Unit 1: Introduction

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Outline

1.1 Advantage of Computer Graphics and Areas of Applications

1.2 Hardware and Software for Computer Graphics. (Hard Copy, Display technologies)

1.3 Random Scan Display System, Video Controller, Random Scan Display Processor

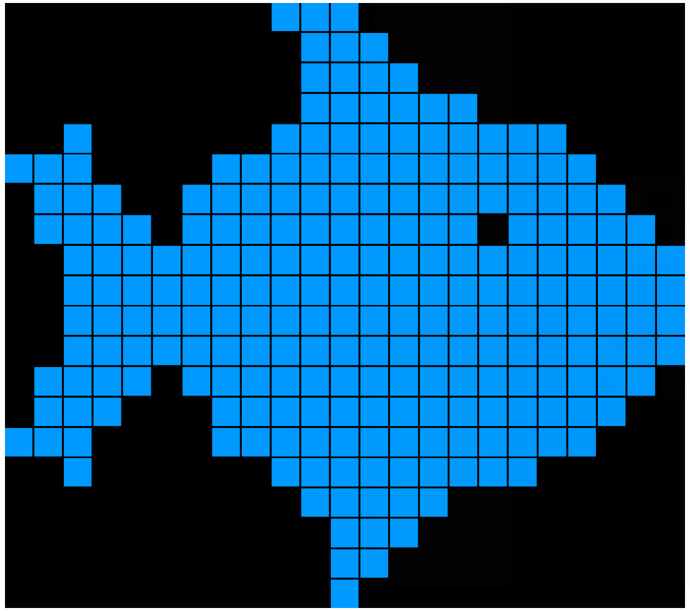
1.4 Raster Graphics

1.5 Scan Conversion Algorithms (Line, Circle Ellipse)

1.6 Area Filling (Rectangle, Ellipse), Clipping (Lines, Circle, Ellipse), Clipping Polygons

What is Computer Graphics?

- Computer graphics is the field or branch of science and technology related to generation (creation), storage and manipulation of graphics (images or pictures) of objects using computer for drawing lines, charts, pictures, etc.
- Computer graphics image is made up of number of pixels.
- Picture Element (Pixel) is the smallest addressable graphical unit represented on the computer screen.



Introduction

- Graphics is to plot some points on graph to make an image or simply, graphics is an image or a visual representation of an object
- Computer graphics is to plot some pixels (points) on the computer screen to make an image. Pixel or picture element is the smallest screen point or elementary part of the computer screen.

Computer + Graphics = Computer Graphics

Computer = System= Hardware + Software

Hardware= Input devices + Output devices

Software = Data structure + Graphics algorithm + languag**e**

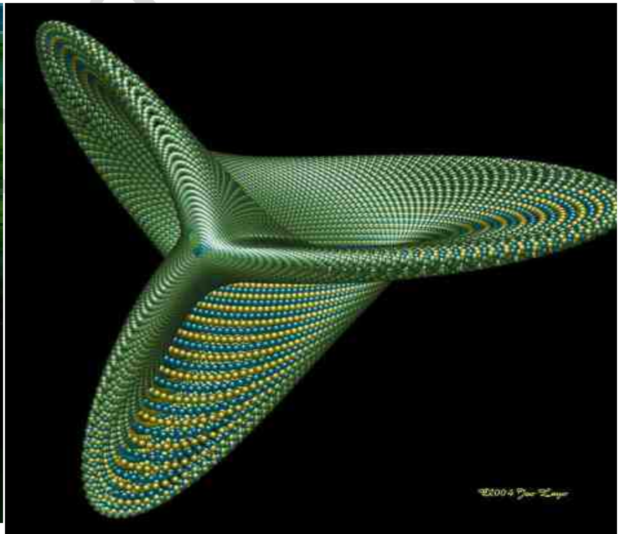
Graph + pics = Graphics

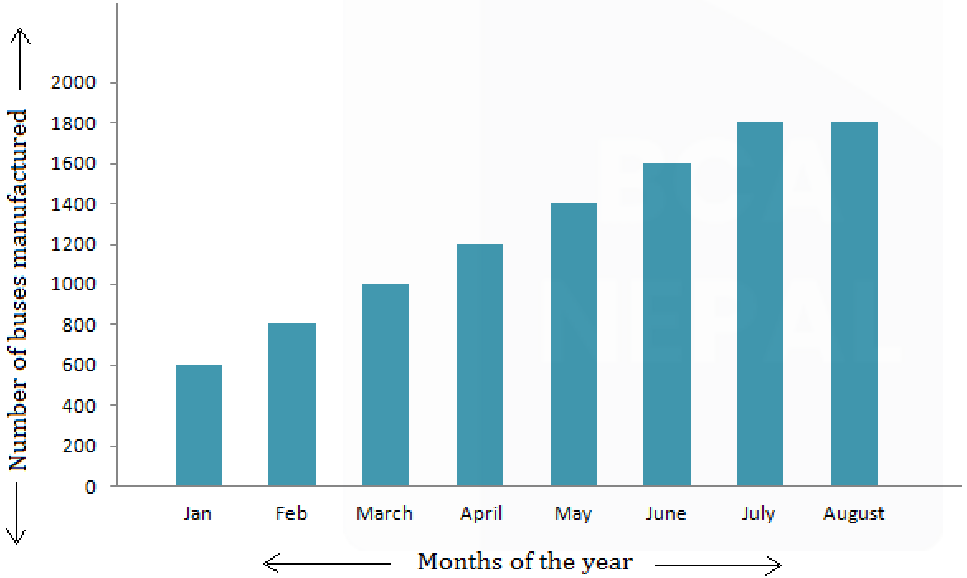
Graph= mathematical figure like line, circle, ellipse

Pics= Images

Advantages of Computer Graphics

- Most effective and commonly used ways of communication with computer.
- It provides tools for producing
  - Picture of “real world”,
  - Synthetic objects such as mathematical surfaces in 3D and
  - Data that have no inherent geometry such as survey result.





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- Possible to produce animations.
- Can move objects as well as observes as per requirement

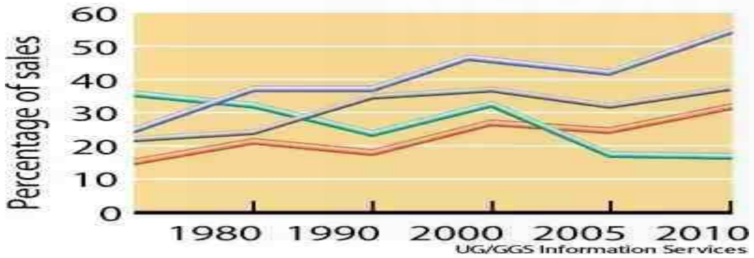
Application of Computer Graphics

1. User Interfaces

- The interface between the human and the computer has been radically changed by the use of computer graphics.
- Button, Icon, Menu, Scroll bar etc.
- Most applications have a Graphical User Interface (GUI) for user friendly and interactive operation.
- Multiple simultaneous activities
- Click facilities

2. Plotting Graphs and charts

- Plotting 2D and 3D graphs such as the histograms, bar and pie charts
- These plotting or visualization of measurement data are useful to analyze meaningfully and concisely the trend and pattern of complex data.



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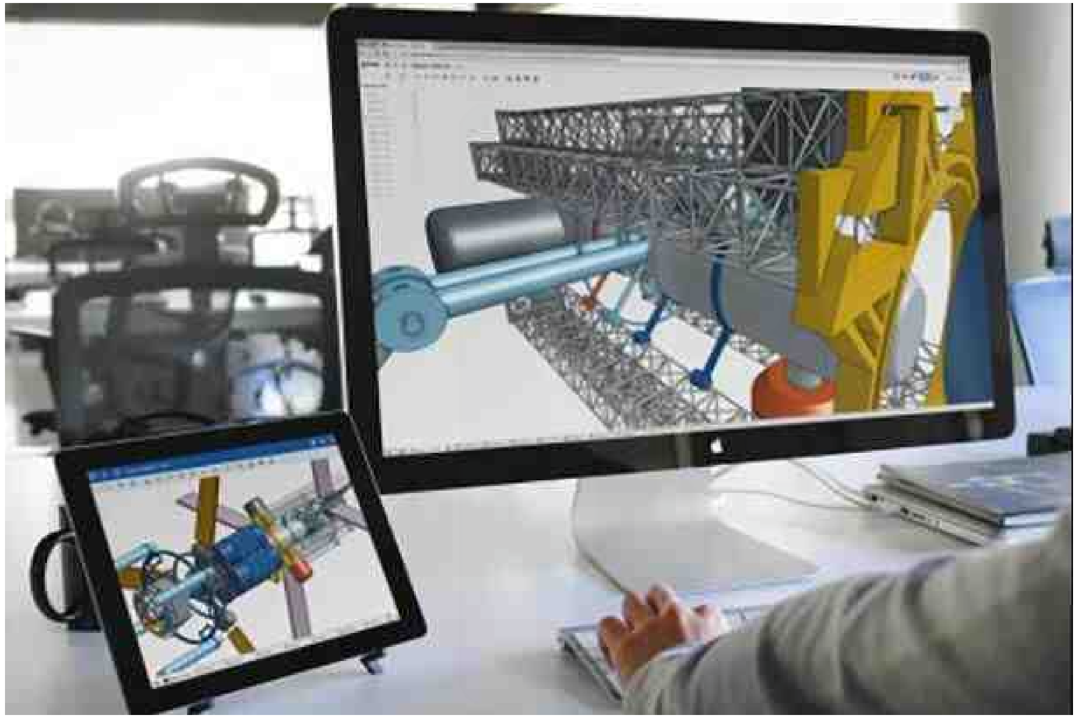
3. Office automation and electronic publishing

- Document that contains text, tables, graphs, drawings, pictures can be easily printed or saved as electronic (softcopy) document
- The office automation and electronic publishing became possible by the development of computer graphics.

4. Computer aided design and drafting

- A major use of computer graphics is in design process
- Computer graphics is used to design components and systems of architectural, mechanical, electrical, electronic devices, auto mobile bodies, aircrafts, watercrafts, spacecrafts, very large scale integrated (VLSI) chips, optical systems, and telephone and computer networks.



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5. Scientific and business visualization

- Scientific visualization means generating computer graphics for scientific works and medical data sets
- Business visualization is generating computer graphics for nonscientific data sets such as share market or economic data set
- Visualization makes easier to understand the trends and pattern inherent in huge amount of data sets. It would otherwise be almost impossible to analyze those data numerically.

6. Simulation and virtual reality

- Simulation is the imitation of the conditions like those, which is encountered in real life
- Virtual reality is an interactive computer-generated experience taking place within a simulated environment, that incorporates mainly auditory and visual, but also other types of sensory feedback like haptic.

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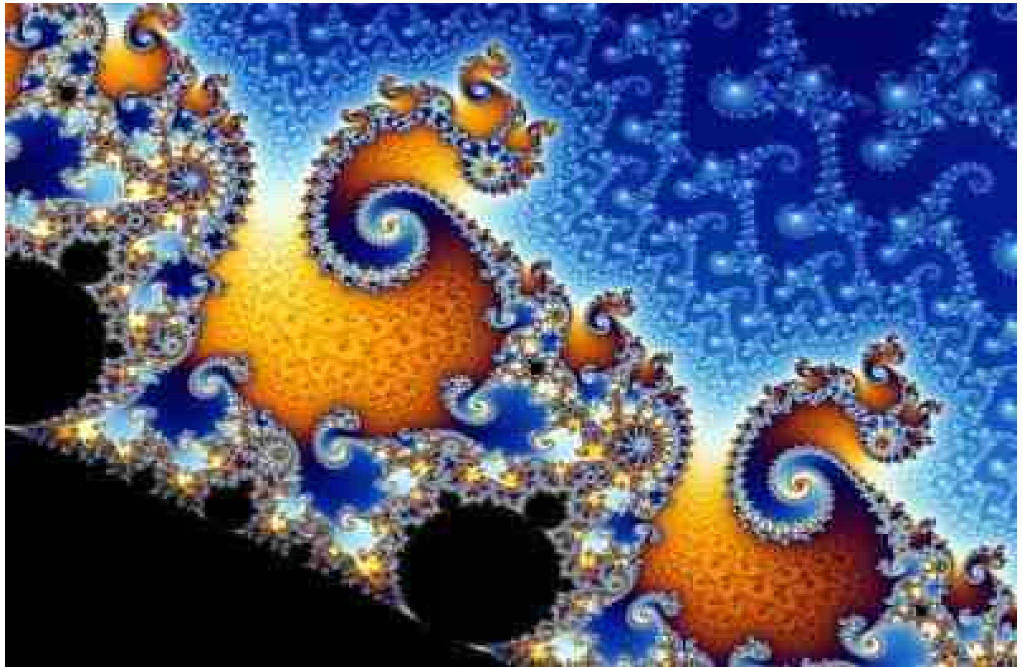
7. Entertainment

- In making games, special effects in movies, music videos, television shows, etc
- FIFA, Formula-1, Doom and Pools, pubg
- The Beauty and the Beast, Bahubali

8. Art and commerce

- Used in both fine art and commercial art.
- These pictures are frequently seen at transportation terminal, super markets, hotel, etc.
- The slide production for commercial, scientific, or education presentation is another cost effective



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9. Cartography

- It includes geographic map, oceanographic chart, weather map, weather map, population density map
- GIS (geographical information system)

10. Medical applications

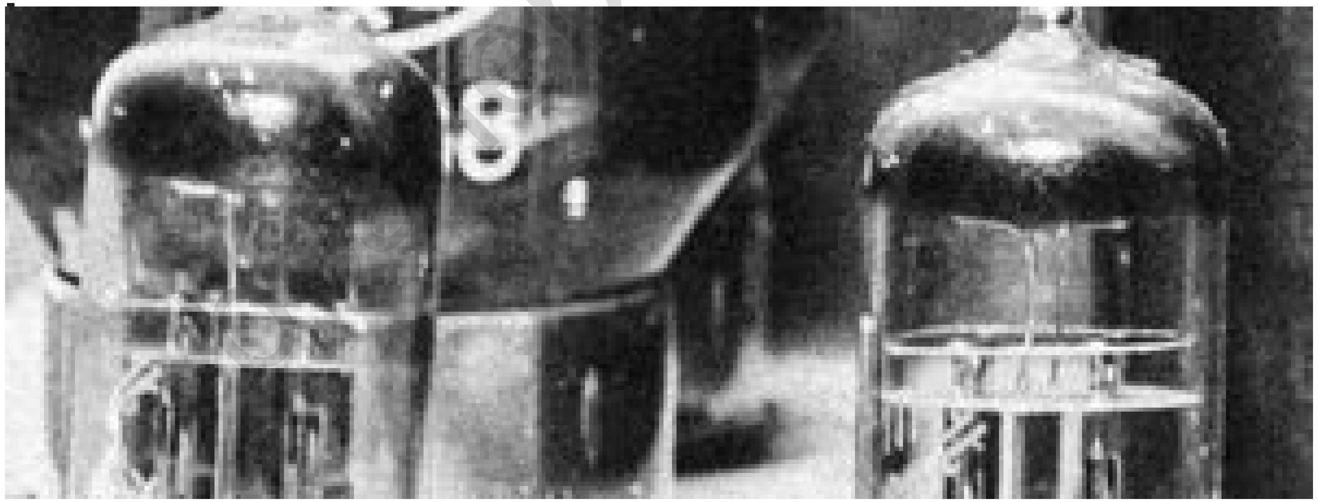
- Powerful tool for diagnosis and treatment in medical fields. X-ray, video x-ray, complex operation, etc.

11. Internet

- Multimedia content available on net.
- Internet became famous because of the development of computer graphics

A historical timeline

- **1897 – The Birth of the Cathode Ray Tube**
- The Cathode Ray Tube was the first display technology and it had an incredibly long run despite being inefficient, bulky, heavy and full of hazardous waste materials. It pretty much owned the 20th century.



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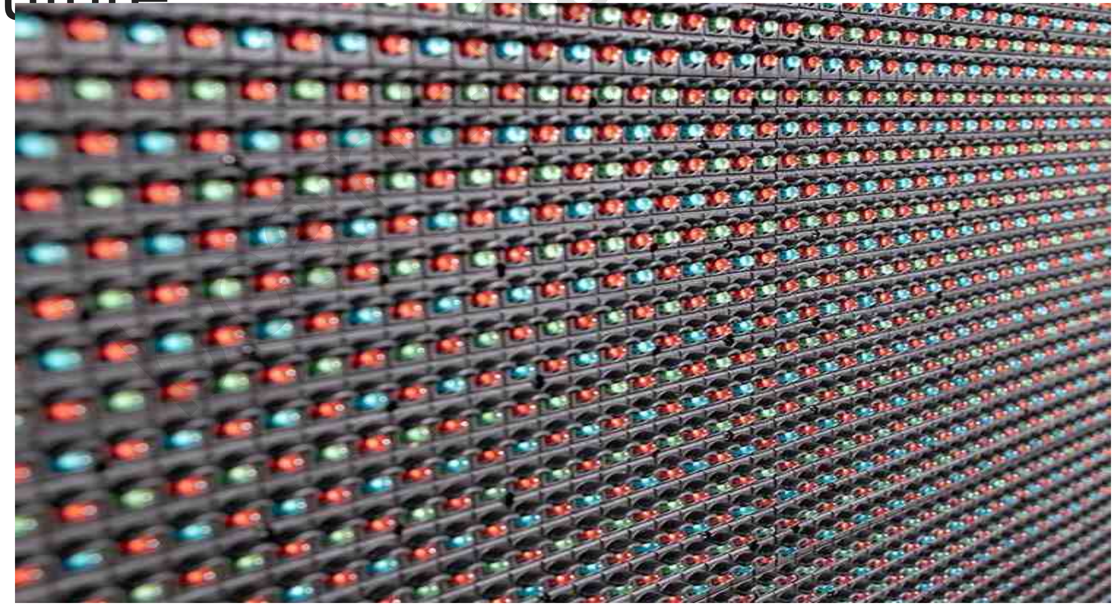
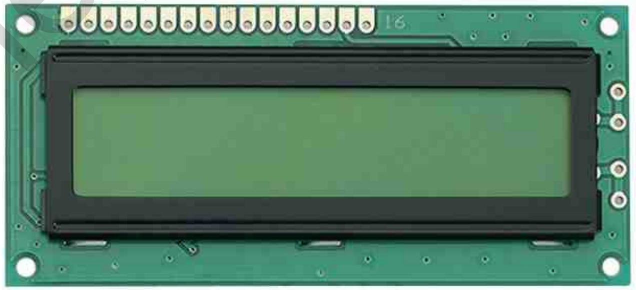
- **1961-1962 – Invention of LED**
- Nick Holonyack invents the light emitting diode (LED) that's visible to the human eye and he is known as "the father of the LED". While initially lacking in efficacy and color options, LED developments have made tremendous strides and are the light source of the future.



- **1964 – Invention of LCD and Plasma**
- Both the first working LCD (liquid crystal display) and the first PDP (plasma display panel) are invented. It would be a while before these technologies evolved into mainstream use (i.e., flat screen televisions didn't start appearing in significant numbers in households). LCDs came to dominate due to mass production, and plasma displays were limited due to their higher weight and lack of s



- **1987 – OLED**
- Researchers at Eastman Kodak invented OLED (organic light-emitting diode) technology, an advancement in electroluminescence. OLEDs are thin, flexible, and remarkably small compared to LEDs. The technology would advance further to challenge LCD with better blacks and a thinner profile. Even after billions of dollars of R&D, OLEDs are still costly to manufacture in high quantity and have limited lifetimes compared to LCD

- **2008 – AMOLED**
- AMOLED, or active-matrix organic light-emitting diode, brings OLEDs a big step further with increased resolution and infinite contrast ratio. When you read about OLED TVs and phones this is the technology that's being used. The backlight is gone and the display is no longer rigid, but organic materials tend to die out, and that becomes the technology's most worrisome flaw for any device designed to last more than a couple years.

- **What’s Next – Moving Way from Backlight**
- Maybe you’ve heard about “secret” production centers Apple has for creating MicroLEDs, a technology that wowed the crowd at the Consumer Electronic Show in 2018. Maybe you’ve seen the high resolution and high color reflective LCD by BOE at the DisplayWeek Show in 2018. And you’ve probably watched the rise of OLEDs, which are flexible, thin and produce better blacks but decay quickly. What it all seems to have in common is a move away from the rigid and inefficient backlight. With FLEx frontlit technologies you can get in front of it and stay there, and you can do it today.

Output devices

- All the **output devices** can be categorized into two categories
- ➢ Hard Copy Devices
- ➢ Soft Copy Devices
- **Hard copy devices** are those that give the output in the tangible form. Printers and Plotters are two common hard copy devices.
- **Soft copy devices** give output in the intangible form or the virtual form, e.g. something displayed on a screen. All the computer monitors are covered under this category.

Printers

All the printers irrespective of the technology used can be categorized as

- Impact Printers
- Non Impact Printers

Impact printers are those printers in which there is a direct contact between the printing head and the paper on which the print is produced.

- They work by striking a head or a needle against an inked ribbon which leaves a mark on the paper.
- These printers produce a lot of noise when printing, because of the head striking the paper.
- Examples are ***Dot Matrix, Daisy Wheel*** and ***Line printers***.

In the case of **non-impact printers** the printing head never comes in direct contact with the paper.

- These printers work by spraying ink on the paper.
- Electrostatic or electromagnetic charge is used in these printers.
- Examples are ***Ink-Jet*** and ***Laser*** printers.

Plotters :

Another hard copy output device is plotter. Plotter is a printing device which can draw continuous lines. This is useful to print vector graphics rather than raster graphics unlike normal printers. Plotters are widely used in applications like CAD.

- Plotters print by moving one or more pen across the surface of a piece of paper. This means that plotters are restricted to line art, rather than raster graphics as with other printers.

- Pen plotters can draw complex line art, including text, but do so slowly because of the mechanical movement of the pens. They are often incapable of efficiently creating a solid region of colour, but can draw an area by drawing a number of close, regular lines.

- Plotters offered the fastest way to efficiently produce very large drawings or colour high-resolution vector-based artwork when computer memory was very expensive and processor power was very limited.

- There are a number of different types of plotters:

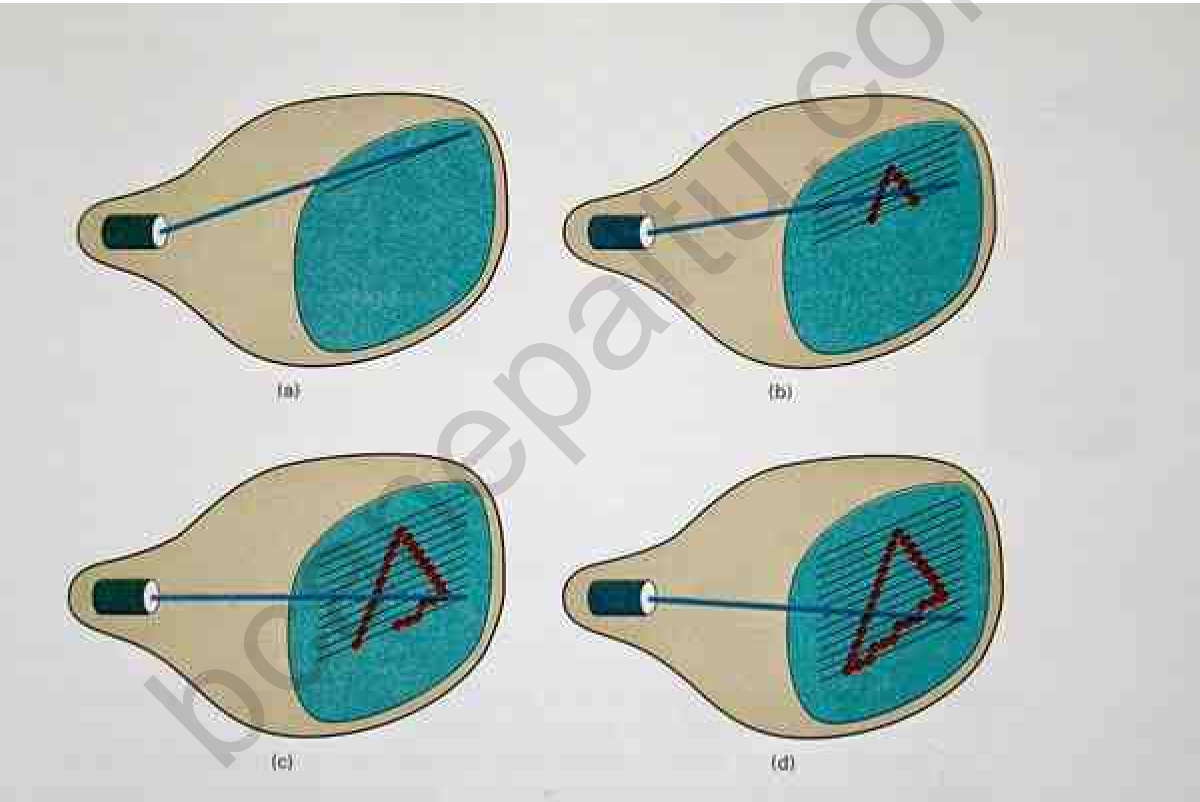
  - A **drum plotter** draws on paper wrapped around a drum which turns to produce one direction of the plot, while the pens move to provide the other direction.

  - A **flatbed plotter** draws on paper placed on a flat surface; and an electrostatic plotter draws on negatively charged paper with positively charged toner.

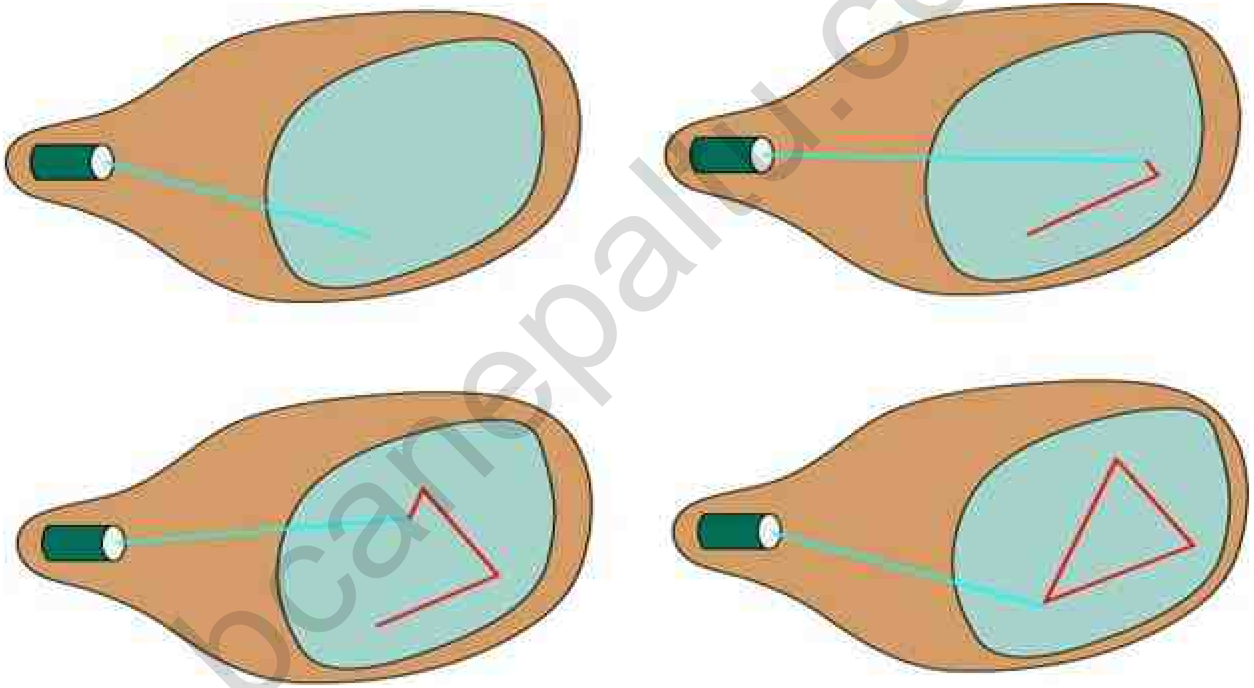
- Pen plotters have essentially become obsolete, and have been replaced by large-format inkjet printers and toner based printers.

- They are most frequently used for CAE (computer-aided engineering) applications, such as CAD (computer-aided design) and CAM (computer-aided manufacturing).

Raster Scan Display



Random (Vector) Scan Display



Difference between Raster and Random (Vector) Scan Display

Base of difference	Raster scan display	Random (vector) scan display
1. Electron beam	The electron beam is swept across the screen one row at a time from top to bottom.	The electron beam is swept to the parts of the screen where a picture is to be drawn.
2. Resolution	It has lower or poor resolution because picture definition is stored as an intensity value.	It has high resolution because it stores picture definition as a set of line commands.
3. Picture definition	Picture definition is stored as a set of intensity values for all screen points (pixels) in a refresh buffer.	Picture definition is stored as a set of line in a display list or file.

Difference between Raster and Random (Vector) Scan Display

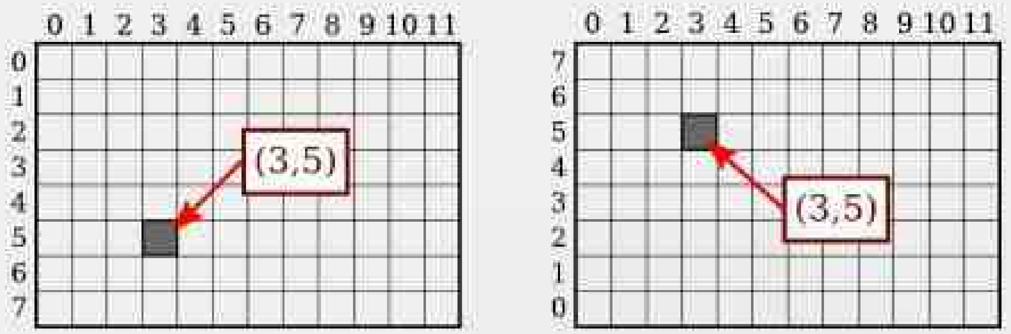
4. Realistic display	The capacity of the system to store intensity values for pixels make it well suited for realistic display with shadow and color pattern.	These systems are designed for line-drawing and can't display realistic shaded scenes.
5. Image drawing	Screen points or pixels are used to draw an image.	Mathematical functions are used to draw an image.
6. Cost	They are cheaper than random display.	It is more expensive than raster-scan display.

Difference between Raster and Random (Vector) Scan Display

Base of difference	Raster scan display	Random (vector) scan display
7. Refresh rate	Refresh rate is 60-80 fps.	All components are drawn 30 to 60 times per second.
8. Interlacing	It uses interlacing.	It doesn't use interlacing.
9. Editing	Editing is difficult.	Editing is easy.
10. Refresh area	Refresh area is independent of picture complexity.	Refresh area depends on complexity of picture.
11. Smoothness	Produce jagged line.	Produce smooth line.
Example:	CRT, TV, Printer	Pen Plotter

Scan Conversion of a point

- Point plotting is done by converting a single co-ordinate position furnished by an application program into appropriate operations for the output device in use.
- Example: Plot point P(3, 2)
- To draw the point on the screen we use function
  - setpixel (x, y)
- To draw the pixel in C language we use function
  - putpixel (x, y)
- Similarly for retrieving color of pixel we have function
  - getpixel (x, y)



12-by-8 pixel grids, shown with row and column numbers. On the left, rows are numbered from top to bottom, on the right, they are numberd bottom to top.

Line

- Line drawing is done by calculating intermediate positions along the line path between two specified endpoint positions.
- The output device is then directed to fill in those positions between the end points with some color.
- For some device (random scan display), a straight line is drawn by joining direct from one point to other.
- Digital devices display a line is drawn by plotting discrete points between the two end points.
- Discrete co-ordinate are calculated from the equation of the line.

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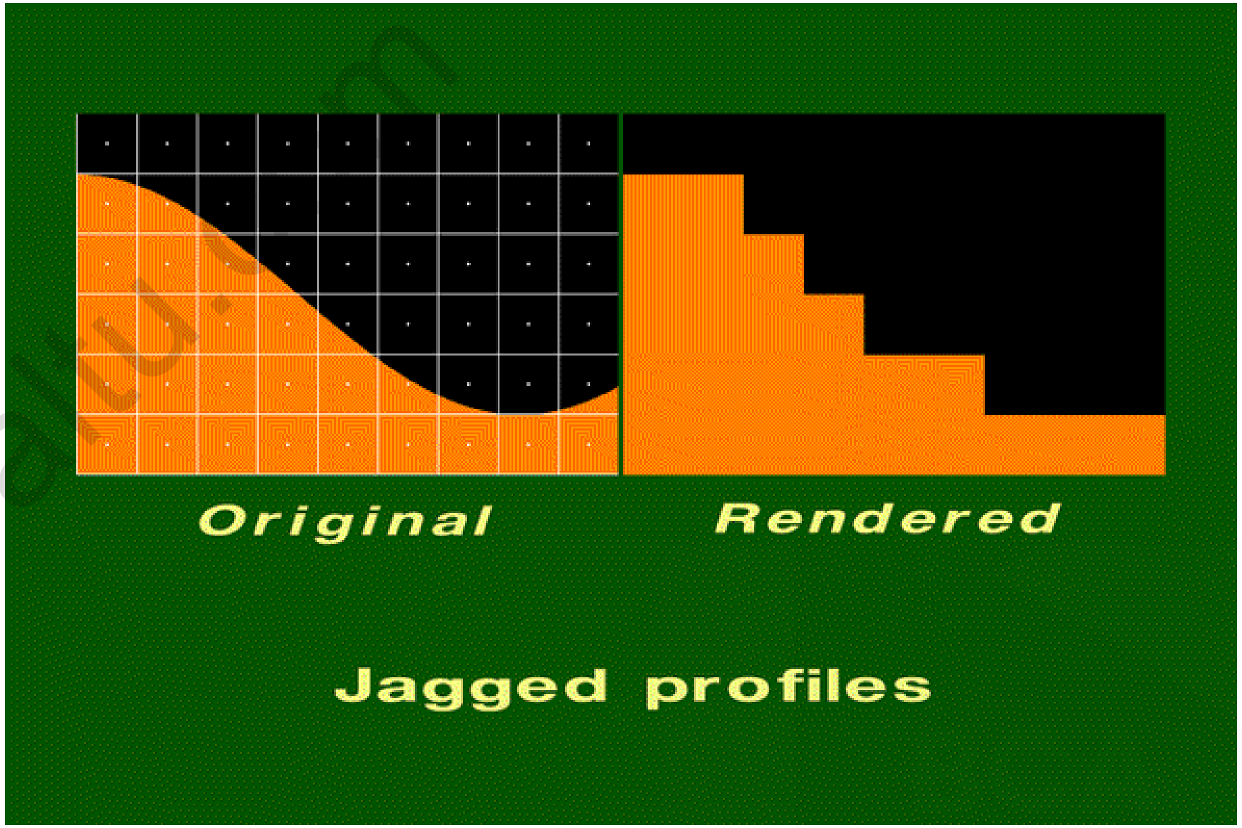
- Screen locations are referenced with integer values.
- So plotted positions may only approximate actual line positions between two specified endpoints. For example line position of (12.36, 23.87) would be converted to pixel position (12, 24).
- This rounding of co-ordinate values to integers causes lines to be displayed with a stair step appearance ("the jaggies").



Line Drawing Algorithms

- The Cartesian slop-intercept equation for a straight line is
  - $y = m x + b$
  - With m representing slope and b as the intercept
- It is possible to draw line using this equation but for efficiency purpose we use different line drawing algorithm.
  - DDA algorithm
  - Bresenham's Line Algorithm
- We can also use this algorithm in parallel if we have more number of processors.

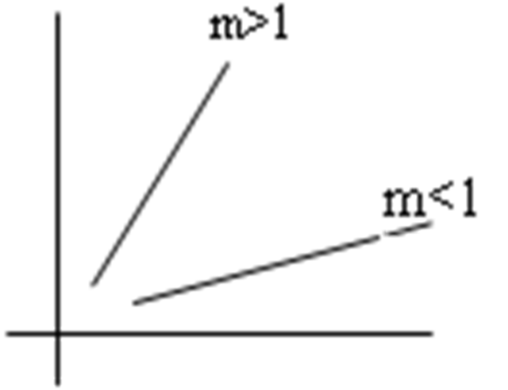
Introduction to DDA algorithm

- Full form of DDA is Digital Differential Analyzer
- DDA is scan conversion line drawing algorithm based on calculating either Δy or Δx using line equation.
- We sample the line at unit intervals in one co-ordinate and find corresponding integer values nearest the line path for the other coordinate.
- Selecting unit interval in either x or y direction based on way we process line.

$m = \Delta y / \Delta x$

$\Delta y = m\,\Delta x$ .... (1)

$\Delta x = \Delta y / m$ .... (2)



Case I: If slope (m) ≤ 1, then sample at unit x intervals ($\Delta x = 1$) and compute each successive y value because the increment in x is more than increment in y.

So, set $\Delta x = 1$.

Now, from equation (1), we get

$\Delta y = m$

i.e., $x_{k+1} = x_k + \Delta x = x_k + 1$ ....(3)

$y_{k+1} = y_k + \Delta y = y_k + m$ ……….......(4)

where subscript k takes integer values starting from 1, for the first point, and increases by 1 unit until the final end point is reached. Since m can be any real number between 0 to 1, the calculated y values must be rounded to the nearest integer.

Case II: If $m > 1$, then the increment in x (Δx) is smaller than increment in y (Δy),

So, set $\Delta y = 1$.

Then,

$\Delta x = 1/m$

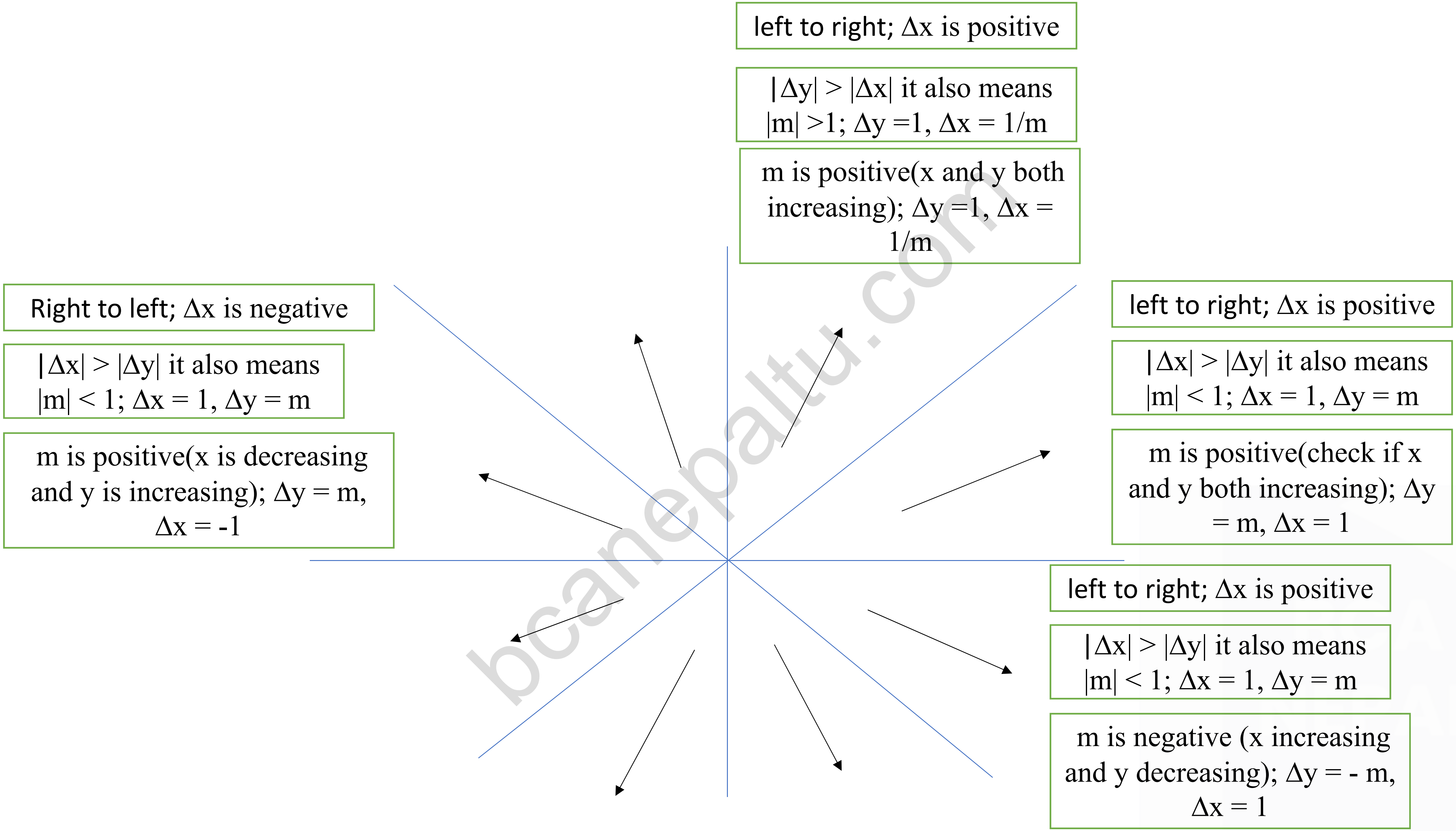
That is,

$x_{k+1} = x_k + 1/m$ ......(5)

$y_{k+1} = y_k + 1$ ....(6)

Here, in the both case $m \leq 1$ and $m>1$, consider the algorithm (i.e., equations 3, 4, 5, 6) based on the assumption that lines are to be processed from the left end point to the right end point.

If this process is reversed (that is, lines are to be processed from right to left), relation required to change in both case.

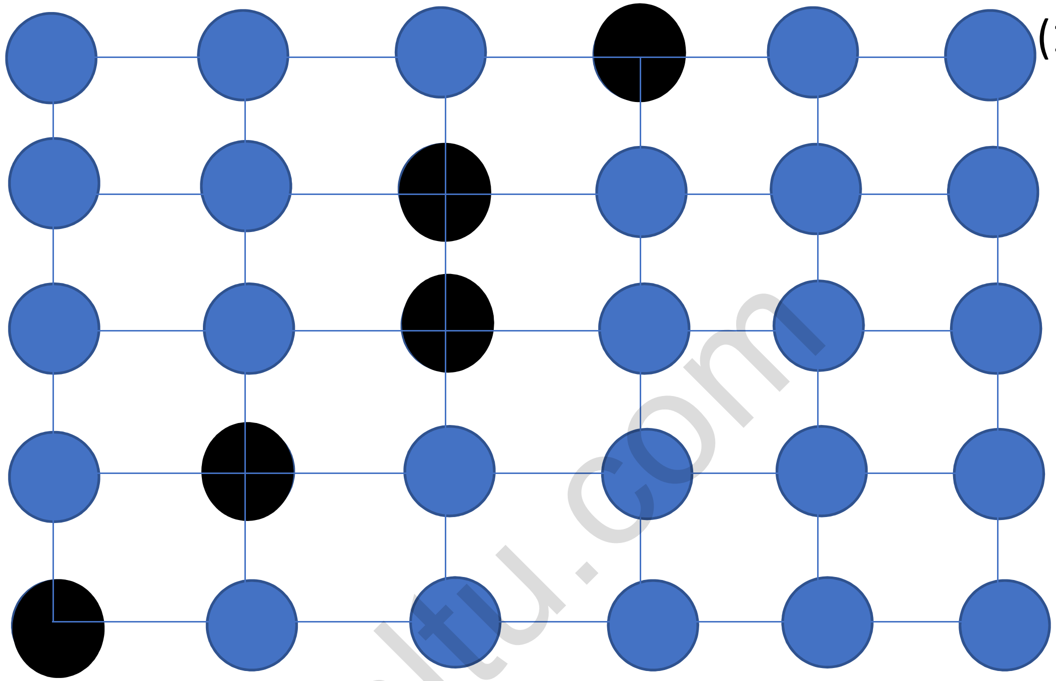


left to right; Δx is positive
$|\Delta y| > |\Delta x|$ it also means $|m| > 1$; $\Delta y = 1$, $\Delta x = 1/m$
m is positive(x and y both increasing); $\Delta y = 1$, $\Delta x = 1/m$
Right to left; Δx is negative
$|\Delta x| > |\Delta y|$ it also means $|m| < 1$; $\Delta x = 1$, $\Delta y = m$
m is positive(x is decreasing and y is increasing); $\Delta y = m$, $\Delta x = -1$
left to right; Δx is positive
$|\Delta x| > |\Delta y|$ it also means $|m| < 1$; $\Delta x = 1$, $\Delta y = m$
m is positive(check if x and y both increasing); $\Delta y = m$, $\Delta x = 1$
left to right; Δx is positive
$|\Delta x| > |\Delta y|$ it also means $|m| < 1$; $\Delta x = 1$, $\Delta y = m$
m is negative (x increasing and y decreasing); $\Delta y = - m$, $\Delta x = 1$

Unit Step Direction in DDA Algorithm

- Processing from left to right.
  - Slope is "+ve"
    - Magnitude is greater than 1 — $\Delta x = 1/m$, $\Delta y = 1$
  - Slope is "+ve"
    - Magnitude is less than 1 — $\Delta x = 1$, $\Delta y = m$
  - Slope is "-ve"
    - Magnitude is less than 1 — $\Delta x = 1$, $\Delta y = -m$
  - Slope is "-ve"
    - Magnitude is greater than 1 — $\Delta x = 1/m$, $\Delta y = -1$

- Processing from right to left.
  - Slope is “+ve”
    - Magnitude is greater than 1 $\Delta x = -1/m$, $\Delta y = -1$
  - Slope is “+ve”
    - Magnitude is less than 1 $\Delta x = m$, $\Delta y = 1$
  - Slope is “-ve”
    - Magnitude is less than 1 $\Delta x = m$, $\Delta y = -1$
  - Slope is “-ve”
    - Magnitude is greater than 1 $\Delta x = 1/m$, $\Delta y = -1$



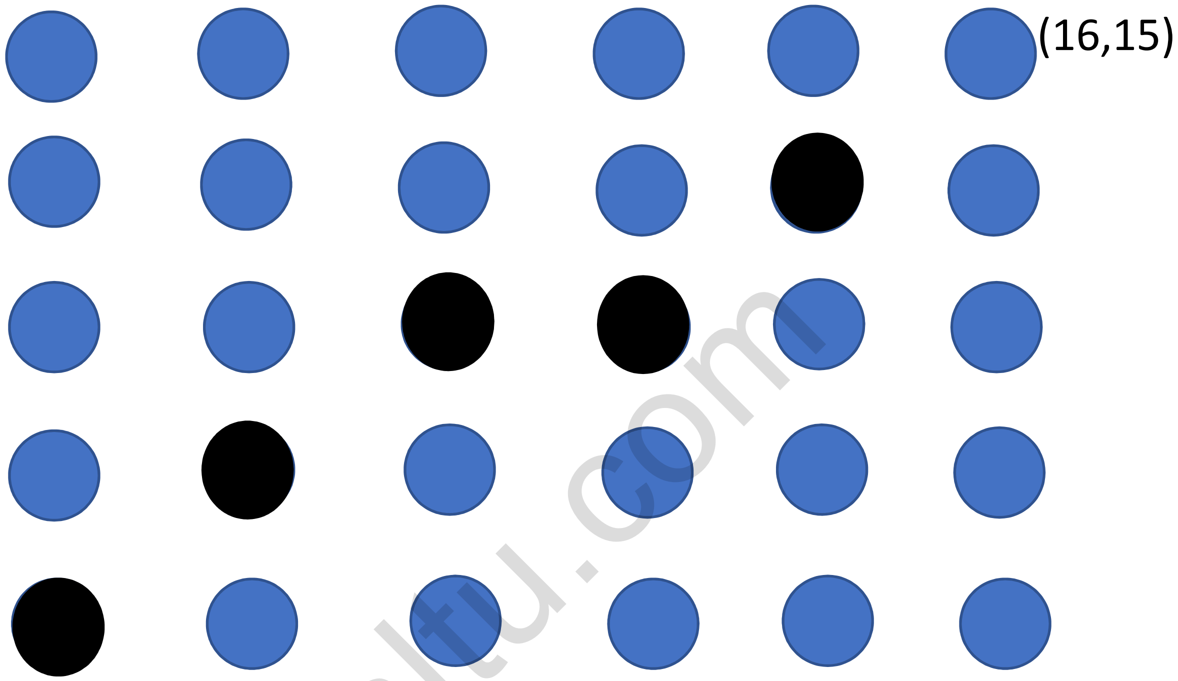
Use DDA to draw line from (10,10) to (13,14)

L→R means X +ve

$|\Delta x| = 3 < |\Delta y| = 4$ means $|m| > 1$;
$\Delta x = 1/m$, $\Delta y = 1$
$m = 4/3 = 1.33$ so
$\Delta x = 1/1.33 = 0.75$

x and y both increasing so slope is +ve.
No change

Iteration	(x,y)		Plot(x,y)
1			(10,10)
2	(10,10)	(10.75,11)	(11,11)
3	(10.75,11)	(11.5,12)	(12,12)
4	(11.5,12)	(12.25,13)	(12,13)
5	(12.25,13)	(13,14)	(13,14)



Use DDA to draw line from (10,10) to (14,13)

L→R means X +ve

$|\Delta x| = 4 > |\Delta y| = 3$ means $|m| < 1$;
$\Delta x = 1$, $\Delta y = m$
$m = 3/4 = 0.75$ so
$\Delta x = 1$, $\Delta y = 0.75$

x and y both increasing so slope is +ve.
No change

Iteration	(x,y)		Plot(x,y)
1			(10,10)
2	(10,10)	(11,10.75)	(11,11)
3	(11,10.75)	(12,11.5)	(12,12)
4	(12,11.5)	(13,12.25)	(13,12)
5	(13,12.5)	(14,13)	(14,13)

Bresenham’s line drawing Algorithm

assignment

Circle

- A circle is defined as the set of points that are all at a given distance r from a center position say (x_c, y_c).
- Whatever is the center, we always start with center as (0,0). With center as origin and radius r, we know some points

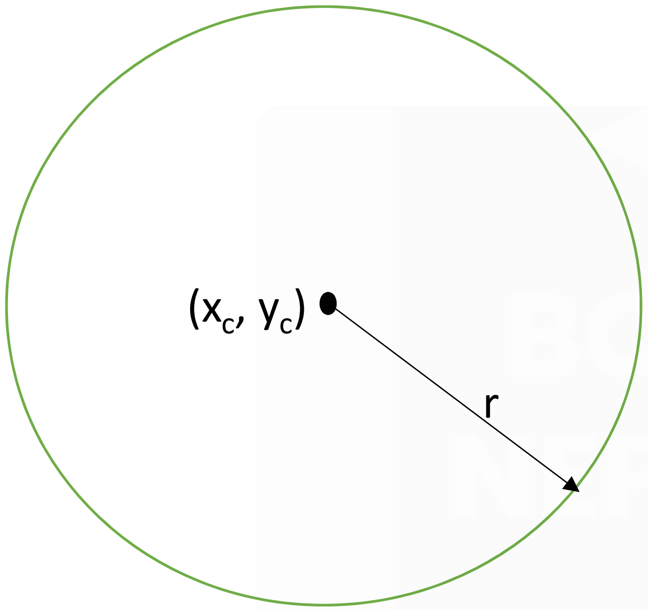
(0, r), (r, 0), (0, -r), (-r, 0)

$$x^2 + y^2 - r^2 = 0$$

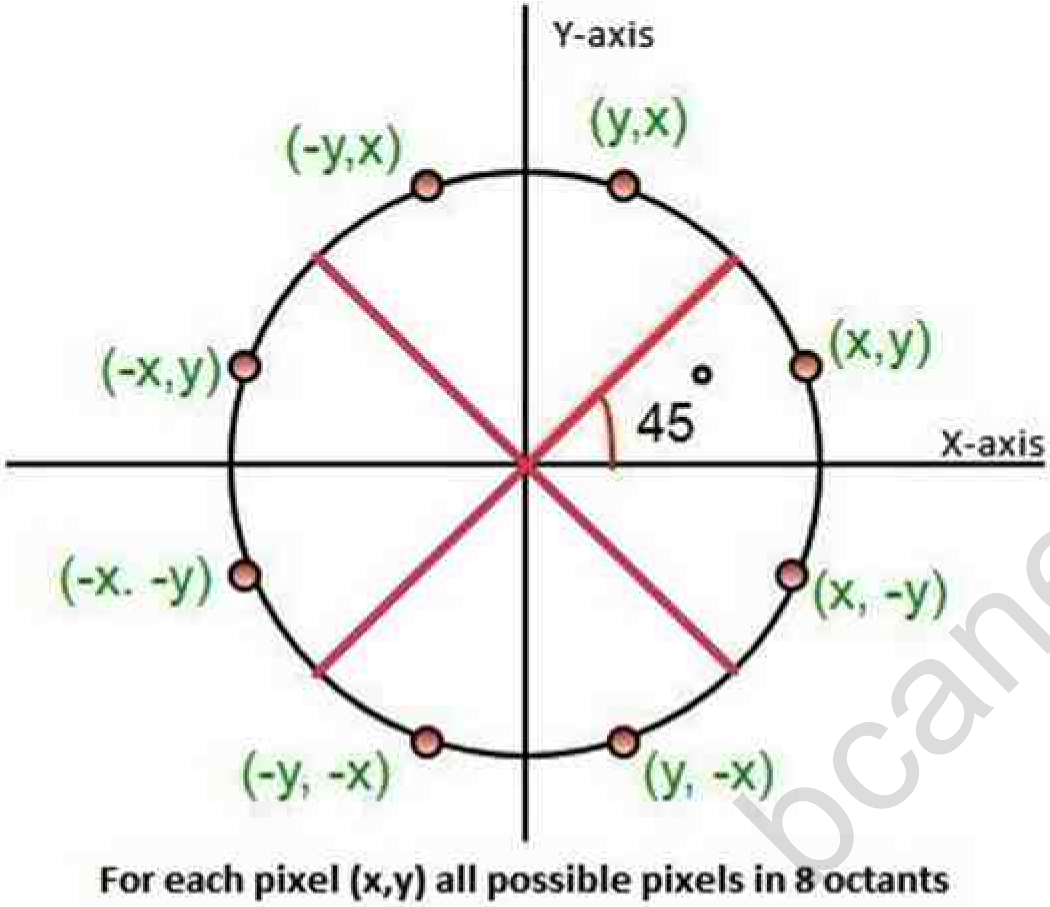
$x_k^2 + y_k^2 - r^2 < 0$, means (x_k, y_k) lies inside the circle boundary

$x_k^2 + y_k^2 - r^2 = 0$, means (x_k, y_k) lies on the circle boundary

$x_k^2 + y_k^2 - r^2 > 0$, means (x_k, y_k) lies outside the circle boundary



Octant symmetry: If we can find co-ordinates in one octant then we can find for all other octants with octant symmetry of circle.



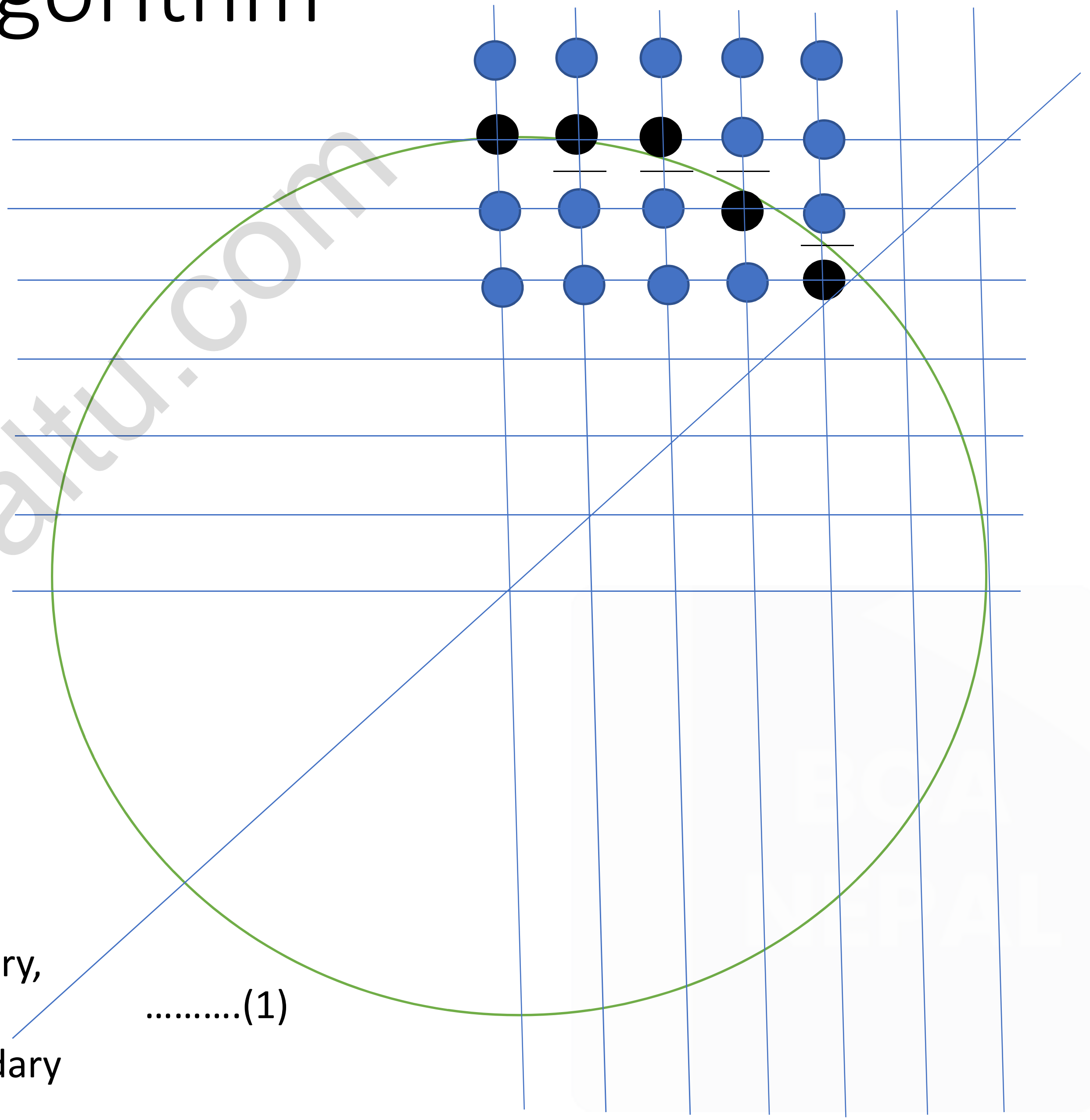
If x_k is current point then, X_{k+1} means next point
If x_k is current point then, x_k+1 means next point in the right of x_k
If x_k is current point then, x_k-1 means next point in the left of x_k

If y_k is current point then, y_{k+1} means next point
If y_k is current point then, y_k+1 means next point in the top of y_k
If y_k is current point then, y_k-1 means next point in the bottom of y_k

Mid-point circle drawing algorithm

Let's start from (r, 0), Determine the steps for point, here $\Delta x = 1$, $\Delta y = -m$

$x_{k+1} = x_k + 1$, $y_{k+1} = y_k - 1/2$



x_k always increments by 1means $x_{k+1} = x_k + 1$ whereas y_k need to be checked with midpoint and y_{k+1} can be either y_k or y_{k-1}

If (x_k, y_k) already plotted then,

$x_{k+1}^2 + y_{k+1}^2 - r^2 < 0$, means (x_{k+1}, y_{k+1}) lies inside the circle boundary,

$x_{k+1}^2 + y_{k+1}^2 - r^2 = 0$, means (x_{k+1}, y_{k+1}) lies on the circle boundary, ...(1)

$x_{k+1}^2 + y_{k+1}^2 - r^2 > 0$, means (x_{k+1}, y_{k+1}) lies outside the circle boundary

Decision parameter Pk,

$p_k = f_{circle}(x_k + 1, y_k - 1/2)^2$

$p_k = (x_k + 1)^2 + (y_k - 1/2)^2 - r^2$...........(2)

if $P_k<0$, then midpoint is inside the circle and y_k is closer to circle boundary. Else, midpoint is outside the circle. And $y_k - 1$ is closer to the circle boundary

Successive decision parameters are obtained using incremental calculations

i.e., next decision parameter is obtained at

$x_{k+1+1} = x_k+2$ and

$y_{k+1} - 1/2$

$\therefore p_{k+1} = f_{circle}(x_{k+1}+1, y_{k+1} - 1/2)^2$

$p_{k+1} = (x_{k+}1+1)^2 + (y_{k+1} - 1/2)^2 - r^2$.......... (3)

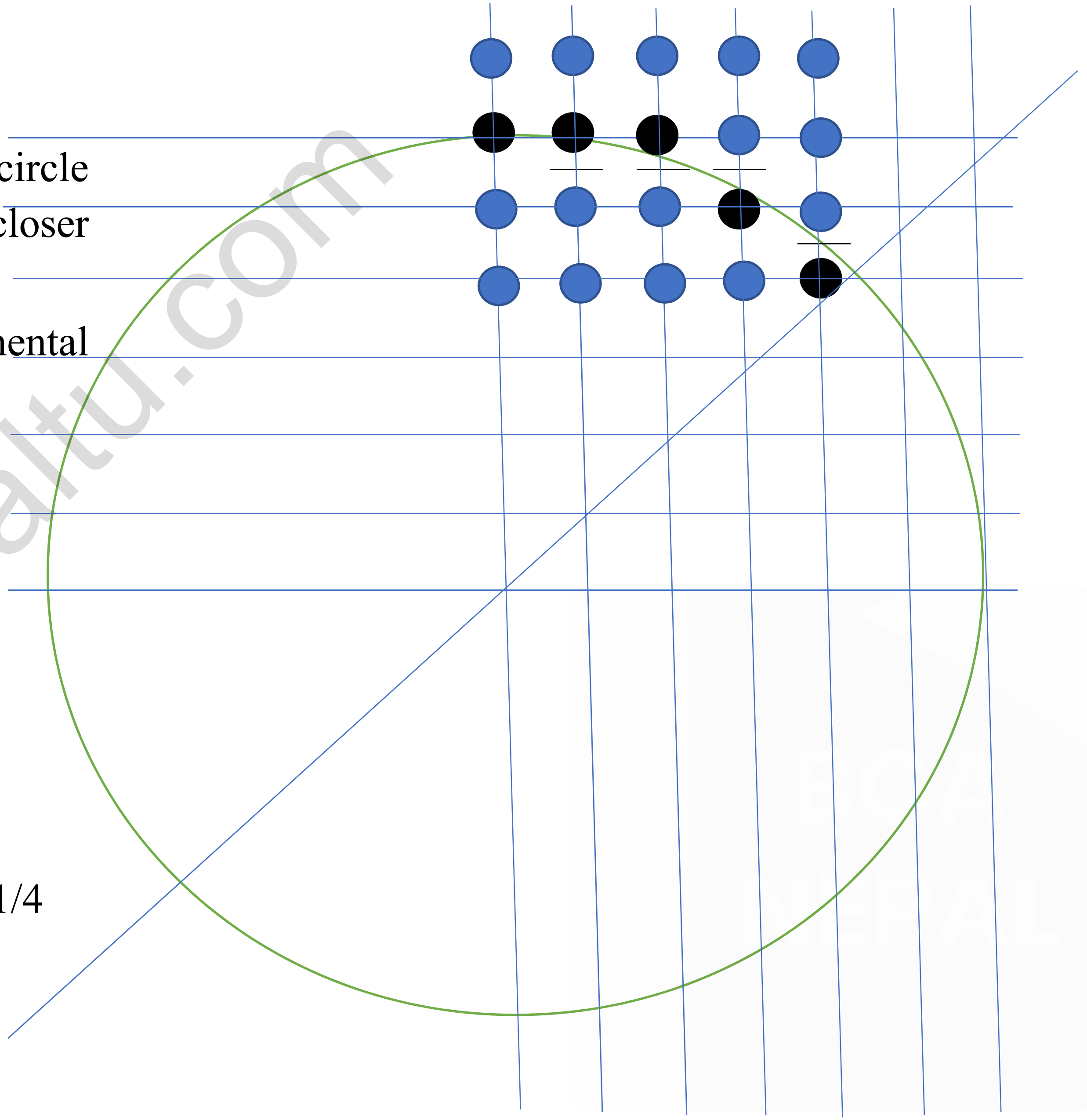
Subtracting equation (2) from (3)

$P_{k+1} - P_k = (x_k+1+1)^2+(y_{k+1}-1/2)^2-r^2-(x_k+1)^2-(y_k-1/2)^2+r^2$

$= (x_k+1)^2+2(x_k+1)+1+y_{k+1}{}^2-2y_{k+1}.1/2+1/4 - (x_k+1)^2-y_k{}^2+2y_k.1/2-1/4$

$= 2(x_k+1)+(y_{k+1}{}^2-y_k{}^2) - (y_{k+1}-y_k) + 1$

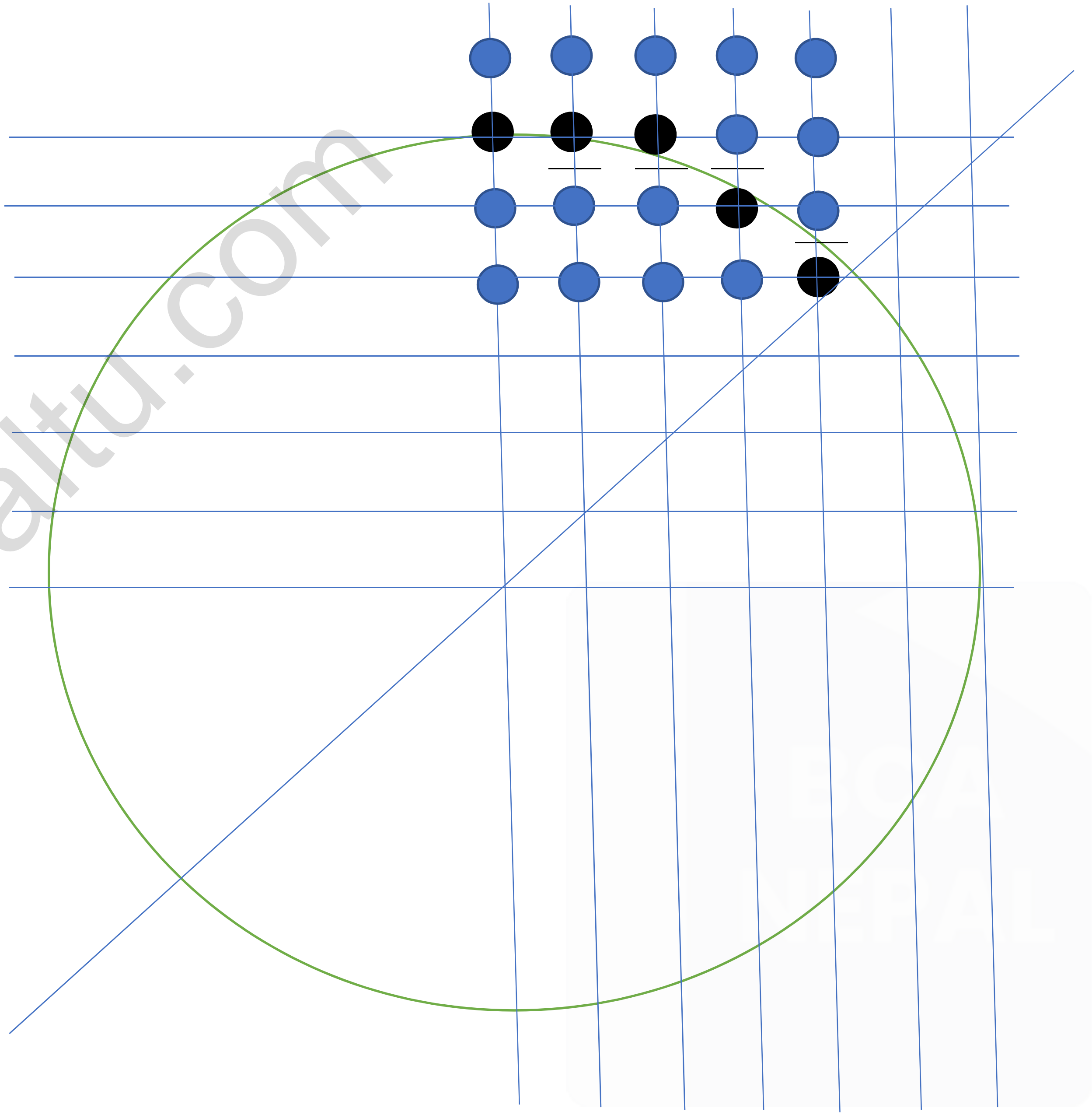
wherey_{k+1} is either y_k or y_k-1 depending on sign of p_k



If $p_k < 0$ then next pixel is at (x_k+1, y_k)

$$p_{k+1} = p_k + 2(x_k+1)+(y_{k+1}{}^2-y_k{}^2) - (y_{k+1}-y_k) + 1$$

$$p_{k+1} = p_k+2.\ x_{k+1} +1$$



If $p_k \geq 0$, then next pixel is at (x_k+1, y_k-1)

$$p_{k+1} = p_k + 2x_{k+1} + [(y_k-1)^2-y_k{}^2]-(y_k-1-y_k)+1$$

$$= p_k+ 2x_{k+1}+(y_k{}^2-2y_k+1-y_k{}^2)+1+1$$

$$= p_k + 2x_{k+1} - 2y_k + 1 + 1 + 1$$

$$= p_k + 2x_{k+1} - 2(y_k - 1) + 1$$

$$= p_k + 2x_{k+1} - 2y_{k+1} + 1$$

Initial decision parameter

The initial decision parameter is obtained by evaluating the circle function at starting point $(x_0, y_0) = (0, r)$.

$$p_0 = f_{circle}(1, r - 1/2)$$

$$= 1^2 + (r - 1/2)^2 - r^2$$

$$= 1 + r^2 - 2r \times + - r^2$$

$$p_0 = 1 - r$$

If the radius r is specified as an integer, we can simply round to $p_0 = 1 - r$

Determine the raster location along the circle octant in the first quadrant for a circle with radius 10 and center at origin.

Solution:

$r = 10$

Initial decision parameter, $p_0 = 1 - r = 1 - 10 = -9$

$(x_0, y_0) = (0, 10)$

$2x_0 = 0, 2y_0 = 20$

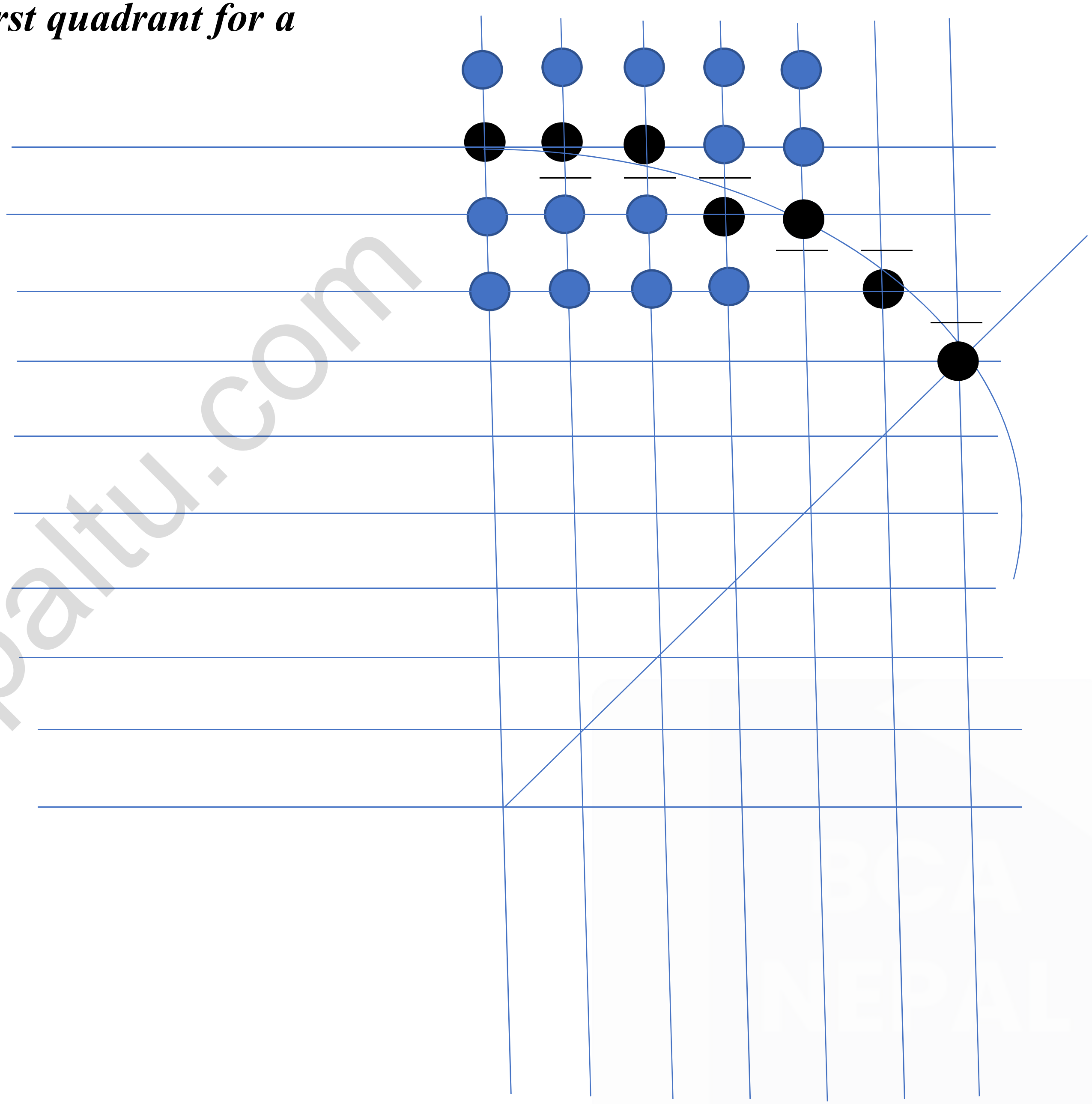
If $p_k < 0$ then next pixel is at (x_k+1, y_k)

$p_{k+1} = p_k + 2.\ x_{k+1} + 1$

If $p_k \geq 0$, then next pixel is at (x_k+1, y_k-1)

$p_{k+1} = p_k + 2x_{k+1} - 2y_{k+1} + 1$

k	p_k	(x_{k+1}, y_{k+1})	$2x_{k+1}$	$2y_{k+1}$
0	–9	(1, 10)	2	20
1	–6	(2, 10)	4	20
2	–1	(3, 10)	6	20
3	6	(4, 9)	8	18
4	–3	(5, 9)	10	18
5	8	(6, 8)	12	16
6	5	(7, 7)	14	14

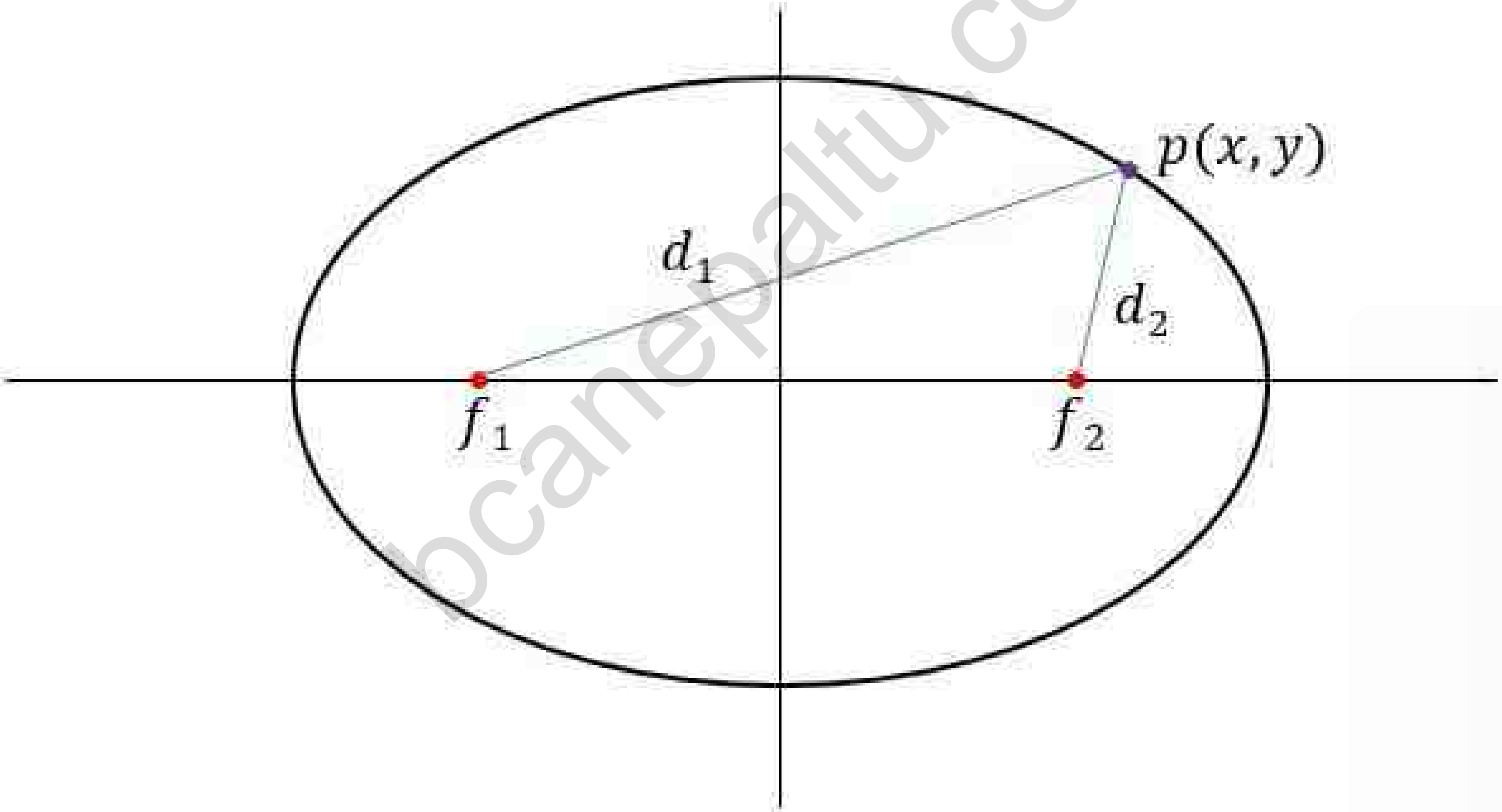


Assignment

1. Using midpoint circle algorithm, calculate the co-ordinate on the first quadrant of a circle having radius 6 and centre (20, 10).

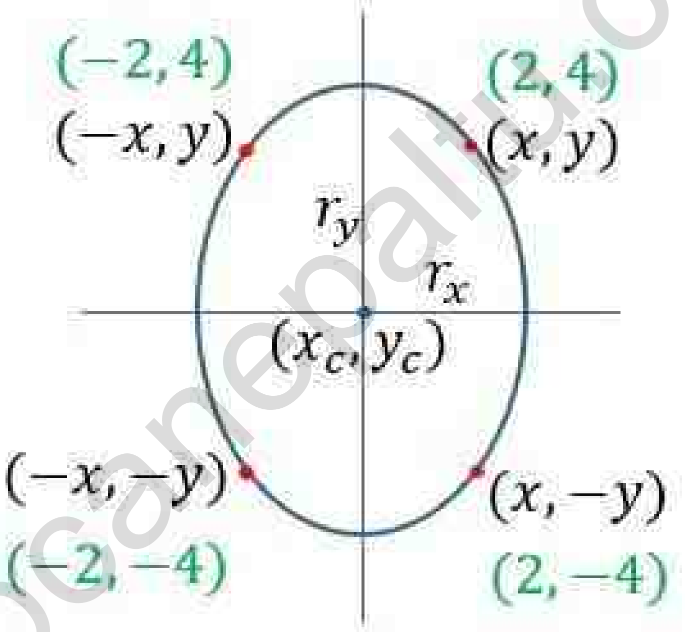
Ellipse

- AN ellipse is defined as the set of points such that the sum of the distances from two fixed positions (**foci**) is same for all points.



Properties of Ellipse-Symmetry

- Symmetry property further reduced computations.
- An ellipse in standard position is symmetric between quadrant.

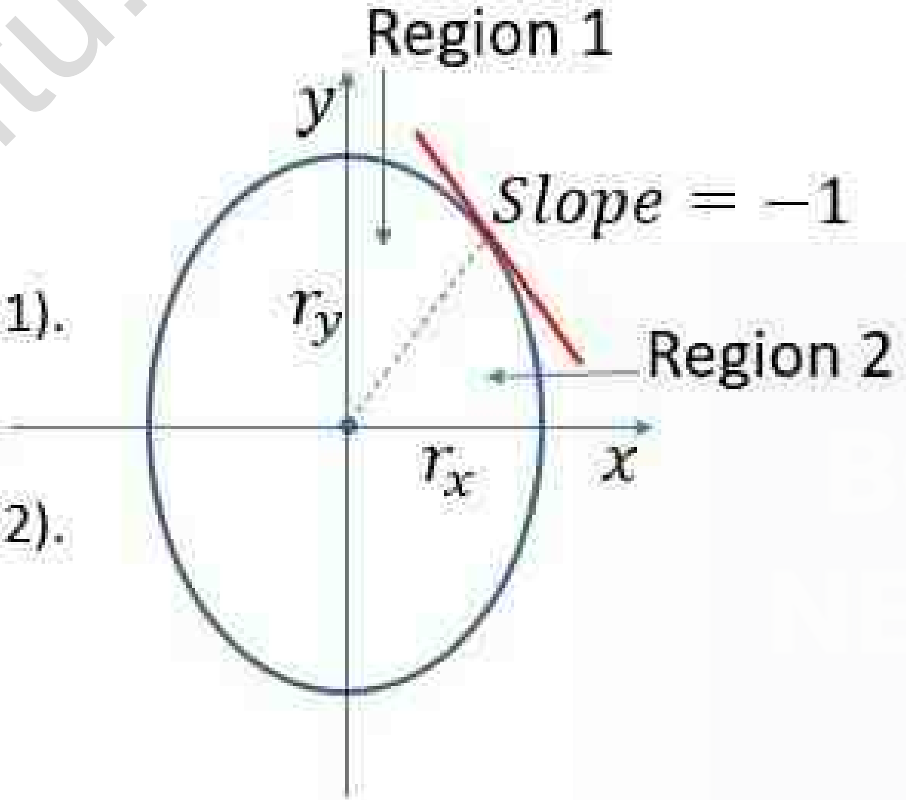


Introduction to Midpoint Ellipse Algorithm

- Given parameters $r_x, r_y, \&\ (x_c, y_c)$.
- We determine points (x, y) for an ellipse in standard position centered on the origin.
- Then we shift the points so the ellipse is centered at (x_c, y_c).
- If we want to display the ellipse in nonstandard position then we rotate the ellipse about its center to align with required direction.
- For the present we consider only the standard position.
- We draw ellipse in first quadrant and than use symmetry property for other three quadrant.

Regions in Midpoint Ellipse Algorithm

- In this method we divide first quadrant into two parts according to the slope of an ellipse
- Boundary divides region at
    - slope = -1.
- We take unit step in X direction
    - If magnitude of ellipse slope < 1 (Region 1).
- We take unit step in Y direction
    - If magnitude of ellipse slope > 1 (Region 2).



Ways of Processing Midpoint Ellipse Algorithm

- We can start from $(0, r_y)$ and step clockwise along the elliptical path in the first quadrant
- Alternatively, we could start at $(r_x, 0)$ and select points in a counterclockwise order.
- With parallel processors, we could calculate pixel positions in the two regions simultaneously.
- Here we consider sequential implementation of midpoint algorithm.
- We take the start position at $(0, r_y)$ and steps along the elliptical path in clockwise order through the first quadrant.

Decision Parameter Midpoint Ellipse Algorithm

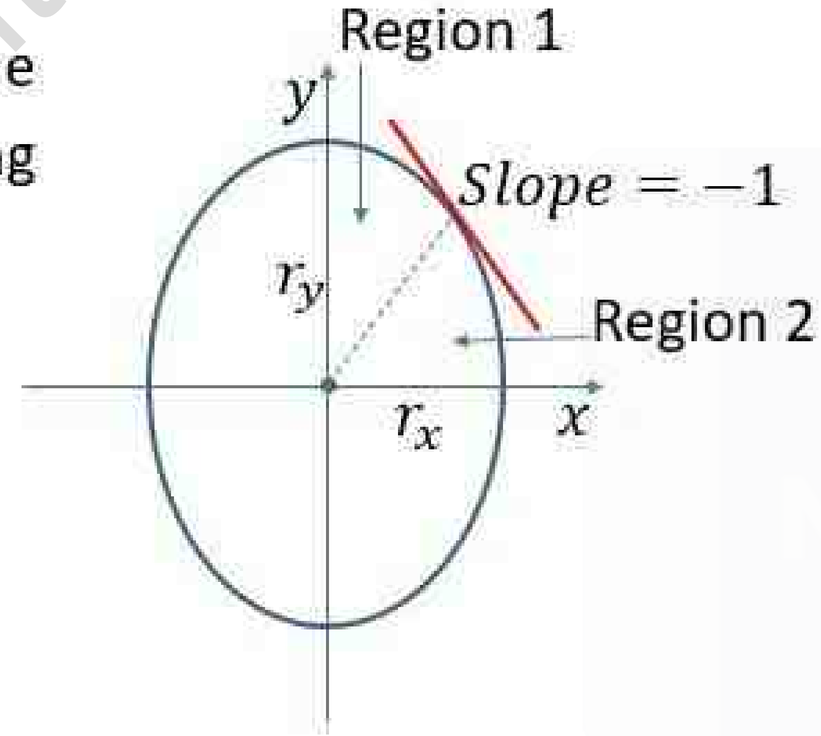
- We define ellipse function for center of ellipse at (0, 0) as follows.
- $f_{ellipse}(x, y) = {r_y}^2x^2 + {r_x}^2y^2 - {r_y}^2{r_x}^2$
- Which has the following properties:

$$f_{ellipse}(x, y) \begin{cases} < 0 & if\ (x, y) is\ inside\ the\ ellipse\ boundary \\ = 0 & if\ (x, y) is\ on\ the\ ellipse\ boundary \\ > 0 & if\ (x, y) is\ outside\ the\ ellipse\ boundary \end{cases}$$

- Thus the ellipse function serves as the decision parameter in the midpoint ellipse algorithm.
- At each sampling position we select the next pixel from two candidate pixel.

Processing Steps of Midpoint Ellipse Algorithm

- Starting at $(0,\ r_y)$ we take unit step in x direction until we reach the boundary between region-1 and region-2.
- Then we switch to unit steps in y direction in remaining portion on ellipse in first quadrant.
- At each step we need to test the value of the slope of the curve for deciding the end point of the region-1.



Decide Boundary between Region 1 and 2

- The ellipse slope is calculated using following equation.
- $\frac{dy}{dx} = -\frac{2{r_y}^2 x}{2{r_x}^2 y}$
- At boundary between region 1 and 2 slope= -1 and equation become.
- $2{r_y}^2 x = 2{r_x}^2 y$
- Therefore we move out of region 1 whenever following equation is false:
- $2{r_y}^2 x \leq 2{r_x}^2 y$

Assuming (x_k, y_k) has been illuminated (selected) we determine the next position along the ellipse path by evaluating the decision parameter at the midpoint $(x_k + 1, y_k - 1/2$)

we have to determine the next point

$(x_k + 1, y_k)$ or $(x_k + 1, y_k - 1)$

We define decision parameter at mid-point as

$p_{1k} = f_{ellipse} (x_k + 1, y_k - 1/2)^2$

$p_{1k} = r_y^2(x_k+1)^2 + r_x^2 (y_k - 1/2)^2 - r_x^2r_y^2$............(iii)

If $p_{1k}<0$, the mid-point is inside the ellipse and the pixel on scan line y_k is closer to the ellipse boundary

Otherwise,

The mid-point is outside or on the boundary and we select the pixel on scan line y_k-1

At the next sampling position $(x_{k+1}+1 = x_k+2)$, the decision parameter for region 1 is evaluated as

$p_{1k+1} = f_{ellipse} (x_{k+1}+1, y_{k+1} - 1/2)$

$p_{1k+1} = r_y^2[(x_k + 1) + 1]^2 + r_x^2 (y_{k+1} - 1/2)^2 - r_x^2r_y^2$.....(iv)

Subtracting equation (iii) from (iv)

$$p_{1k+1} - p_{1k} = r_y^2[(x_k + 1 + 1)^2 - (x_k+1)^2] + r_x^2[(y_{k+1} - 1/2)^2 - (y_k - 1/2)^2] - r_x^2 r_y^2 + r_x^2 r_y^2$$

$$= r_y^2[(x_k + 1)^2 + 2(x_k + 1) + 1 - (x_k+1)^2] + r_x^2[y_{k+1}^2 - 2y_{k+1} . 1/2 + 1/4 - y_k^2 + 2y_k . 1/2 - 1/4]$$

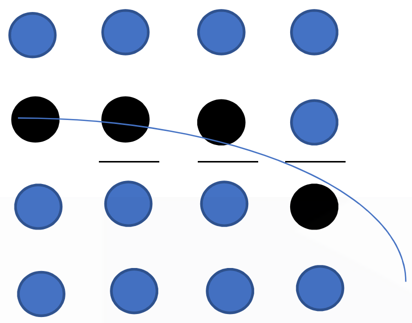
$$= r_y^2 .2(x_k + 1) + r_y^2 + r_x^2 [(y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k)]$$

$$= 2r_y^2(x_k + 1) + r_y^2 + r_x^2[(y_{k+1}^2 - y_k^2) - (y_{k+1} - y_k)]$$

Where y_{k+1} is either y_k or $y_k - 1$, depending on the sign of p_{1k}

$p_{1k} \leq 0$ i.e. $y_{k+1} = y_k$ then decision parameter is

$$p_{1k+1} = p_{1k} + 2r_y^2 (x_k +1) + r_y^2$$



If $p_{1k} > 0$, i.e. $y_{k+1} = y_k - 1$, then decision parameter is

$$p_{1k+1} = p_{1k} + 2r_y^2(x_k + 1) + r_y^2 - 2r_x^2 y_{k+1}$$

The initial decision parameter is evaluated at start position $(x_0, y_o) = (0, r_y)$ as

$$P_{10} = f_{ellipse}(1, r_y - 1/2)$$

$$= 1. r_y^2 + r_x^2 (r_y - 1/2)^2 - r_x^2 r_y^2$$

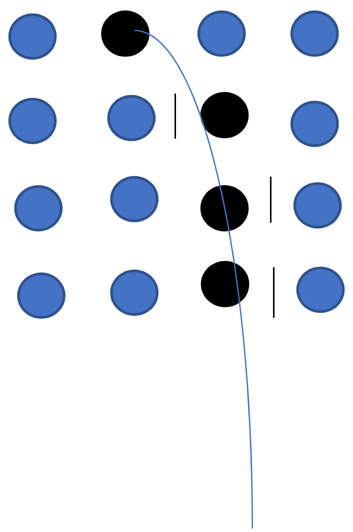
$$P_{10} = r_y^2 - r_x^2(r_y^2 - 2. r_y . 1/2 + ¼) + r_x^2 r_y^2$$

$$P_{10} = r_y^2 + r_x^2 r_y - ¼ \ r_x^2 \quad .... \quad (v)$$

Derivation for Region 2

- $p2_k = f_{ellipse}\left(x_k + \frac{1}{2}, y_k - 1\right)$
- $p2_k = r_y{}^2\left(x_k + \frac{1}{2}\right)^2 + r_x{}^2(y_k - 1)^2 - r_x{}^2 r_y{}^2$
- If $p2_k > 0$ the midpoint is outside the ellipse boundary, and we select the pixel at x_k.
- If $p2_k \le 0$ the midpoint is inside or on the ellipse boundary and we select $x_k + 1$.
- At the next sampling position decision parameter for region 2 is evaluated as.
- $p2_{k+1} = f_{ellipse}\left(x_{k+1} + \frac{1}{2}, y_{k+1} - 1\right)$
- $p2_{k+1} = r_y{}^2\left(x_{k+1} + \frac{1}{2}\right)^2 + r_x{}^2[(y_k - 1) - 1]^2 - r_x{}^2 r_y{}^2$
- Now subtract $p2_k$ from $p2_{k+1}$
- $p2_{k+1} - p2_k = r_y{}^2\left(x_{k+1} + \frac{1}{2}\right)^2 + r_x{}^2[(y_k - 1) - 1]^2 - r_x{}^2 r_y{}^2 - r_y{}^2\left(x_k + \frac{1}{2}\right)^2 - r_x{}^2(y_k - 1)^2 + r_x{}^2 r_y{}^2$

- $p2_{k+1} - p2_k = r_y{}^2\left(x_{k+1} + \frac{1}{2}\right)^2 + r_x{}^2[(y_k - 1) - 1]^2 - r_x{}^2 r_y{}^2 - r_y{}^2\left(x_k + \frac{1}{2}\right)^2 - r_x{}^2(y_k - 1)^2 + r_x{}^2 r_y{}^2$
- $p2_{k+1} - p2_k = r_y{}^2\left(x_{k+1} + \frac{1}{2}\right)^2 + r_x{}^2(y_k - 1)^2 - 2r_x{}^2(y_k - 1) + r_x{}^2 - r_y{}^2\left(x_k + \frac{1}{2}\right)^2 - r_x{}^2(y_k - 1)^2$
- $p2_{k+1} - p2_k = r_y{}^2\left(x_{k+1} + \frac{1}{2}\right)^2 - 2r_x{}^2(y_k - 1) + r_x{}^2 - r_y{}^2\left(x_k + \frac{1}{2}\right)^2$
- $p2_{k+1} - p2_k = -2r_x{}^2(y_k - 1) + r_x{}^2 + r_y{}^2\left[\left(x_{k+1} + \frac{1}{2}\right)^2 - \left(x_k + \frac{1}{2}\right)^2\right]$
- $p2_{k+1} - p2_k = -2r_x{}^2(y_k - 1) + r_x{}^2 + r_y{}^2\left[\left(x_{k+1} + \frac{1}{2}\right)^2 - \left(x_k + \frac{1}{2}\right)^2\right]$
- Now making $p2_{k+1}$ as subject.
- $p2_{k+1} = p2_k - 2r_x{}^2(y_k - 1) + r_x{}^2 + r_y{}^2\left[\left(x_{k+1} + \frac{1}{2}\right)^2 - \left(x_k + \frac{1}{2}\right)^2\right]$
- Here x_{k+1} is either x_k or $x_k + 1$, depends on the sign of $p2_k$.



If $p_{2k} > 0$, then

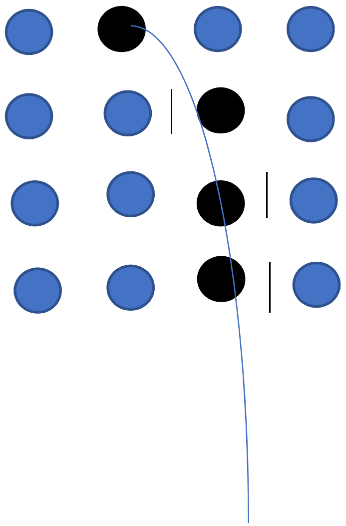
$x_{k+1} = x_k$

$p_{2k+1} = p_{2k} - 2r_x^2 y_{k+1} + r_x^2$

If $p_k \leq 0$, then

So $x_{k+1} = x_k + 1$

$p_{2k+1} = p_{2k} + 2r_y^2 x_{k+1} - 2r_y^2 y_{k+1} + r_x^2$



- In region-2 initial position is selected which is last position of region one and the initial decision parameter is calculated as follows.
- $p2_0 = f_{ellipse}\left(x_0 + \frac{1}{2}, y_0 - 1\right)$
- $p2_0 = r_y{}^2\left(x_0 + \frac{1}{2}\right)^2 + r_x{}^2(y_0 - 1)^2 - r_x{}^2 r_y{}^2$
- For simplify calculation of $p2_0$ we could also select pixel position in counterclockwise order starting at $(r_x, 0)$.

Thank you