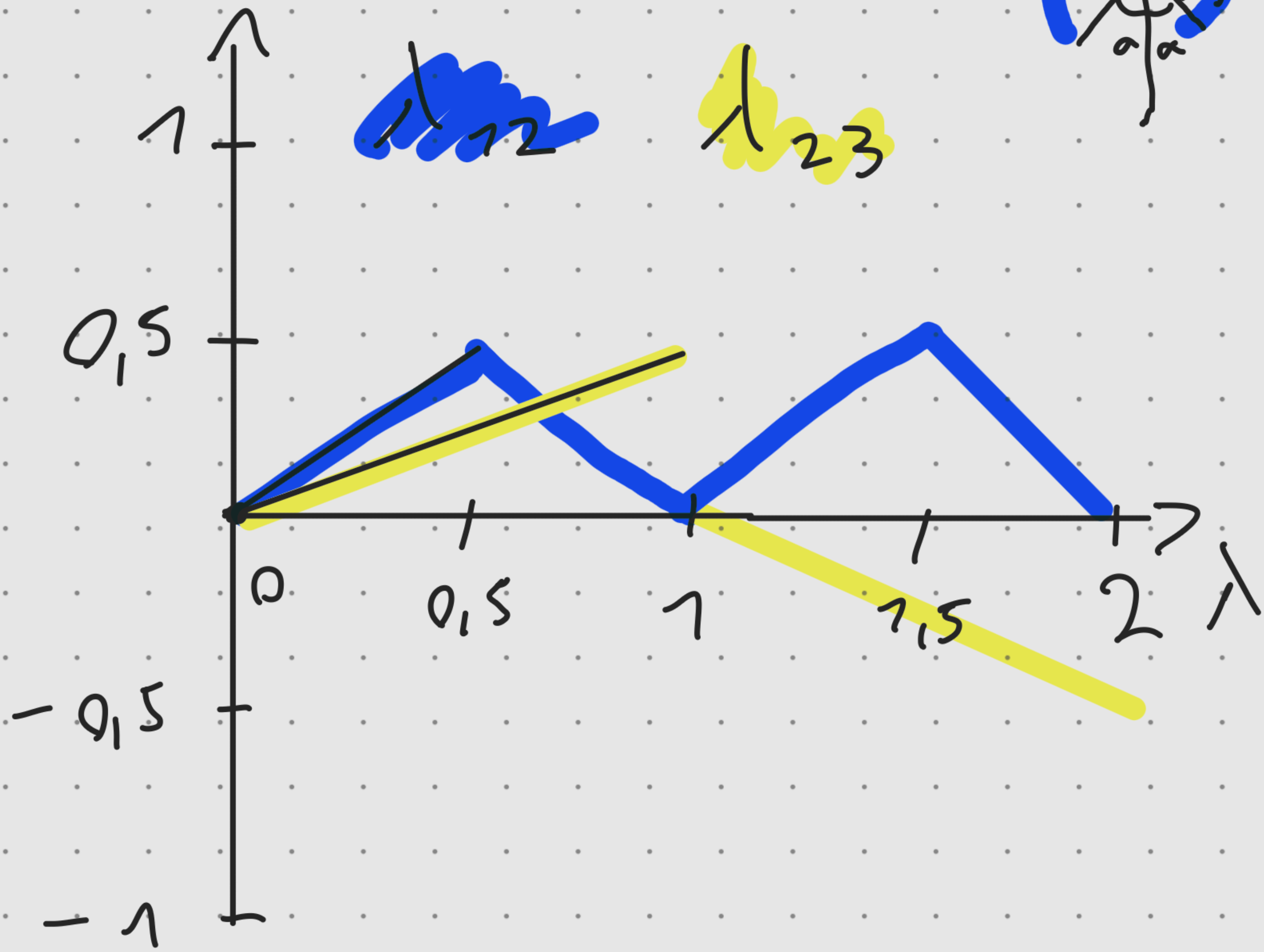
$\lambda' = -0{,}25$

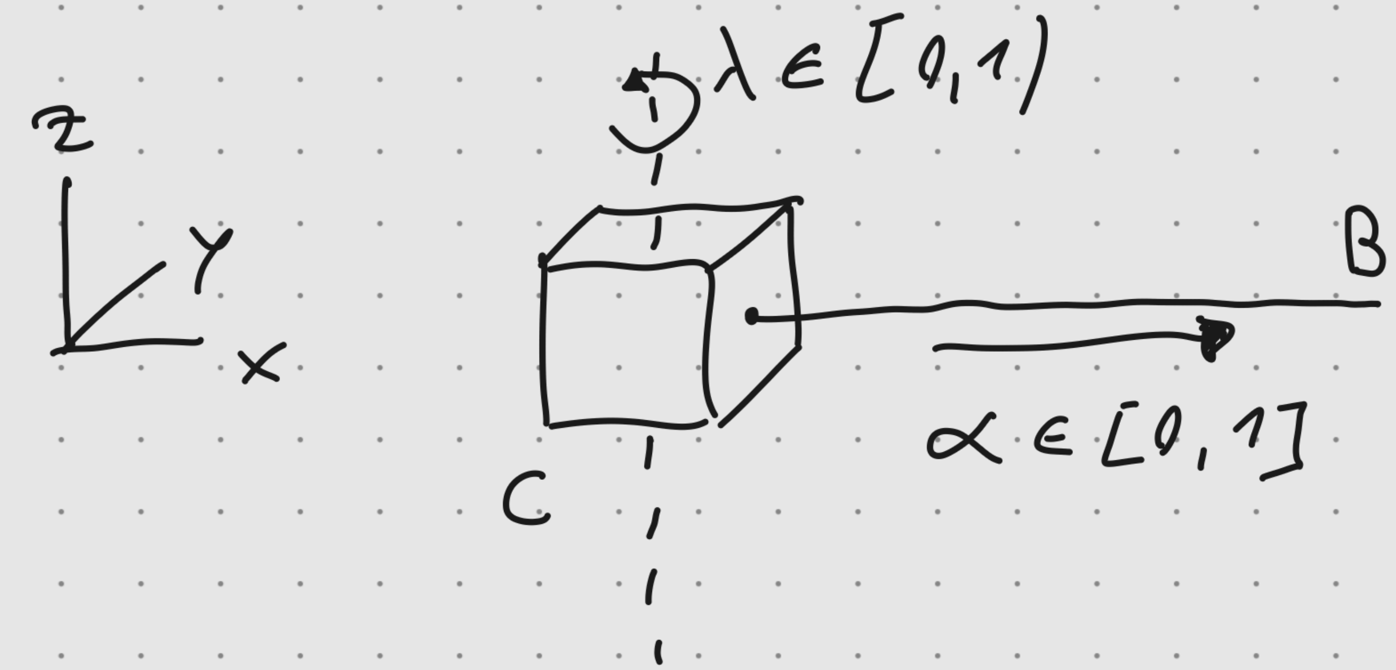


$\lambda = 1$

$\lambda' = 0$







$$C \longrightarrow R_z(\lambda)\, C\, \tilde{R}_z(\lambda)$$

$$B \longrightarrow R_z(\lambda,\alpha)\, B(\alpha)\, \tilde{R}_z(\lambda,\alpha)$$

2D:

$$S(\lambda,\alpha) = R_z(\lambda,\alpha)$$

$$= R_z(\lambda_z)$$

$$\lambda_z = (1-\alpha)\cdot \begin{cases} \lambda & 0 \le \lambda < 0{,}5 \\ \lambda - 1 & 0{,}5 \le \lambda < 1 \end{cases}$$

Flipping sides

Flipping sides with R_x

$$S(\lambda, \alpha)$$

$$= R_x(\lambda_x) \cdot R_z(\lambda_z)$$

$$\lambda_z = (1-\alpha) \cdot \begin{cases} \lambda & 0 \leq \lambda < 0{,}5 \\ 1-\lambda & 0{,}5 \leq \lambda \leq 1 \\ \lambda - 1 & 1 < \lambda \leq 1{,}5 \\ 2-\lambda & 1{,}5 < \lambda < 2 \end{cases}$$

$$\lambda_x = \frac{1}{2} \begin{cases} \lambda & 0 \leq \lambda < 1 \\ 1-\lambda & 1 \leq \lambda < 2 \end{cases}$$

2 Possibilities:

Flipping through positive z with λ_x

" negative z with λ_x

$\longrightarrow$ 4π symmetry

Correcting phases:

$$R_x = e^{-e_{23}\pi\lambda_x}, \quad R_z = e^{-e_{12}\pi\lambda_z}$$

$$S(\lambda, \alpha)$$

$$e^{\pm i\pi} = -1$$
$$e^{\pm 2\pi i} = 1$$

$0 \le \lambda < 0,5$

$$= e^{-e_{23}\pi\frac{1}{2}\lambda}\, e^{-e_{12}\pi(1-\alpha)\lambda}\ ]\ \lambda - \alpha\lambda$$

$0,5 \le \lambda \le 1$

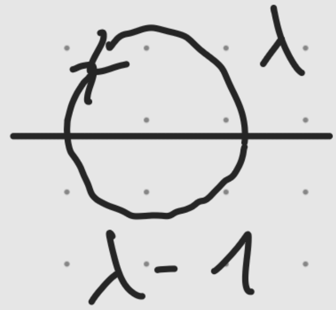
$$= e^{-e_{23}\frac{\pi}{2}\lambda}\, e^{-e_{12}\pi\overbrace{(1-\alpha)(1-\lambda)}^{1-\alpha-\lambda+\alpha\lambda}}$$

$$= e^{-e_{23}\frac{\pi}{2}\lambda}\, e^{-e_{12}\pi}$$

$1 < \lambda \le 1,5$

$$= e^{-e_{23}\frac{\pi}{2}(1-\lambda)}\, e^{-e_{12}\pi(1-\alpha)(\lambda-1)}$$

$$= e^{-e_{23}\frac{\pi}{2}}\, e^{e_{23}\frac{\pi}{2}\lambda} \qquad e^{-e_{12}\pi(1-\alpha)}$$



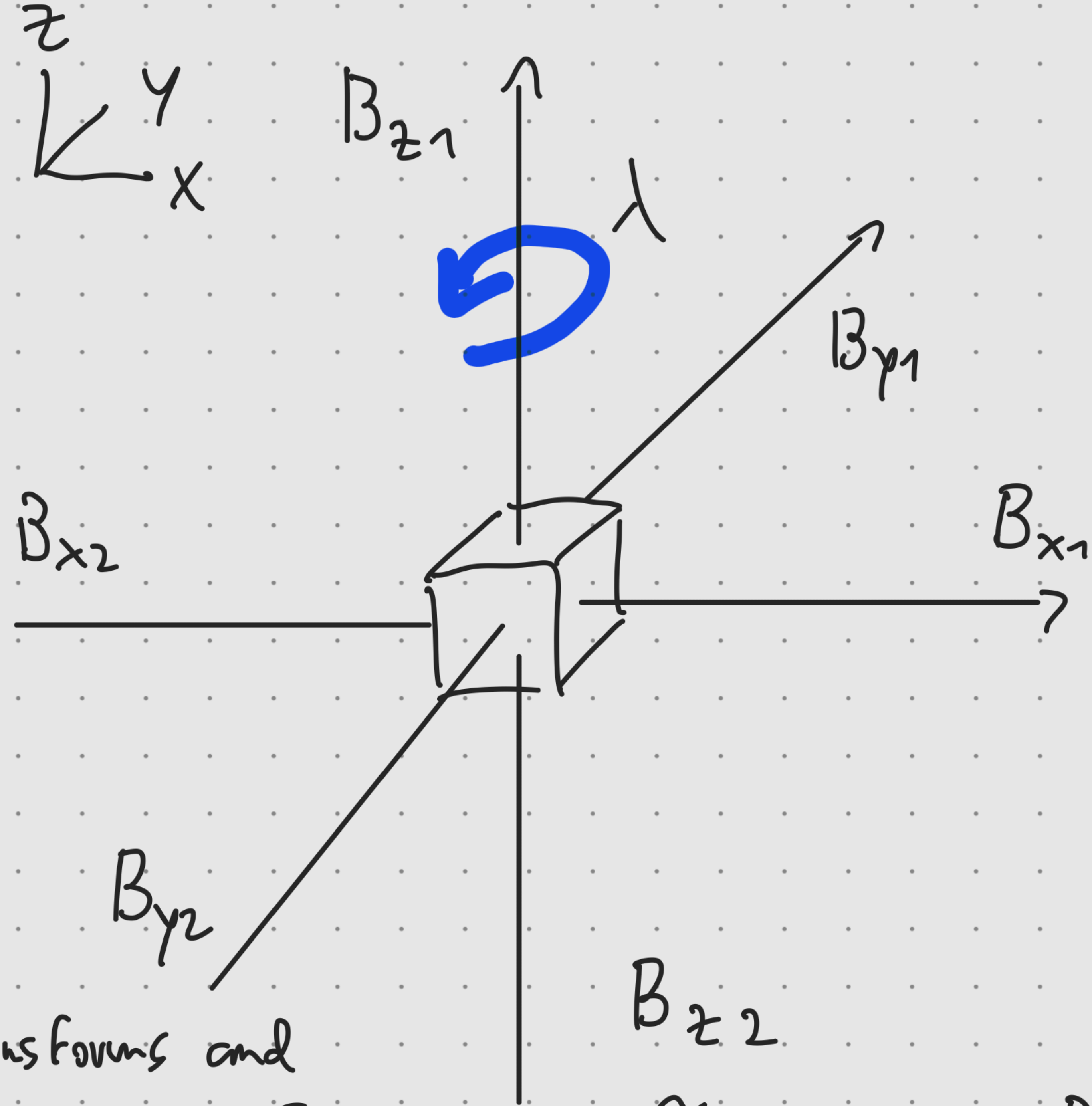
$e^{i\pi\lambda}$

$$e^{i\pi(\lambda-1)}$$
$$= e^{i\pi\lambda}\, e^{-i\pi} = -e^{i\pi\lambda}$$

$$
\begin{aligned}
[e_{12}, e_{23}] &= e_{12}e_{23} - e_{23}e_{12} \\
&= e_{13} - e_{31} \\
&= e_{13} + e_{13} = 2e_{13}
\end{aligned}
$$

$$
\begin{aligned}
e_{12}e_{23} &= e_{13} \\
&= -e_{31} \\
&= -e_{32}e_{21} \\
&= -e_{23}e_{12}
\end{aligned}
$$

$$
\begin{aligned}
[e_{12}, e_{13}] &= e_{12}e_{13} - e_{13}e_{12} \\
&= -e_{2\cancel{11}3} + e_{3\cancel{11}2} \\
&= -e_{23} - e_{23} = -2e_{23}
\end{aligned}
$$



Transforms and Phases:

$$S_{\lambda+1}\, B_{x1}\, \tilde{S} = B_{x1}$$

$$R_z\left(\tfrac{1}{2}\right) S_{\lambda}\, B_{x1}\, \tilde{S}\, \tilde{R}_z = B_{x2}$$

$$R_z\left(\tfrac{1}{4}\right) S_{\lambda+\frac{1}{2}}\, B_{x1}\, \tilde{S}\, \tilde{R}_z = B_{y1}$$

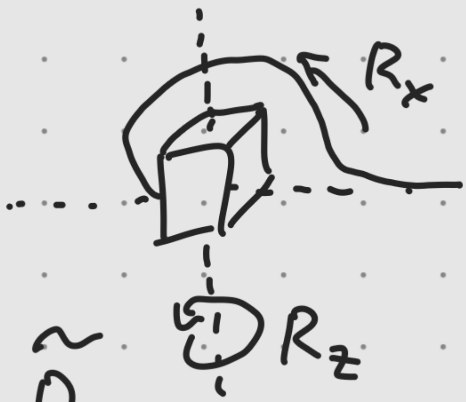
$$R_z\left(\tfrac{3}{4}\right) S_{\lambda+\frac{3}{2}}\, B_{x1}\, \tilde{S}\, \hat{R}_z = B_{y2}$$

$$R_z\left(\tfrac{1}{4}+\tfrac{1}{2}\lambda\right) R_y\left(\tfrac{1}{4}\right) S_{\frac{1}{2}}\, B_{x1}\, \tilde{S}\, \tilde{R}_y\, \tilde{R}_z(\lambda) = B_{z1}$$

$$R_z\left(\tfrac{1}{4}+\tfrac{1}{2}\lambda\right) R_y\left(\tfrac{1}{4}\right) R_z\left(\tfrac{1}{2}\right) S_{\frac{3}{2}}\, B_{x1}\, \tilde{S}\, \tilde{R}_z\, \tilde{R}_y\, \tilde{R}_z(\lambda) = B_{z2}$$

↳ general form of S ?

such that: $S \ldots R \, B_{x1} \, \tilde{R} \ldots \tilde{S}$?



$$S_z = R_x R_z$$

$$B_{ij}(\lambda) = \underbrace{R \, S_z}_{} \, B_{x1} \, \underbrace{\tilde{S}_z \, \tilde{R}}_{}$$

$$B_{ij}(\lambda) = S \underbrace{R \, B_{x1} \, \tilde{R}}_{B_{ij}} \tilde{S} = S \, B_{ij} \, \tilde{S}$$

$$S R = R S_z$$

$$S = R S_z \tilde{R}$$

$$= R R_x R_z \tilde{R}$$

i	j	R	λ
x	1	1	$\lambda + 1$
x	2	$1 \cdot R_z\left(\frac{1}{2}\right)$	λ
y	1	$R_z\left(\frac{1}{4}\right)$	$\lambda + \frac{1}{2}$
y	2	$R_z\left(\frac{1}{4}\right) \cdot R_z\left(\frac{1}{2}\right)$	$\lambda + \frac{3}{2}$
z	1	$R_z\left(\frac{\lambda}{2}\right) R_y\left(\frac{1}{4}\right) R_z\left(\frac{1}{2}\right)$	$\frac{3}{2}$
z	2	$R_z\left(\frac{\lambda}{2} + \frac{1}{4}\right) R_y\left(\frac{1}{4}\right)$	$\frac{1}{2}$

$$\hat{=} \; R_z\left(\frac{1}{4}\right) R_y\left(\frac{1}{4}\right) \; \hat{=}$$

$$R_z\left(\frac{3}{4}\right) R_y\left(\frac{3}{4}\right)$$

i	j	R	λ
x	1	1	$\lambda+1$
x	2	$\cancel{R_z\left(\frac{1}{2}\right)}$	λ
y	1	$R_z\left(\frac{1}{4}\right)$	$\lambda+\frac{1}{2}$
y	2	$R_z\left(\frac{3}{4}\right)$	$\lambda+\frac{3}{2}$
z	1	$R_z\left(\frac{\lambda}{2}\right) R_x\left(\frac{3}{4}\right) R_z\left(\frac{3}{4}\right)$	$\frac{3}{2}$
z	2	$R_z\left(\frac{\lambda}{2}\right) R_x\left(\frac{3}{4}\right) R_z\left(\frac{1}{4}\right)$	$\frac{1}{2}$

$\to$ only positive dir $j=1$ matter (other: $R_z\left(\frac{1}{2}\right)$)

i	R	λ
x	1	$\lambda+1$
y	$R_z\left(\frac{1}{4}\right)$	$\lambda+\frac{1}{2}$
z	$R_z\left(\frac{\lambda}{2}\right) R_x\left(\frac{3}{4}\right) R_z\left(\frac{3}{4}\right)$	$\frac{3}{2}$

$$B_x(\lambda) = S_z(\lambda+1)\, B_x\, \tilde{S}_z(\lambda+1)$$

$$B_y(\lambda) = R_z\left(\tfrac{1}{4}\right) S_z\left(\lambda+\tfrac{1}{2}\right) B_x \;\tilde{\ldots}$$

$$B_z(\lambda) = R_z\left(\tfrac{\lambda}{2}\right) R_x\left(\tfrac{3}{4}\right) R_z\left(\tfrac{3}{4}\right) S\left(\tfrac{3}{2}\right) B_x \;\tilde{\ldots}$$

$$R\, S\, B\, \tilde{S}\, \tilde{R} \longrightarrow S' R\, B\, \tilde{R}\, \tilde{S}'$$

$$S_z' R = R\, S_z$$

$$S_z' = R\, S_z\, \tilde{R}$$

$$= R\, R_x\, R_z\, \tilde{R}$$

$$S_z^x = S_z(\lambda+1)$$

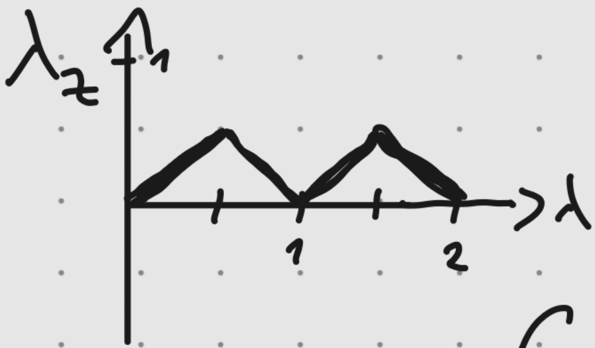
$$S_z^y = R_z\left(\tfrac{1}{4}\right) \quad S_z\left(\lambda+\tfrac{1}{2}\right) \tilde{\ldots}$$

$$S_z^z = R_z\left(\tfrac{\lambda}{2}\right) R_x\left(\tfrac{3}{4}\right) R_z\left(\tfrac{3}{4}\right) S_z\left(\tfrac{3}{2}\right) \tilde{\ldots}$$

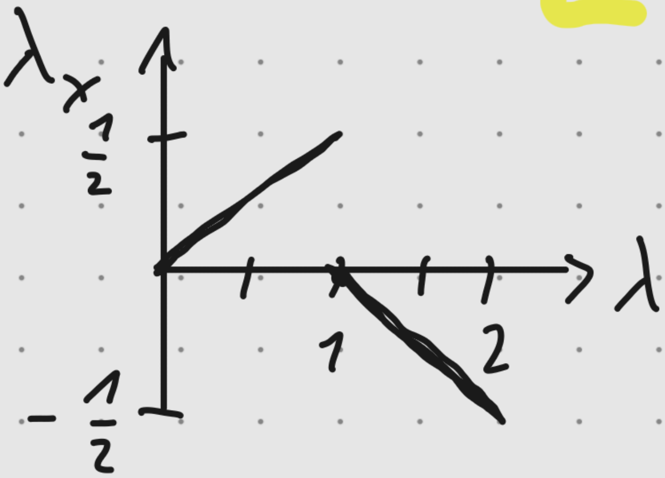
$$S_z(\lambda, \alpha)$$

$$= R_x(\lambda_x) \cdot R_z(\lambda_z)$$

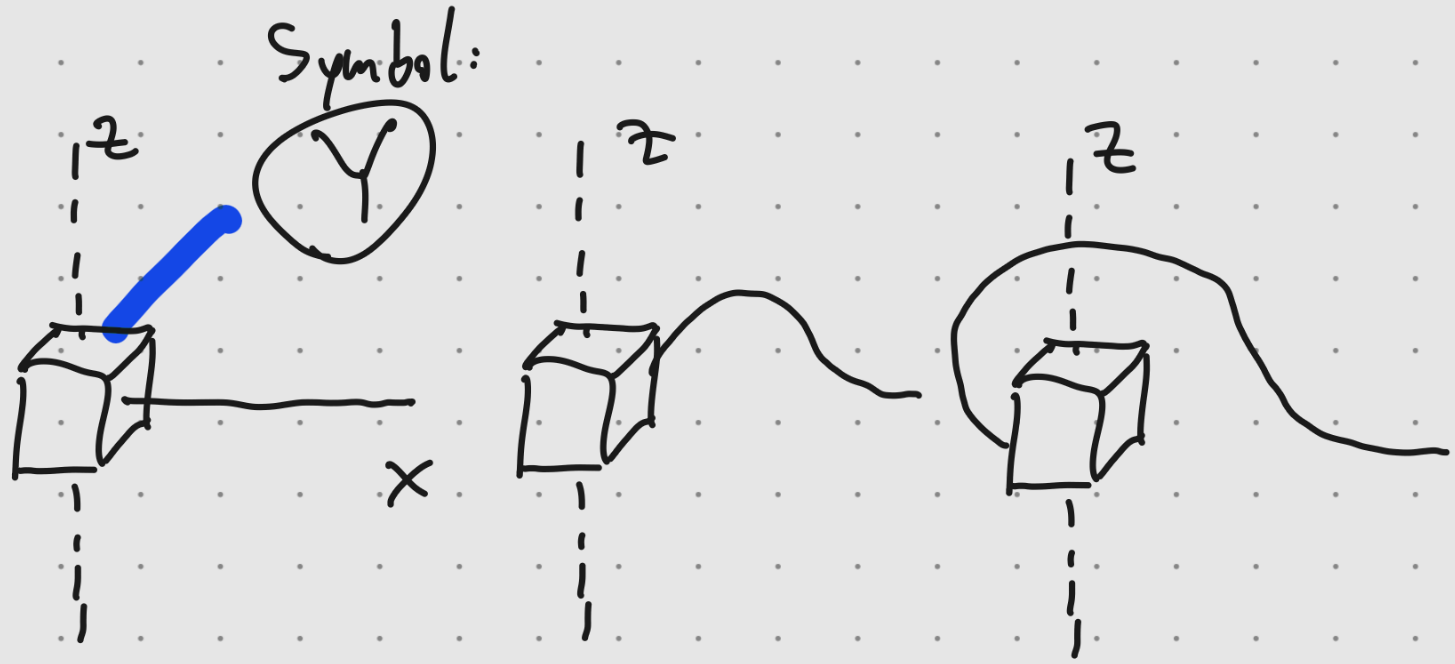
$$\lambda_z = (1-\alpha) \cdot \begin{cases} \lambda & 0 \le \lambda < 0{,}5 \\ 1-\lambda & 0{,}5 \le \lambda \le 1 \\ \lambda - 1 & 1 < \lambda \le 1{,}5 \\ 2-\lambda & 1{,}5 < \lambda < 2 \end{cases}$$

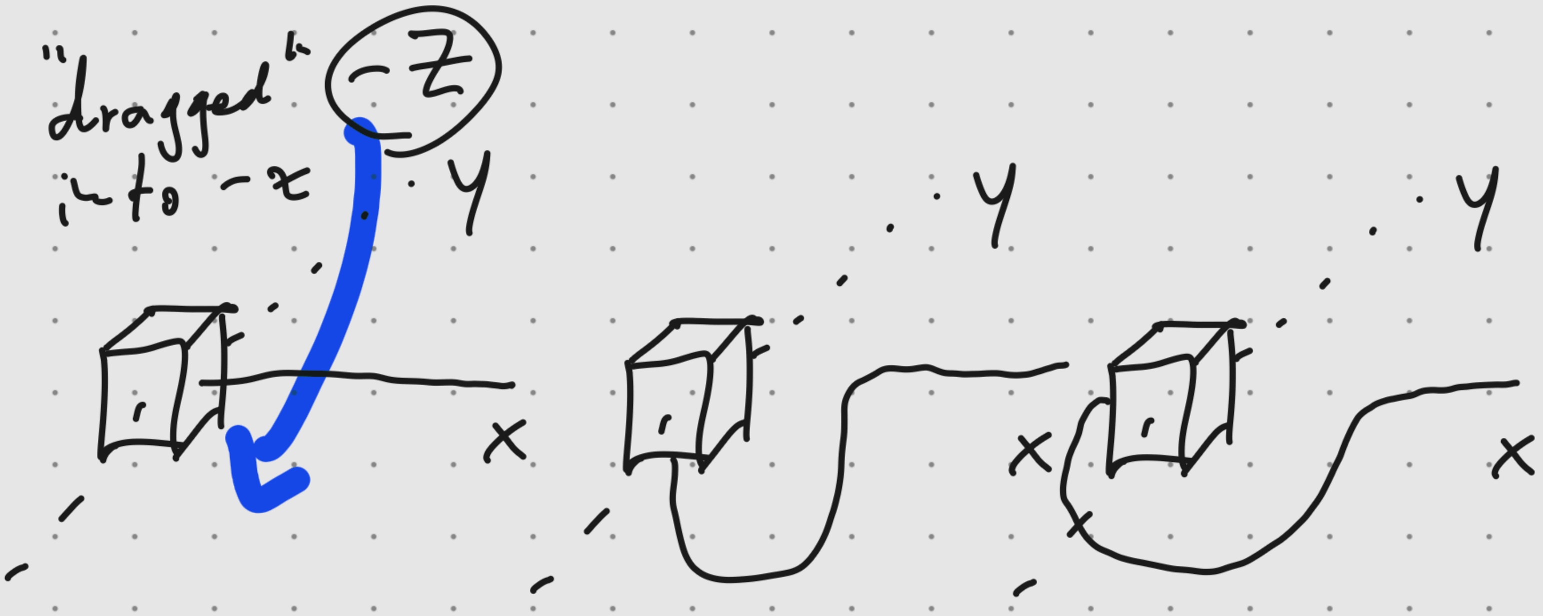


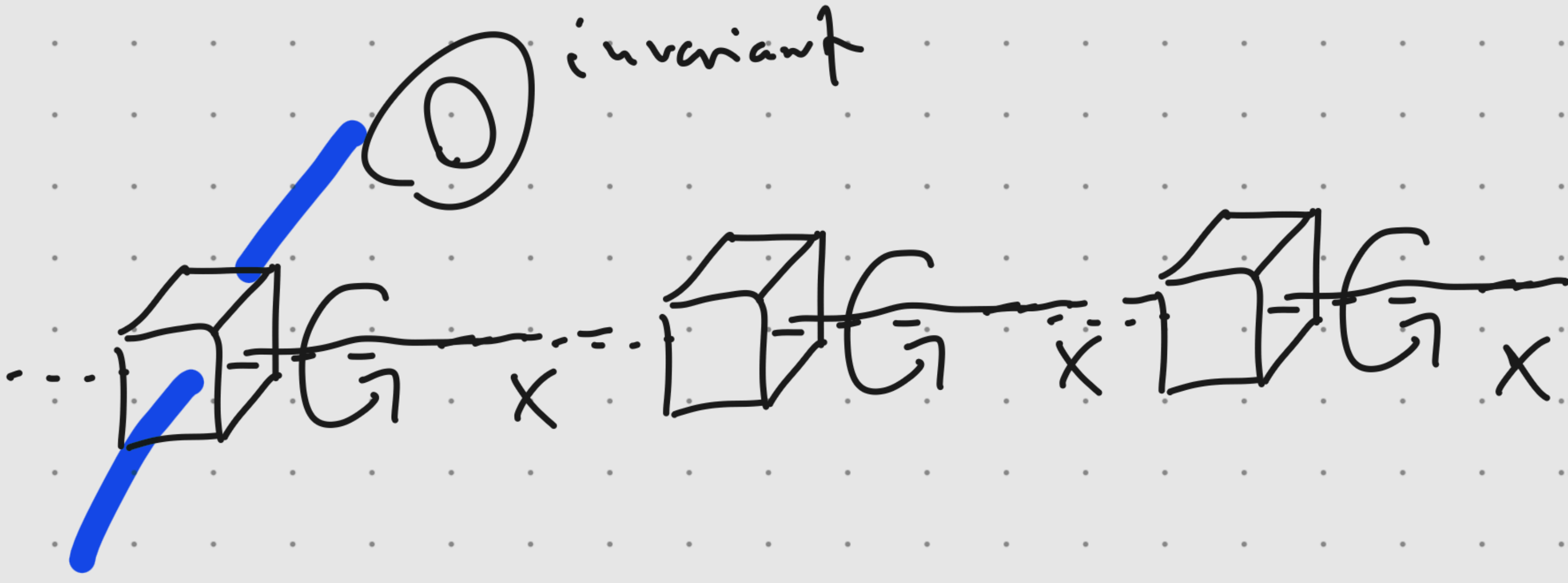
$$\lambda_x = \frac{1}{2} \begin{cases} \lambda & 0 \le \lambda < 1 \\ 1-\lambda & 1 \le \lambda < 2 \end{cases}$$



3 Cases for x:







Symbols:

Rotation \ Axis	x	y	z
x	0	z	−Y
y	−z	0	X
z	Y	−X	0

$$S^{x\;(=\text{axis})}_{z\;(=\text{rot})} = S_z$$

ax \ rot	1	2	3
1	0	-3	2
2	3	0	-1
3	-2	1	0

$$S^{Y}_{z} = S^{x}_{z}\begin{pmatrix} x \to Y \\ Y \to -X \end{pmatrix}$$

$$S^{z}_{z} = R\left(\frac{\lambda}{2}\right) \quad \text{invar}$$

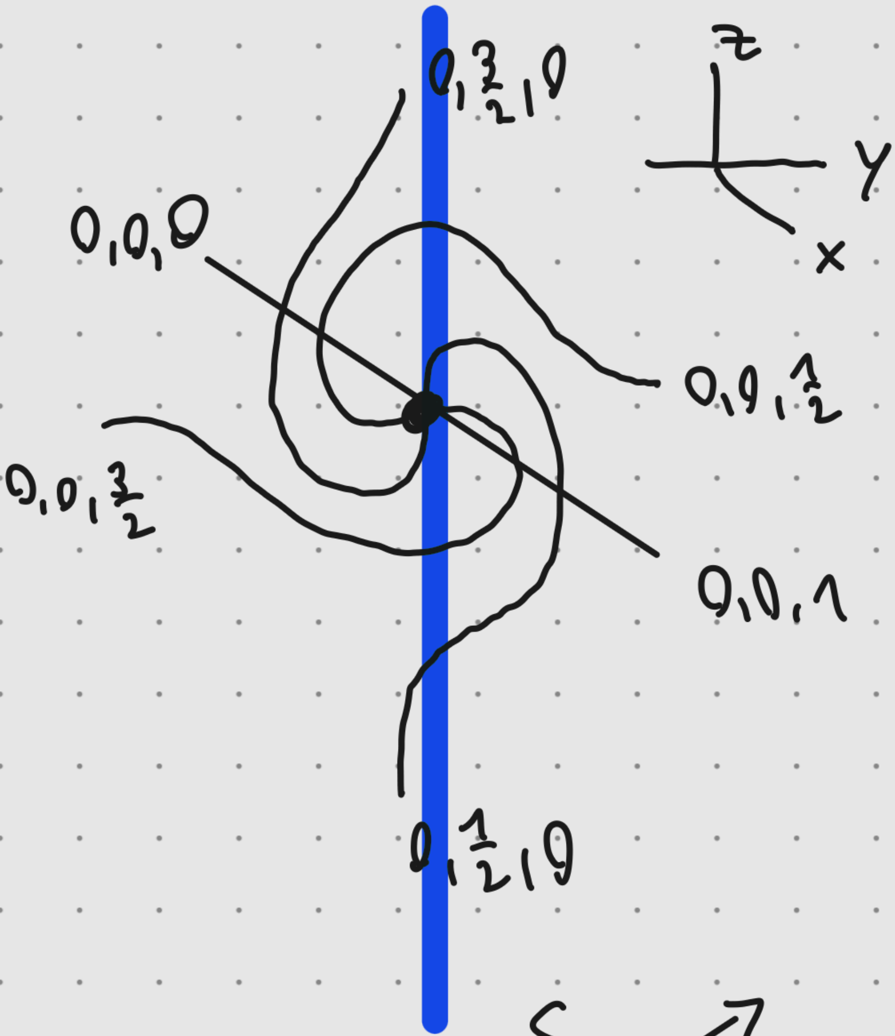
$$S_y^x = S_z^x \begin{pmatrix} z \to y \\ y \to -z \end{pmatrix}$$

$$S_x^x = R\left(\frac{\lambda}{2}\right) \quad \text{invar}$$

$$S_y^z = S_z^y \begin{pmatrix} -x \to x \\ z \to y \\ y \to z \end{pmatrix}$$

e_i

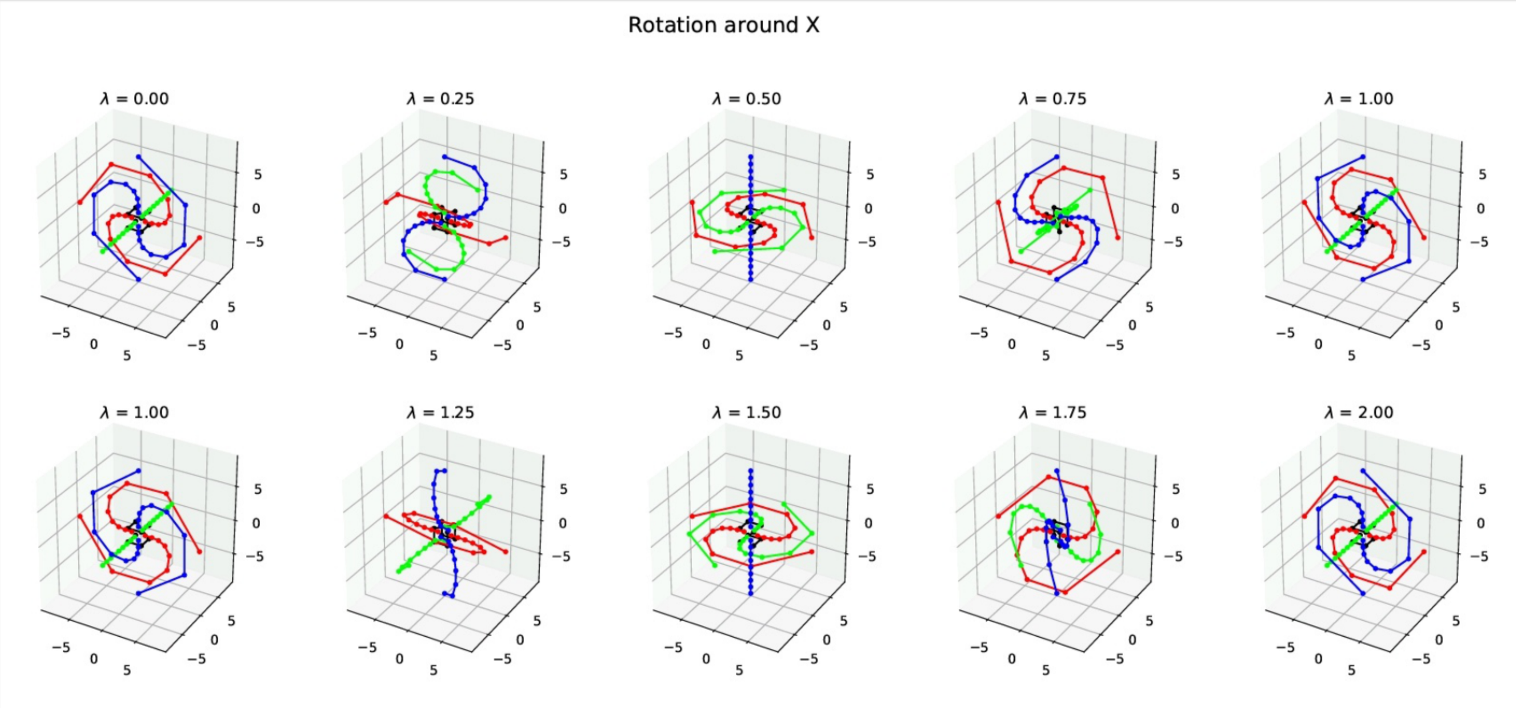
$$\begin{array}{c|ccc} & & \text{rot} & \\ & 1 & 2 & 3 \\ \hline \text{ax } 1 & 0 & -3 & 2 \\ 2 & 3 & 0 & -1 \\ 3 & -2 & 1 & 0 \end{array} \;\hat{=}\; e_j \begin{array}{c|ccc} & e_1 & e_2 & e_3 \\ \hline e_1 & & & \\ e_2 & & -\frac{e_i \wedge e_j}{e_{123}} & \\ e_3 & & = e_{ij}^* & \end{array}$$

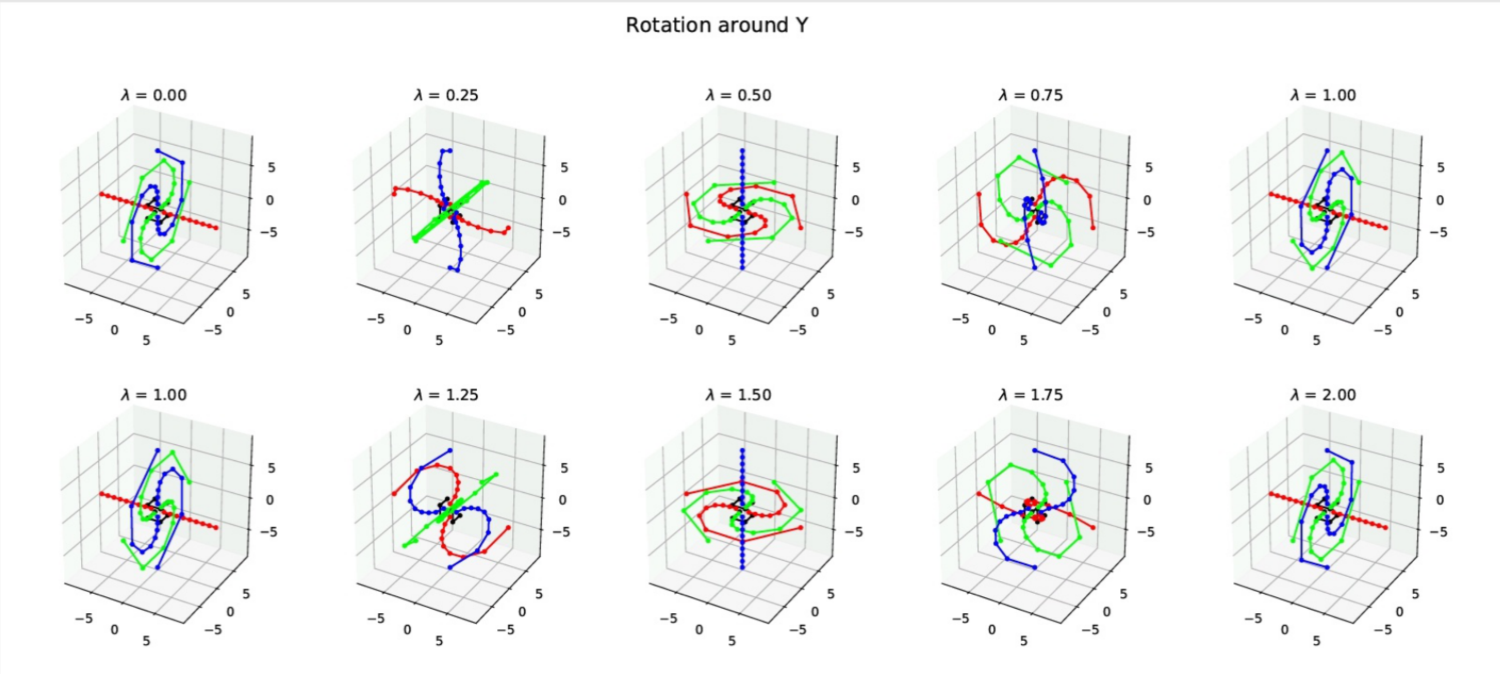


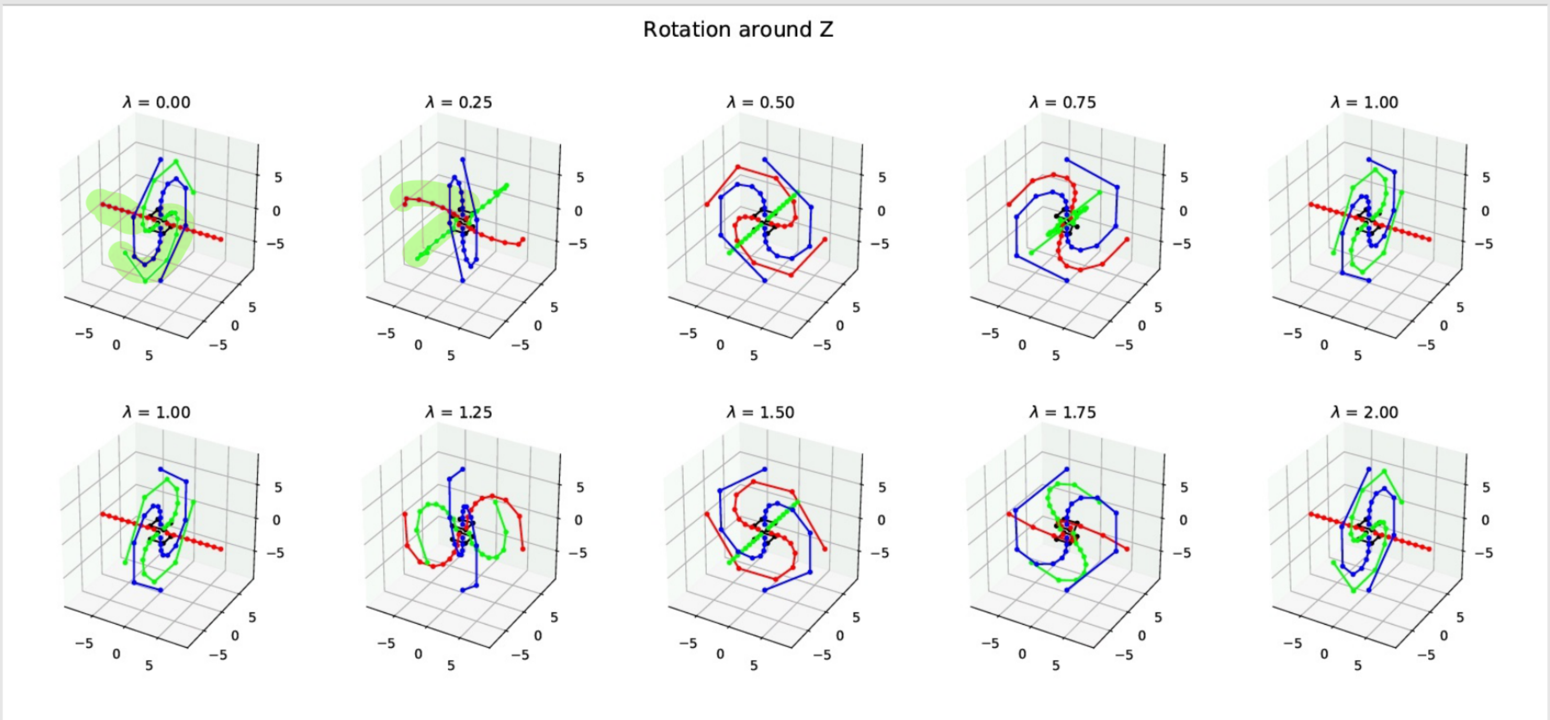
$\lambda_x, \lambda_y, \lambda_z$

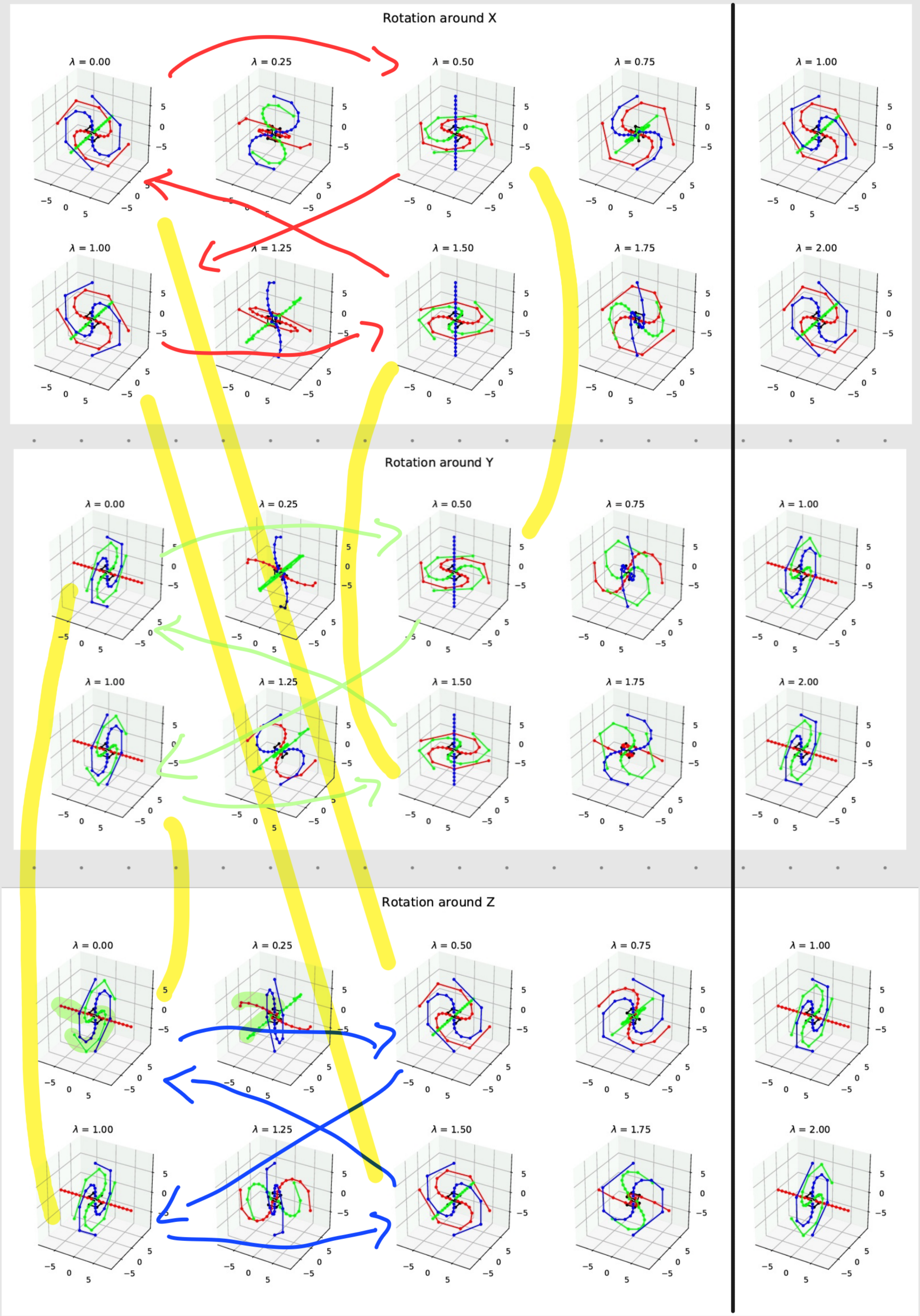
S_x, S_y, S_z

$$
\begin{tikzcd}
 & & & \\
0,0,0 \arrow[ur, "S_y"] \arrow[rr, "S_z"] & & 0,0,\frac{1}{2} \arrow[ur, "S_x"] \arrow[d, "S_z"] & \\
0,0,\frac{3}{2} \arrow[u, "S_z"] \arrow[ur, "S_x"] & & 0,0,1 \arrow[ll, "S_z"] \arrow[ur, "S_y"] &
\end{tikzcd}
$$

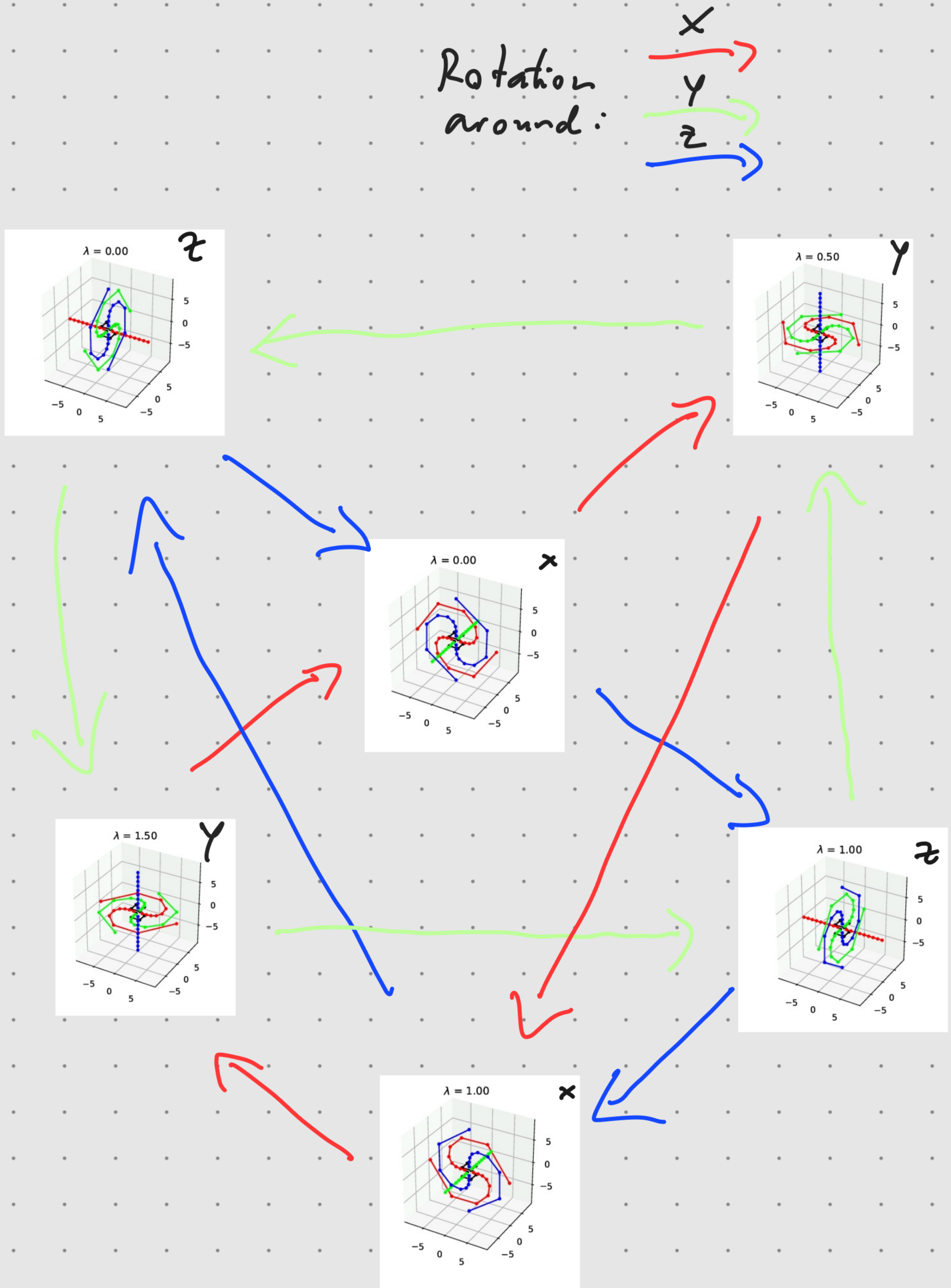

Rotation around X
λ = 0.00
λ = 0.25
λ = 0.50
λ = 0.75
λ = 1.00
λ = 1.00
λ = 1.25
λ = 1.50
λ = 1.75
λ = 2.00

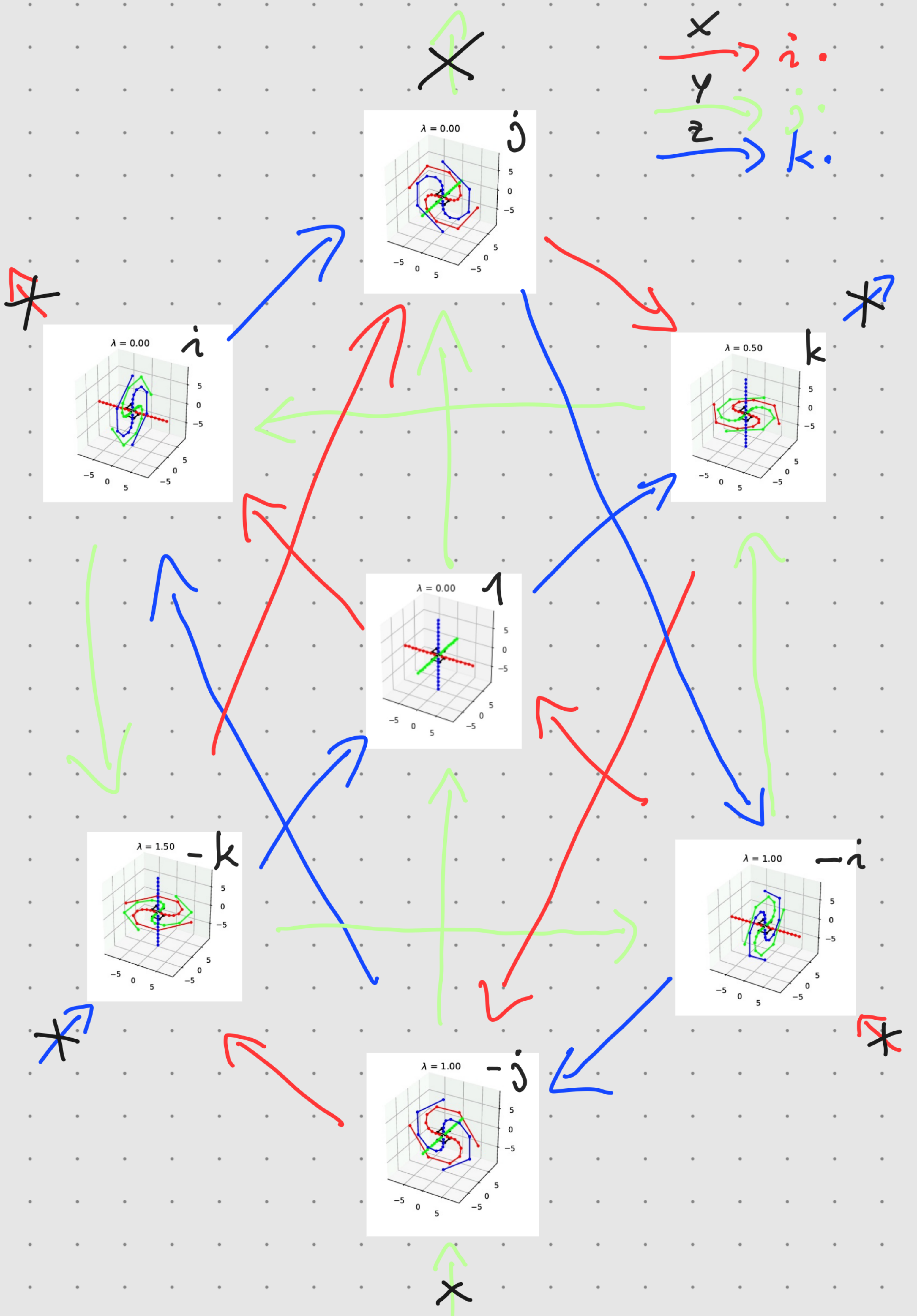

Rotation around Y
λ = 0.00
λ = 0.25
λ = 0.50
λ = 0.75
λ = 1.00
λ = 1.00
λ = 1.25
λ = 1.50
λ = 1.75
λ = 2.00


Rotation around Z
λ = 0.00
λ = 0.25
λ = 0.50
λ = 0.75
λ = 1.00
λ = 1.00
λ = 1.25
λ = 1.50
λ = 1.75
λ = 2.00



Rotation around X
λ = 0.00
λ = 0.25
λ = 0.50
λ = 0.75
λ = 1.00
λ = 1.00
λ = 1.25
λ = 1.50
λ = 1.75
λ = 2.00
Rotation around Y
λ = 0.00
λ = 0.25
λ = 0.50
λ = 0.75
λ = 1.00
λ = 1.00
λ = 1.25
λ = 1.50
λ = 1.75
λ = 2.00
Rotation around Z
λ = 0.00
λ = 0.25
λ = 0.50
λ = 0.75
λ = 1.00
λ = 1.00
λ = 1.25
λ = 1.50
λ = 1.75
λ = 2.00


Rotation around:
x
y
z
λ = 0.00
z
λ = 0.50
y
λ = 0.00
x
λ = 1.50
y
λ = 1.00
z
λ = 1.00
x



x → i
y → j
z → k
λ = 0.00
j
λ = 0.00
i
λ = 0.50
k
λ = 0.00
1
λ = 1.50
-k
λ = 1.00
-i
λ = 1.00
-j

Phases

R_{plane} $\quad S^{\text{axis}}_{\text{rotation plane}}$

X rotation:

X $\quad R_{23}(\frac{\lambda}{2}) S^{x}_{12}(\frac{3}{2})$ $\quad = R_{23}(\frac{\lambda}{2}+\frac{1}{4}) S^{x}_{12}(\frac{1}{2})$

$\quad R_{23}(\frac{\lambda}{2}) S^{-x}_{12}(\frac{1}{2})$ $\quad R_{23}(\frac{\lambda}{2}+\frac{1}{4}) S^{-x}_{12}(\frac{3}{2})$

Y $\quad S^{Y}_{23}(\lambda+1)$

$\quad S^{-Y}_{23}(\lambda+0)$

Z $\quad S^{Z}_{23}(\lambda+\frac{1}{2})$

$\quad S^{-Z}_{23}(\lambda+\frac{3}{2})$

Y rotation:

X $\quad S^{X}_{31}(\lambda+0)$

$\quad S^{-X}_{31}(\lambda+1)$

Y $\quad R_{31}(\frac{\lambda}{2}) S^{Y}_{12}(\frac{3}{2})$ $\quad R_{31}(\frac{\lambda}{2}+\frac{1}{4}) S^{Y}_{23}(\frac{3}{2})$

$\quad R_{31}(\frac{\lambda}{2}) S^{-Y}_{12}(\frac{1}{2})$ $\quad = R_{31}(\frac{\lambda}{2}+\frac{1}{4}) S^{Y}_{23}(\frac{1}{2})$

Z $\quad S^{Z}_{31}(\lambda+\frac{1}{2})$

$\quad S^{-Z}_{31}(\lambda+\frac{3}{2})$

Z rotation:

$\lambda \to \lambda+1$

X $\quad S^{X}_{12}(\lambda+1)$ $\quad S^{X}_{12}(\lambda+0)$

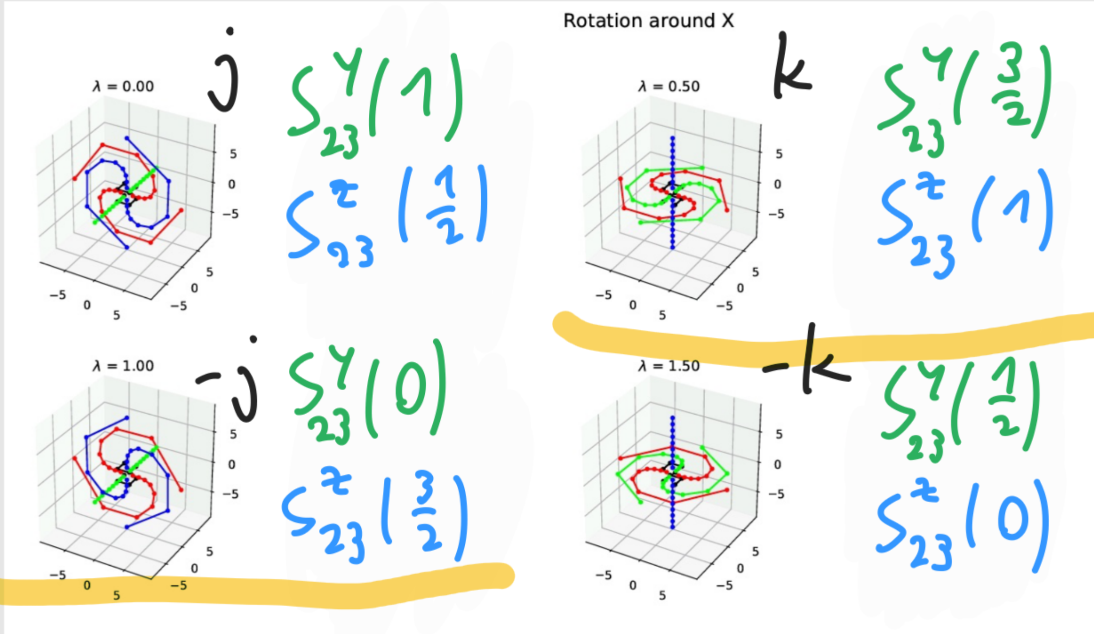
$\quad S^{-X}_{12}(\lambda+0)$ $\quad S^{-X}_{12}(\lambda+1)$

Y $\quad S^{Y}_{12}(\lambda+\frac{1}{2})$ $\quad S^{Y}_{12}(\lambda+\frac{3}{2})$

$\quad S^{-Y}_{12}(\lambda+\frac{3}{2})$ $\quad S^{Y}_{12}(\lambda+\frac{1}{2})$

Z $\quad R_{12}(\frac{\lambda}{2}) S^{Z}_{31}(\frac{3}{2})$ $\quad R_{12}(\frac{\lambda}{2}+\frac{1}{2}) S^{Z}_{31}(\frac{3}{2})$ $\quad = R_{12}(\frac{\lambda}{2}+\frac{3}{4}) S^{Z}_{31}(\frac{1}{2})$

$\quad R_{12}(\frac{\lambda}{2}) S^{-Z}_{31}(\frac{1}{2})$ $\quad R_{12}(\frac{\lambda}{2}+\frac{1}{2}) S^{-Z}_{31}(\frac{1}{2})$ $\quad R_{12}(\frac{\lambda}{2}+\frac{3}{4}) S^{-Z}_{31}(\frac{3}{2})$

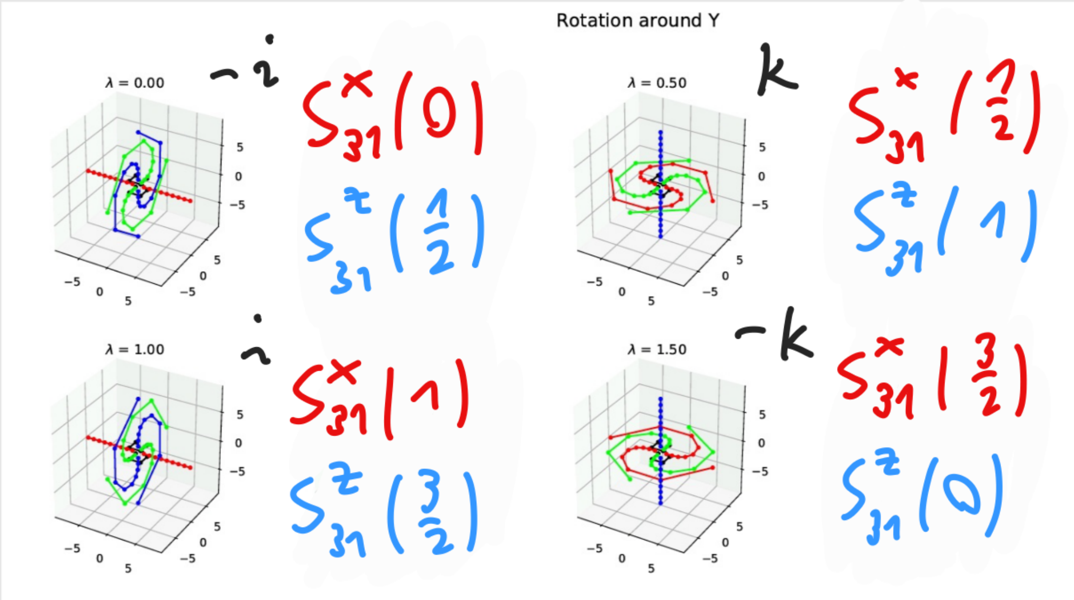


j $S^{Y}_{23}(1)$ $S^{Z}_{23}(\frac{1}{2})$

k $S^{Y}_{23}(\frac{3}{2})$ $S^{Z}_{23}(1)$

$-j$ $S^{Y}_{23}(0)$ $S^{Z}_{23}(\frac{3}{2})$

$-k$ $S^{Y}_{23}(\frac{1}{2})$ $S^{Z}_{23}(0)$

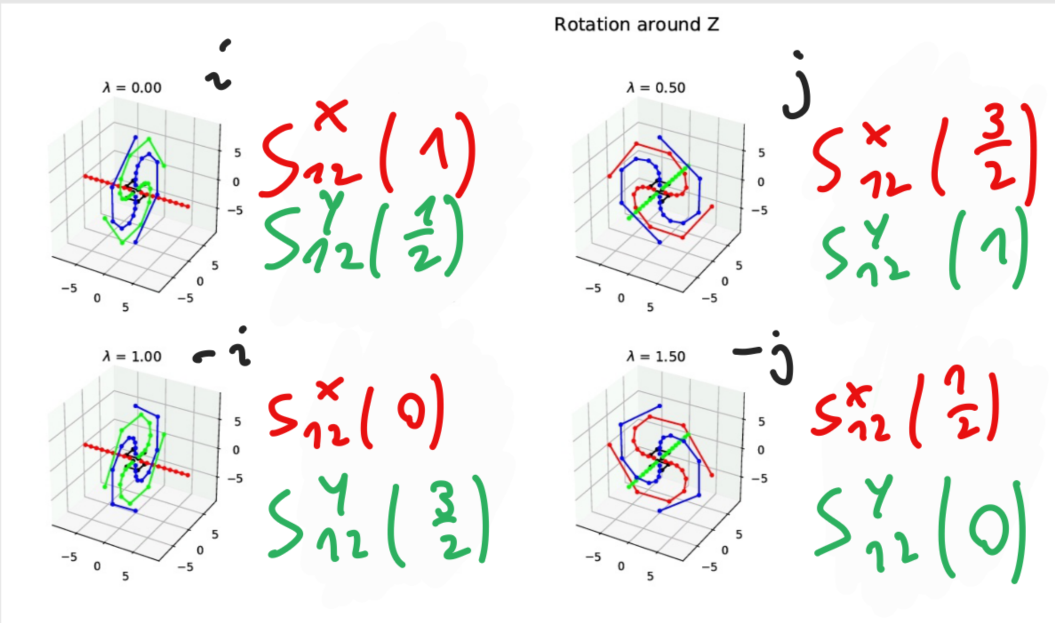


$-i$ $S^{X}_{31}(0)$ $S^{Z}_{31}(\frac{1}{2})$

k $S^{X}_{31}(\frac{1}{2})$ $S^{Z}_{31}(1)$

i $S^{X}_{31}(1)$ $S^{Z}_{31}(\frac{3}{2})$

$-k$ $S^{X}_{31}(\frac{3}{2})$ $S^{Z}_{31}(0)$

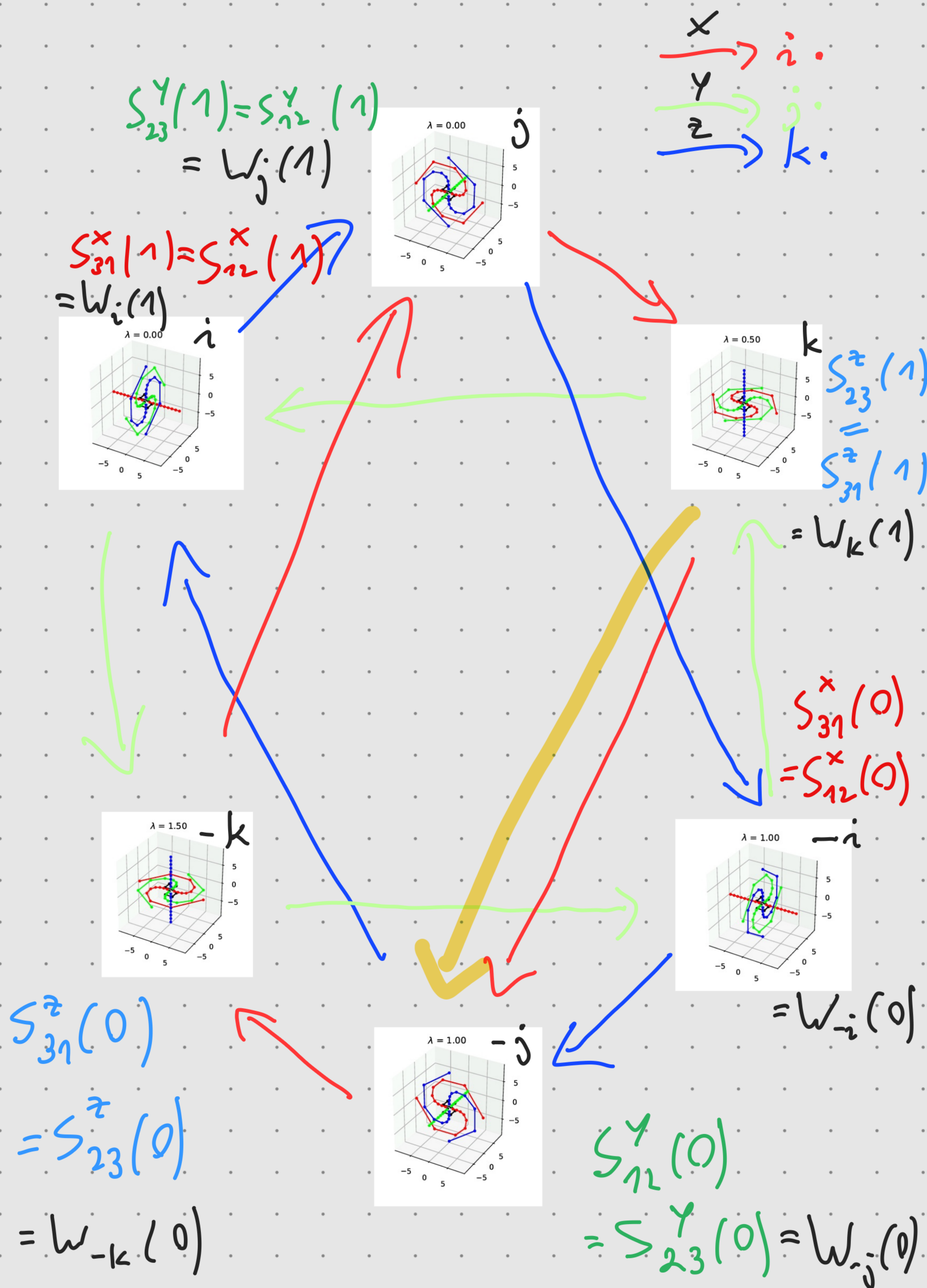


i $S^{X}_{12}(1)$ $S^{Y}_{12}(\frac{1}{2})$

j $S^{X}_{12}(\frac{3}{2})$ $S^{Y}_{12}(1)$

$-i$ $S^{X}_{12}(0)$ $S^{Y}_{12}(\frac{3}{2})$

$-j$ $S^{X}_{12}(\frac{1}{2})$ $S^{Y}_{12}(0)$



$x \to i$
$y \to j$
$z \to k$
$S^y_{23}(1) = S^y_{12}(1) = W_j(1)$
$\lambda = 0.00$ j
$S^x_{31}(1) = S^x_{12}(1) = W_i(1)$
$\lambda = 0.00$ i
$\lambda = 0.50$ k
$S^z_{23}(1) = S^z_{31}(1) = W_k(1)$
$S^x_{31}(0) = S^x_{12}(0)$
$\lambda = 1.50$ $-k$
$\lambda = 1.00$ $-i$
$= W_{-i}(0)$
$\lambda = 1.00$ $-j$
$S^z_{31}(0) = S^z_{23}(0) = W_{-k}(0)$
$S^y_{12}(0) = S^y_{23}(0) = W_{-j}(0)$

"wrap"

$$S^x_{12}(0) = S^x_{31}(0) \rightarrow W_{-i}(0)$$

$$S^x_{12}(1) = S^x_{31}(1) \rightarrow W_{i}(1)$$

$$S^y_{23}(0) = S^y_{12}(0) \rightarrow W_{-j}(0)$$

$$S^y_{23}(1) = S^y_{12}(1) \rightarrow W_{j}(1)$$

$$S^z_{31}(0) = S^z_{23}(0) \rightarrow W_{-k}(0)$$

$$S^z_{31}(1) = S^z_{23}(1) \rightarrow W_{k}(1)$$

i :

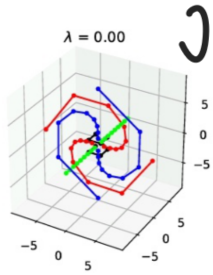
j :

k :

$$S^z_{23}\left(\tfrac{1}{2}\right)$$

$R_{23}(\frac{\lambda}{2})\, S^{x}_{12}(\frac{3}{2})$

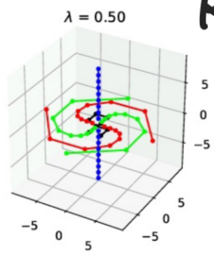
$i\,\cdot$



j

$S^{y}_{23}(1)$

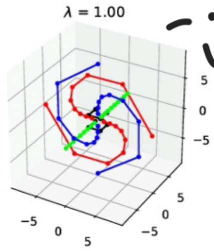
$S^{z}_{23}(\frac{1}{2})$



k

$S^{y}_{23}(\frac{3}{2})$

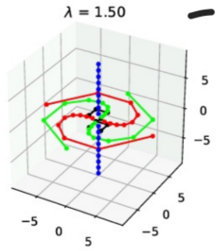
$S^{z}_{23}(1)$



$-j$

$S^{y}_{23}(0)$

$S^{z}_{23}(\frac{3}{2})$



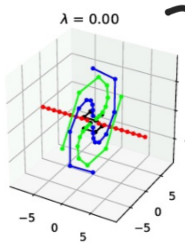
$-k$

$S^{y}_{23}(\frac{1}{2})$

$S^{z}_{23}(0)$

$R_{31}(\frac{\lambda}{2})\, S^{y}_{12}(\frac{3}{2})$

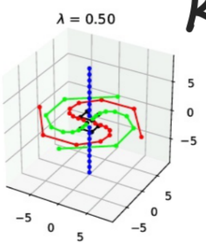
$j\,\cdot$



$-i$

$S^{x}_{31}(0)$

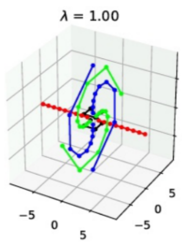
$S^{z}_{31}(\frac{1}{2})$



k

$S^{x}_{31}(\frac{1}{2})$

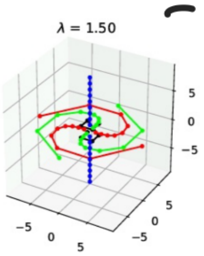
$S^{z}_{31}(1)$



i

$S^{x}_{31}(1)$

$S^{z}_{31}(\frac{3}{2})$



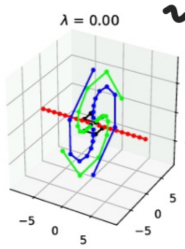
$-k$

$S^{x}_{31}(\frac{3}{2})$

$S^{z}_{31}(0)$

$R_{12}(\frac{\lambda}{2})\, S^{z}_{31}(\frac{3}{2})$

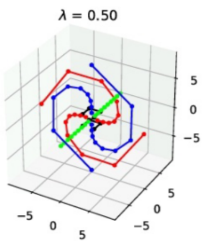
$k\,\cdot$



i

$S^{x}_{12}(1)$

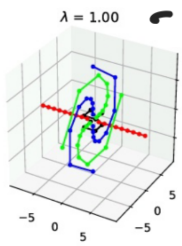
$S^{y}_{12}(\frac{1}{2})$



j

$S^{x}_{12}(\frac{3}{2})$

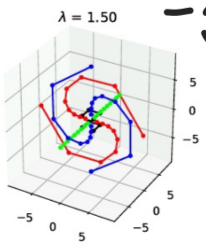
$S^{y}_{12}(1)$



$-i$

$S^{x}_{12}(0)$

$S^{y}_{12}(\frac{3}{2})$



$-j$

$S^{x}_{12}(\frac{1}{2})$

$S^{y}_{12}(0)$

state a $-a$

2 representations:

if next is $(a-1)\cdot$ $(a-1)\cdot$ | if next is $(a+1)\cdot$ $(a+1)\cdot$

$$R_{a(a+1)}\left(\frac{\lambda}{2}\right) S^{a-1}_{(a-1)a}\left(\frac{3}{2}\right) \qquad R_{(a-1)a}\left(\frac{\lambda}{2}\right) S^{a+1}_{a(a+1)}\left(\frac{3}{2}\right)$$

$$S^{a}_{a(a+1)}(\lambda+1)+0 \qquad S^{a}_{(a-1)a}(\lambda+1)+0$$

$$S^{a+1}_{a(a+1)}\left(\lambda+\frac{1}{2}\right)^{-\frac{1}{2}} \qquad S^{a-1}_{(a-1)a}\left(\lambda-\frac{1}{2}\right)^{+\frac{1}{2}}$$

in program:

$$S(\text{rot}, \text{string})$$

e.g.

$$S^{x}_{12} \to S(e_3, e_1)$$

$$S^{a}_{(a-1)a} \to S(e_{a+1}, e_a) \Big/ S^{a}_{a(a+1)} \to S(e_{a-1}, e_a)$$

state a $-a$

2 representations:

if next is

$(a-1)\cdot$ $(a-1)\cdot$

$$R_{a(a+1)}\left(\frac{\lambda}{2}\right) S^{a-1}_{a-1}\left(\frac{3}{2}\right)$$

$$S^{a}_{a-1}(\lambda+1)+0$$

$$S^{a+1}_{a-1}\left(\lambda+\frac{1}{2}\right)^{-\frac{1}{2}}$$

if next is

$(a+1)\cdot$ $(a+1)\cdot$

$$R_{(a-1)a}\left(\frac{\lambda}{2}\right) S^{a+1}_{a+1}\left(\frac{3}{2}\right)$$

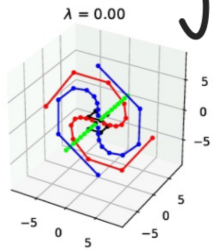
$$S^{a}_{a+1}(\lambda+1)+0$$

$$S^{a-1}_{a+1}\left(\lambda-\frac{1}{2}\right)^{+\frac{1}{2}}$$

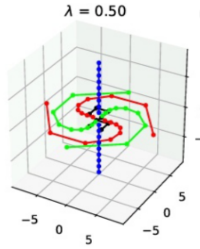
$i \cdot \longrightarrow$

$-i \cdot \longleftarrow$

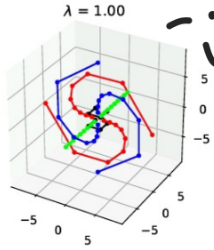
$R_{23}(\frac{\lambda}{2})\, S^x_{12}(\frac{3}{2})$



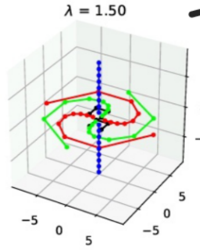
j $\quad \dfrac{S^z_{23}(\frac{1}{2})}{S^y_{23}(1)}$



k $\quad \dfrac{S^z_{23}(1)}{S^y_{23}(\frac{3}{2})}$



$-j$ $\quad \dfrac{S^z_{23}(\frac{3}{2})}{S^y_{23}(0)}$

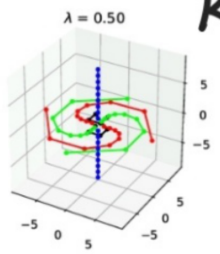


$-k$ $\quad \dfrac{S^z_{23}(0)}{S^y_{23}(\frac{1}{2})}$

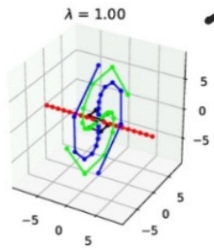
$j \cdot \longrightarrow$

$-j \cdot \longleftarrow$

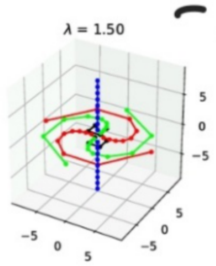
$R_{31}(\frac{\lambda}{2})\, S^y_{12}(\frac{3}{2})$



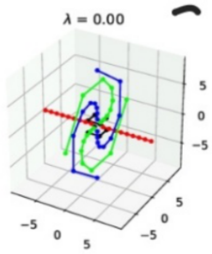
k $\quad \dfrac{S^x_{31}(\frac{1}{2})}{S^z_{31}(1)}$



i $\quad \dfrac{S^x_{31}(1)}{S^z_{31}(\frac{3}{2})}$



$-k$ $\quad \dfrac{S^x_{31}(\frac{3}{2})}{S^z_{31}(0)}$

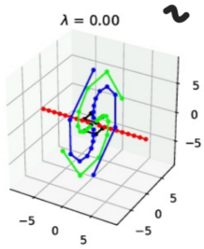


$-i$ $\quad \dfrac{S^x_{31}(0)}{S^z_{31}(\frac{1}{2})}$

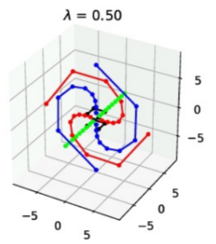
$k \cdot \longrightarrow$

$-k \cdot \longleftarrow$

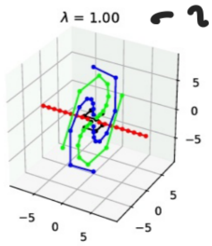
$R_{12}(\frac{\lambda}{2})\, S^z_{31}(\frac{3}{2})$



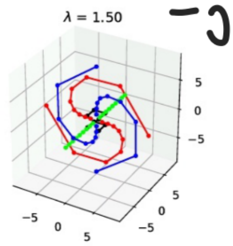
i $\quad \dfrac{S^y_{12}(\frac{1}{2})}{S^x_{12}(1)}$



j $\quad \dfrac{S^y_{12}(1)}{S^x_{12}(\frac{3}{2})}$



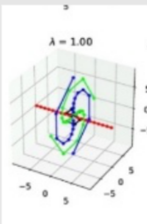
$-i$ $\quad \dfrac{S^y_{12}(\frac{3}{2})}{S^x_{12}(0)}$



$-j$ $\quad \dfrac{S^y_{12}(0)}{S^x_{12}(\frac{1}{2})}$

i

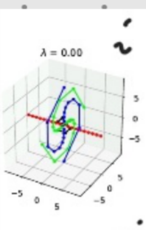
$j \cdot$



i $\dfrac{S^{x}_{31}(1)}{S^{z}_{31}(\frac{3}{2})}$

$R_{31}(\frac{\lambda}{2})\, S^{Y}_{12}(\frac{3}{2})$

$k \cdot$

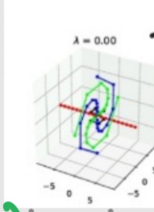


i $\dfrac{S^{y}_{12}(\frac{1}{2})}{S^{x}_{12}(1)}$

$R_{12}(\frac{\lambda}{2})\, S^{z}_{31}(\frac{3}{2})$

$-i$

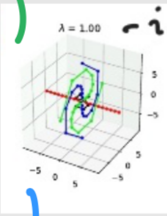
$j \cdot$



$-i$ $\dfrac{S^{x}_{31}(0)}{S^{z}_{31}(\frac{1}{2})}$

$R_{31}(\frac{\lambda}{2})\, S^{Y}_{12}(\frac{3}{2})$

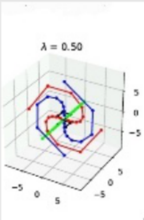
$k \cdot$



$-i$ $\dfrac{S^{y}_{12}(\frac{3}{2})}{S^{x}_{12}(0)}$

$R_{12}(\frac{\lambda}{2})\, S^{z}_{31}(\frac{3}{2})$

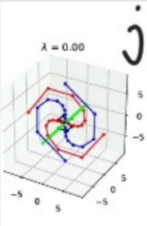
j



j $\dfrac{S^{y}_{12}(1)}{S^{x}_{12}(\frac{3}{2})}$

$k \cdot$

$R_{12}(\frac{\lambda}{2})\, S^{z}_{31}(\frac{3}{2})$

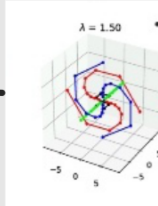


j $\dfrac{S^{z}_{23}(\frac{1}{2})}{S^{y}_{23}(1)}$

$i \cdot$

$R_{23}(\frac{\lambda}{2})\, S^{x}_{12}(\frac{3}{2})$

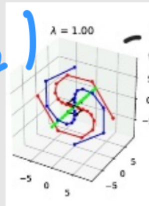
$-j$



$-j$ $\dfrac{S^{y}_{12}(0)}{S^{x}_{12}(\frac{1}{2})}$

$k \cdot$

$R_{12}(\frac{\lambda}{2})\, S^{z}_{31}(\frac{3}{2})$

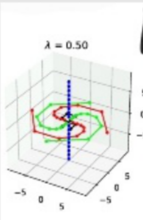


$-j$ $\dfrac{S^{z}_{23}(\frac{3}{2})}{S^{y}_{23}(0)}$

$i \cdot$

$R_{23}(\frac{\lambda}{2})\, S^{x}_{12}(\frac{3}{2})$

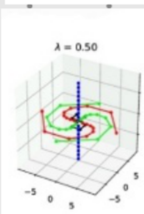
k



k $\dfrac{S^{z}_{23}(1)}{S^{y}_{23}(\frac{3}{2})}$

$i \cdot$

$R_{23}(\frac{\lambda}{2})\, S^{x}_{12}(\frac{3}{2})$

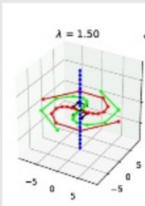


k $\dfrac{S^{x}_{31}(\frac{1}{2})}{S^{z}_{31}(1)}$

$j \cdot$

$R_{31}(\frac{\lambda}{2})\, S^{Y}_{12}(\frac{3}{2})$

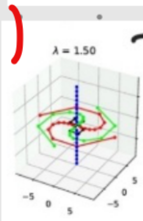
$-k$



$-k$ $\dfrac{S^{z}_{23}(0)}{S^{y}_{23}(\frac{1}{2})}$

$i \cdot$

$R_{23}(\frac{\lambda}{2})\, S^{x}_{12}(\frac{3}{2})$



$-k$ $\dfrac{S^{x}_{31}(\frac{3}{2})}{S^{z}_{31}(0)}$

$j \cdot$

$R_{31}(\frac{\lambda}{2})\, S^{Y}_{12}(\frac{3}{2})$

Rule :

$-a$

$$(a+1)\cdot \begin{array}{l} S^{a}_{(a-1)a}\left(1\ (+1)\right) \\ R_{(a-1)a}\left(\tfrac{1}{2}\right) S^{a+1}_{(a+1)(a-1)}\left(\tfrac{3}{2}\right) \\ S^{(a-1)}_{(a-1)a}\left(\tfrac{3}{2}\ (+1)\right) \end{array} \quad \begin{array}{|l} \text{string axis:} \\ a \\ a+1 \\ a-1 \end{array}$$

$$(a-1)\cdot R \begin{array}{l} S^{(a+1)}_{a(a+1)}\left(\tfrac{1}{2}\ (+1)\right) \\ {}_{a(a+1)}\left(\tfrac{1}{2}\right) S^{a-1}_{(a+1)(a-1)}\left(\tfrac{3}{2}\right) \\ S^{a}_{a(a+1)}\left(1\ (+1)\right) \end{array} \quad \begin{array}{|l} a+1 \\ a-1 \\ a \end{array}$$

$-a$

$$(a \pm 1)\cdot \left| \begin{array}{l} S^{a+1}_{(a-1)a\,(a+1)}\left(\tfrac{1}{2}\left(+\tfrac{1}{2}\right)(+1)\right) \\ R_{(a-1)a\,(a+1)}\left(\tfrac{1}{2}\right) S^{a\pm1}_{(a+1)(a-1)}\left(\tfrac{3}{2}\right) \\ S^{a-1}_{(a-1)a\,(a+1)}\left(1\left(+\tfrac{1}{2}\right)(+1)\right) \end{array} \right| \begin{array}{l} a+1 \\ a+1-1 \\ a-1 \end{array}$$

After simplifying:

a : current State

b : rotation axis

$$\longrightarrow \quad c = \begin{cases} a-1 & \text{if } b=a+1 \\ a+1 & b=a-1 \end{cases}$$

$$S = \begin{cases} +1 & \text{sign}(a) > 0 \\ -1 & \text{sign}(a) < 0 \end{cases}$$

$$P = \begin{cases} +1 & b=a-1 \\ -1 & b=a+1 \end{cases}$$

Stringlist

$$= \left(\text{String}_a, \text{String}_b, \text{String}_c \right)$$

$$\lambda \in [0, 0{,}5]$$

$$R = \left(S^{a+1}_{b+1} \left(\lambda + 1 + \begin{cases} 1 & \text{sign}<0 \\ 0 \end{cases} \right) \right.$$

$$R_b \left(\frac{\lambda}{2} + \frac{1}{4} + \begin{cases} \frac{1}{2} & \text{sign}<0 \\ 0 \end{cases} \right) S^{b+1}_{a+1} \left(\frac{3}{2} \right),$$

$$\left. S^{c+1}_{b+1} \left(\lambda + \frac{P}{2} + \begin{cases} 1 & \text{sign}<0 \\ 0 \end{cases} \right) \right)$$

Connection to Cube

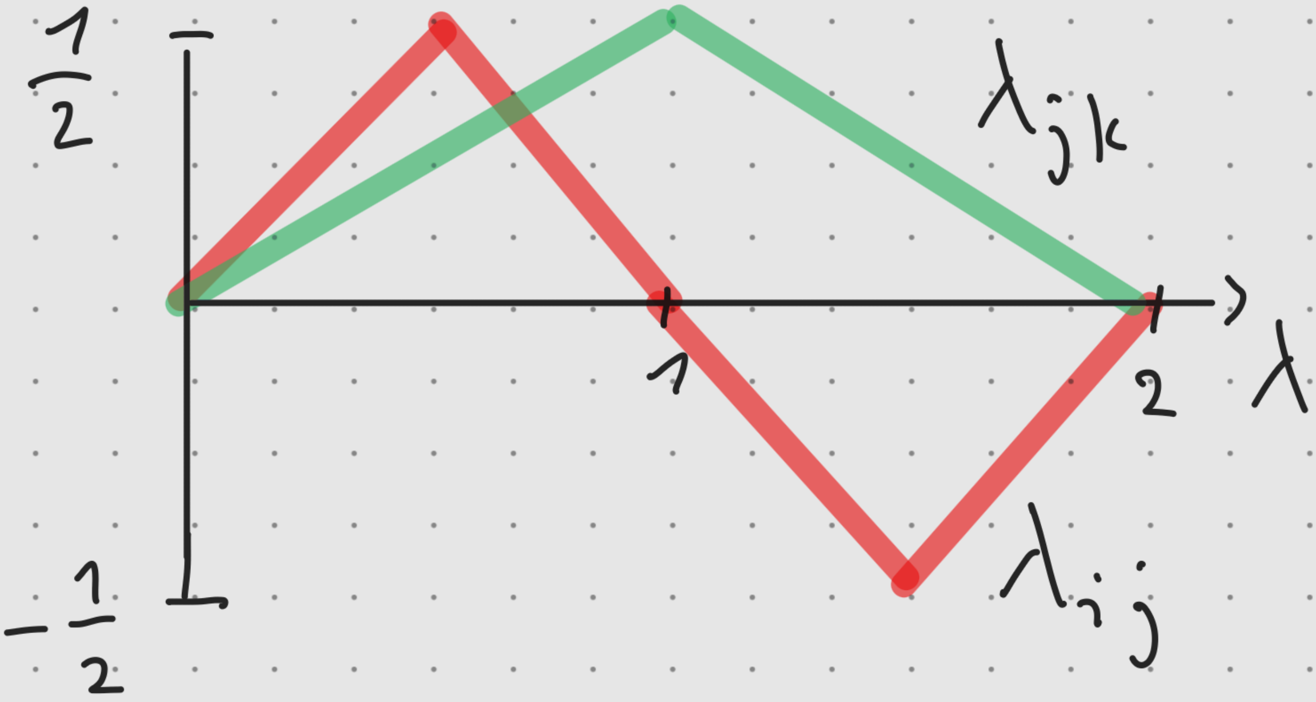
String axis: i $\longrightarrow$ $e_j = (e_i \wedge e_k)^*$
rot. axis: k

$$S(\lambda, \alpha)$$

$$= R_{jk}(\lambda_{jk}) \cdot R_{ij}(\lambda_{ij})$$

$$\underline{\lambda_{ij}} = (1-\alpha) \cdot \begin{cases} \lambda & 0 \le \lambda < 0{,}5 \\ 1-\lambda & 0{,}5 \le \lambda \le 1 \\ 1-\lambda & 1 < \lambda \le 1{,}5 \\ \lambda - 2 & 1{,}5 < \lambda < 2 \end{cases}$$

$$\underline{\lambda_{jk}} = \frac{1}{2} \begin{cases} \lambda & 0 \le \lambda < 1 \\ 2-\lambda & 1 \le \lambda < 2 \end{cases}$$



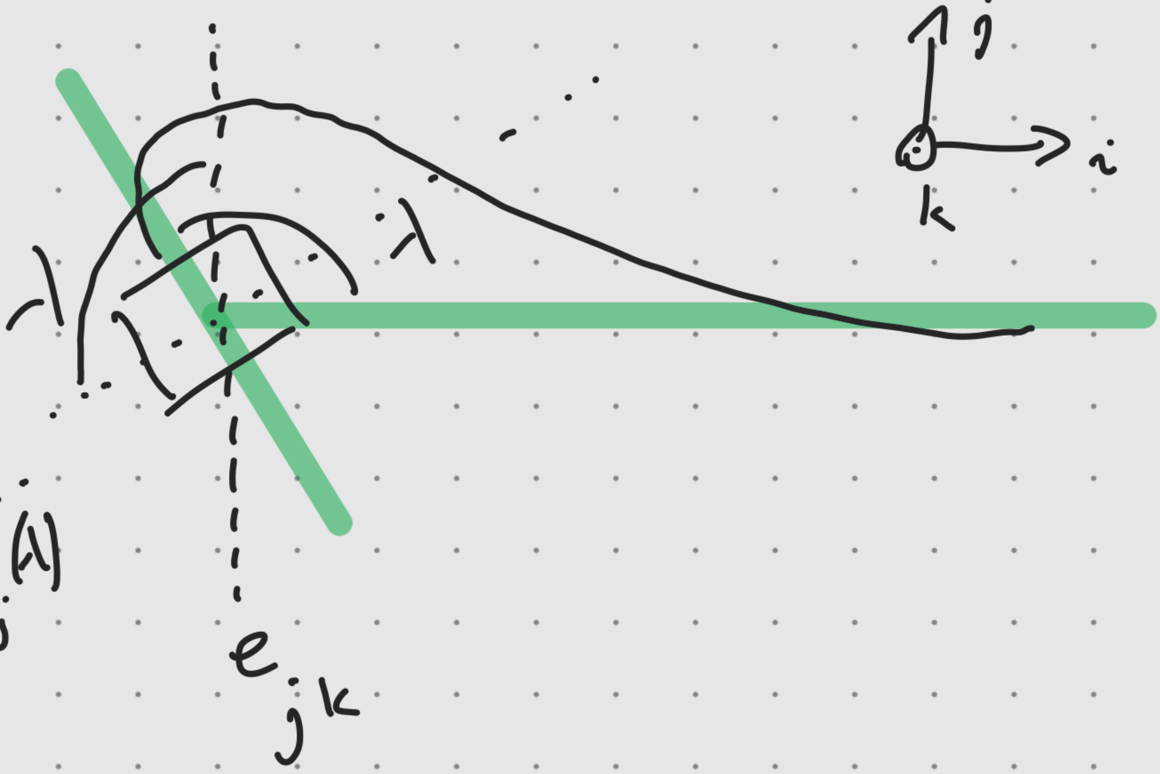
		plane
String axis:	i	jk
rot. axis:	k	ij

$\alpha = 0 \longrightarrow \alpha = 1$

$$R_{ij}(\lambda_{ij}) \;\Big|\; R_{jk}(\lambda_{jk}) R_{ij}(\lambda_{ij}) \;\Big|\; R_{jk}(\lambda_{jk})$$

$R\left(\frac{\lambda}{2}\right)$

$R_{ij}(\lambda) e_{jk} \tilde{R}_{ij}(\lambda)$



- Berry Phase
- Motor Paper
- Chladni Paper