GAME THEORY

HISTORY:

- Game theory came in to existence in 20th Century.
- In 1944 John Von Neumann and Oscar Morgenstern published a book Theory of game and Economic Behaviour,in which they discussed how businesses of all types may use this technique to determine the best strategies given a competitive business environment.

DEFINITION :

In business and economics literature, the term 'game' refers to a situation of conflict and competition in which two or more competitor(or participant) are involved in the decision making process in anticipation of certain outcome over a period of time.

BASIC TERM USED IN GAME THOERY

1.Player
2.Two-person game/n-person game
3.Zero-sum game/non-zero sum game
4.Strategy
   a. Pure Strategy
   b. Mixed Strategy
5.Saddle point

PLAYER :

The competitor are referred to as player. A player may be individual, a group of individual, or an organization.

TWO-PERSON GAME/ N-PERSON GAME :

- If a game involves only two player(competitors),then it is called a two-person game.
- If number of player are more than two, then game is referred to as n-person game.

ZERO SUM GAME/NON-ZERO SUM GAME :

In a game ,if sum of the gain to one player is exactly equal to the sum of losses to another player, so that the sum of the gains and losses equals to zero, then the game is said to be a zero-sum game. Otherwise it is said to be non zero-sum game.

STRATEGY :

- The decision rule by which a player determines his course of action is called a strategy.
- The strategy for a player is the list of all possible action that he will take for every payoff(outcome) that might arise.

PURE STRATEGY :

- This is a decision rule that is always used by the player to select the particular strategy(course of action).thus each player know in advance all strategies, out of which he always selects only one particular startegy,regardess of the other player's strategy
- The objective of the player is to maximize their gains or minimize their losses.

MIXED STRATEGY

- If a player decide in advance, to use all or some of his available courses of action in some fixed proportion, he is said to use mixed strategy.
- Thus mixed strategy is a selection among pure strategies with some fixed probabilities(proportions).

OPTIMUM STRATEGY :

- The particular strategy by which a player optimizes his gains or losses, without knowing the competitors strategies, is called optimal strategy

PAYOFF MATRIX

The payoff (a quantitative measure of satisfaction that a player gets at the end of the play) in terms of gains and losses, when player select their particular strategies, can be represented in the form of a matrix, called the payoff matrix.

PLAYER Y

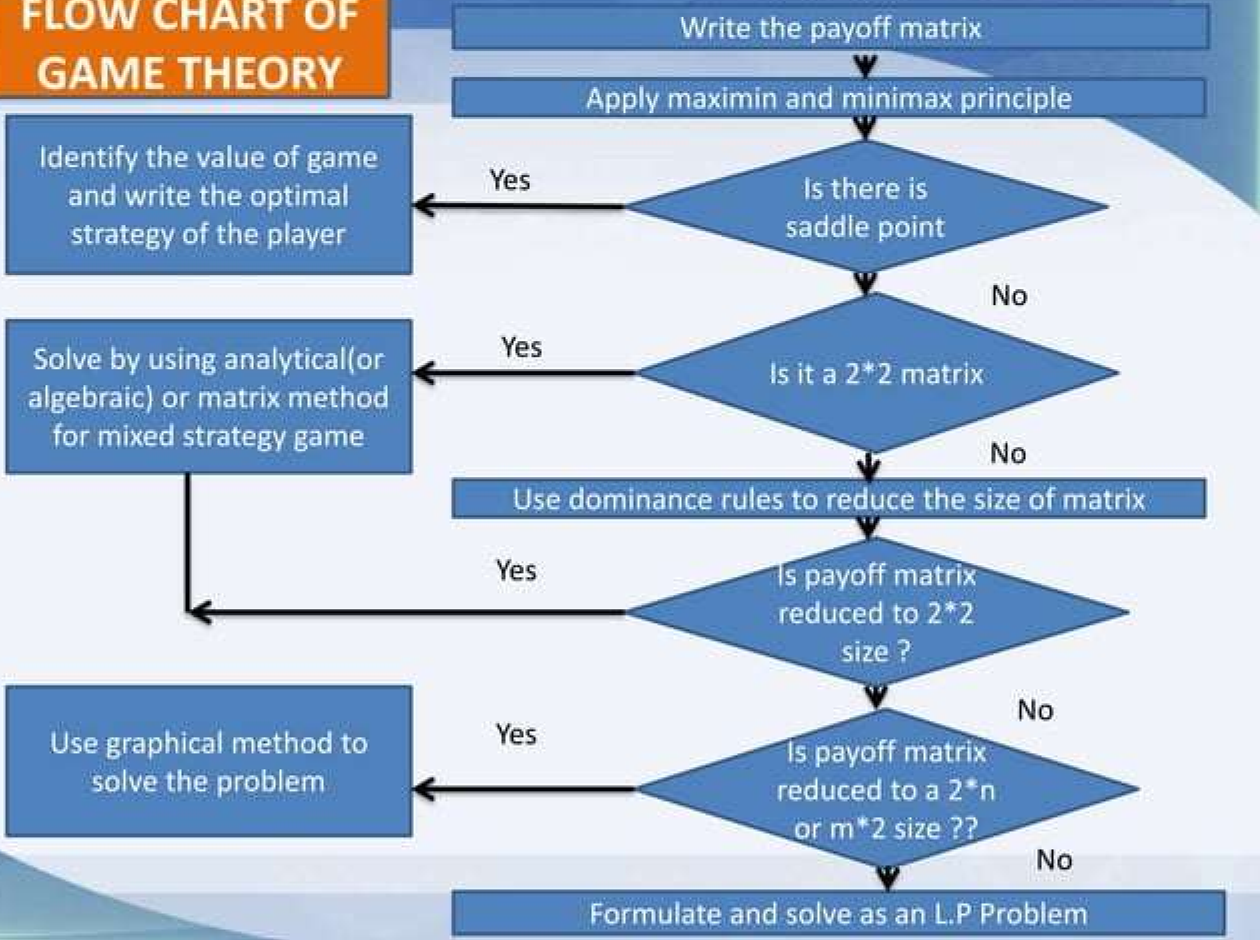
PLAYER X	Y1	Y2	Y3
X1	**24**	**36**	**8**
X2	32	20	16

In this pay-off matrix, positive pay-off is the gain to maximizing player (X) and loss to minimizing player (Y).E.g., if X chooses strategy X1 and Y chooses strategy Y1, then X's gain is 32 and Y's loss is 32.

ASSUMPTIONS OF THE GAME

- There are finite number of competitors.
- There is conflict of interests between them.
- Each player has available with him finite number of possible strategies(courses of action).
- One player attempts to maximize gains and the other attempts to minimize losses.
- Players know all possible available choices but does not know which one is going to be chosen.
- Players simultaneously select their respective courses of action.
- The payoff is fixed and determined in advance.
- Players have to make individual decisions without direct communication.

FLOW CHART OF GAME THEORY



Method to find value of game under decision making environment of certainty are as follows

- Decision Making Environment
  - Certainty
    - Non-Competitive Situation
    - Competitive Situation
      - Pure Strategy
        - Minimax and Maximin Principle(saddle point)
      - Mixed Strategy
        - 2*2 order payoff matrix
          - Algebraic Method
          - Analytical Method
          - Matrix Method
          - Short-cut Method
        - 2*n or m*2 order payoff matrix
          - Graphical Method
        - m*n order payoff matrix
          - LP Method
  - Uncertainty
  - Risk

RULES FOR GAME THEORY

RULE 1: Look for pure Strategy (Saddle point)

RULE 2: Reduce game by Dominance

If no pure strategies exist, the next step is to eliminate certain strategies(row/column) by law of Dominance.

RULE 3: Solve for mixed Strategy

A mixed strategy game can be solved by different solution method, such as

1. Arithmetic Method
2. Algebraic Method
3. Graphical method
4. Matrix Method
5. Short Cut Method

PURE STRATEGIES (MINIMAX AND MAXIMIN PRINCIPLE) :

Maximin Principle

- Maximize the player's minimum gains. That means select the strategy that gives the maximum gains among the row minimum value.

Minimax Principle

- Minimize the player's maximum gains. That means ,select the strategy that gives the minimum loss among the column maximum values.

Saddle Point

- If the maximin value equals the minimax value, the the game is said to have a saddle (equilibrium) point and the corresponding strategies are called "Optimal Strategies"

Value of game

This is the expected payoff at the end of the game, when each player uses his optimal strategy

- A game is said to be a fair game if the lower (maximin) and upper (minimax) value of the game are equal and both equals zero.
- A game is said to be a strictly determinable game if the lower (maximin) and upper (minimax) value of the game are equal and both equal the value of game.

RULES TO FINDOUT SADDLE POINT

Example:

	B1	B2	Minimum of row	
A1	4	6	4	Maximin
A2	3	5	3	
Maximum of column	4	6		Saddle point =4
	Minimax			

- Select the minimum (lowest) element in each row of the payoff matrix and write them under 'Minimum of row' heading. Then, select the largest element among these element and enclose it in a rectangle.
- Select the maximum (largest) element in each column of the pay off matrix and write them under 'column maxima' heading. Then select the lowest element among these elements and enclose it in a circle.
- Find out the element(s) that is same in the circle as the well as rectangle and mark the position of such element(s) in the matrix . This element represents the value of the game and is called the saddle (or equilibrium) point.

RULES OF DOMINANCE

- The rule of dominance are used to reduce the size of payoff matrix. These rule helps in deleting certain rows and columns of the payoff matrix that are inferior(less attractive) to at least one of the remaining rows and columns.
- Rows and columns once deleted can never be used for determining the optimum strategy for both players.
- The rule of dominance are especially used for the evaluation of 'two-person zero-sum' games without a saddle point(equilibrium) point.

Example :

Player B

Player A

	B_1	B_2	B_3
A_1	-5	10	20
A_2	5	-10	-10
A_3	5	-20	-20

Player B

Player A	B_1	B_2	B_3	Minimum of row	
A_1	-5	10	20	-5	Maximin
A_2	5	-10	-10	-10	
A_3	5	-20	-20	-20	
Maximum of column	5 Minimax	10	20		

- Given game have no saddle point because Maximin is not equal to Minimax. Therefore, further we Must try to reduce the size of the given payoff matrix as further as possible .

Note that : every element of column B3 is more than or equal to every corresponding element of Row B2 Evidently , the choice of strategy B3, will always result in more losses as compared to that of selecting the strategy B2.Thus , strategy B3 is inferior to B2. Hence, delete The B3 strategy from the payoff matrix. Reduced payoff matrix is …

Player B

Player A	B1	B2
A1	-5	10
A2	5	-10
A3	5	-20

- Now, it may be noted that strategy A_2 of player A is dominated by his A_3 strategy (every element of column A2 is more than or equal to every corresponding element of Row A3), since the profit due to strategy A2 is greater than or equal to the profit due to strategy A3. Hence, strategy A3(row 3) can be deleted from further consideration.

  Thus the reduced payoff matrix becomes:

Player B

Player A		B1	B2	Minimum of row	
	A1	-5	10	-5	→ Maximin
	A2	5	-10	-10	
Maximum of column		5 ↓ Minimax	10		

Note : the maximin value is not equal to minimax value, hence there is no saddle point. For this type of game situation, it is possible to obtain a solution by applying the concept of mixed strategies.

MIXED STRATEGY: GAME WITHOUT SADDLE POINT

In certain case, there is no pure strategy solution for a game ,i.e. no saddle point exists.

Algebraic Method

This method is used to determine the probability of using different strategies by players A and B. This method becomes quite lengthy when a number of Strategies for both the players are more than two.

- Players selects each of the available strategies for certain proportion of time i.e., each player selects a strategy with some probability.

It could be stated specifically with the help of following example :

	Player B		
	B1	B2	
Player A A1	5	2	p1
A2	3	4	p2
	q1	q2	

As it can be seen that saddle point does not exist, we follow following method :
Let,

p1= probability of selecting the strategy A1,
p2= probability of selecting the strategy A2,
q1= probability of selecting the strategy B1
q2= probability of selecting the strategy B2

And V be the value of game,

5p1+3p2=v---------------------(1)

2p1+4p2=v---------------------(2)

p1+p2=1-------------------------(3)

After solving above three equation,

$P1=1/4$ | $P2=3/2$ | $V=7/2$

Therefore optimum strategy for player A is (1/4,3/4).player A should play strategy A1 25% time and A2 75% time in order to maximize is expected game by 7/2 units

In the same way for player B

$5q1+2q2=V$----------(1)
$3q1+4q2=V$----------(2)
$q1+q2=1$--------------(3)

After solving above equation,

$q1=1/2$ | $q2=1/2$ | $V=7/2$

Therfore optimum strategies for player B is(1/2,1/2) player b should play strategy B1 50% time and B2 50% time in order to minimize is expected game by 7/2 unit.

Arithmetic Method

The arithmetic method (also known as short cut method) provide an easy method for finding optimal strategies for each player in a payoff matrix of size 2*2,without saddle point.
The step of this method are as follows-

Step 1: Find the differences between the two values in the first row and put it against the second row of the matrix, neglecting the negative sign (if any).

Step 2 : Find the differences between the two values in the second row and put it against the first row of the matrix, neglecting the negative sign (if any).

Step 3 : Repeat step 1 and step 2 for two column also.

The value obtained by 'swapping the difference' represent the optimal relative frequency of play for both players strategies. These may be converted to probabilities by dividing each of them by their sum.

Example: no saddle point in below payoff matrix

Player B

Player A

	B1	B2
A1	2	-1
A2	-1	0

Neglect negative sign.

-1-0=-1 , 1/1+3=1/4

2-(-1)=3 , 3/1+3=3/4

-1-0=-1 2-(-1)=3

1/1+3=1/4 3/1+3=3/4

Using A's oddments

Player B plays B1

2*1/4+(-1)*3/4=-1/4

Player B plays B2

(-1)*1/4+0*3/4=-1/4

Using B's oddments

Player A plays A1

$$2*1/4+(-1)*3/4=-1/4$$

Player A plays A2

$$(-1)*1/4+0*3/4=-1/4$$

Therefore the optimum strategy for player A is (1/4,3/4) and player b is (1/4,3/4) and value of game is -1/4

GRAPHICAL METHOD

The graphical method is useful for the game where the payoff matrix is of the size 2*n or m*2 .i.e. the game with mixed strategies that has only two undominated pure strategies for one of the player in two-person zero-sum game.

Example:

Player A \ Player B	B1	B2	B3	B4	
A1	2	2	3	-2	p1
A2	4	3	2	6	p2

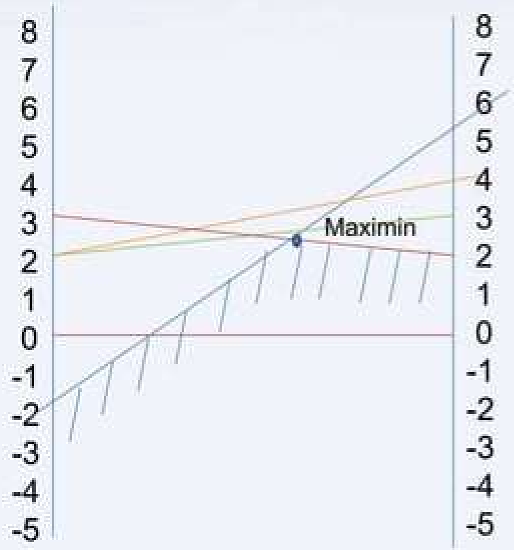
Step 1 : As this 2*3 matrix does not have saddle point. Therefore graphic method is to be used to reduce it to 2*2 game.

Step 2 : let p1 and p2 be the probabilities with which player A uses his pure strategies, The expected pay-off of player A corresponding t player B's pure strategies is :

B's Pure Strategies	E(v) A's expected pay-off
B1	2p1+4p2
B2	2p1+3p2
B3	3p1+2p2
B4	-2p1+6p2

These three expected pay-off lines are plotted on the graph to solve the game

Step 3 : Graph for player A. draw two parallel lines apart from each other and mark a scale on each. These two represent the two strategies of player A. The graph is :



From above graph given payoff matrix is reduced to 2*2 matrix and it can be solved by using arithmetic matrix.

Player B

		B3	B4		
Player A	A1	3	-2	4	4/9
	A2	2	6	5	5/9
		8	1		
		8/9	1/9		

Value of game=3*4/9+2*5/9=22/9

Optimum strategy of A is (4/9,5/9) and B is(0,0,8/9,1/9) and value of game is 22/9.

SIGNIFICANCE

- **Helps in decision making** : Game theory develops a framework for analyzing decision-makings under the situations of inter-dependence of firms with existing uncertainties about the competitor's reactions to any course of action adopted by a firm.

- **Provide scientific quantitative technique**: This theory outlines a scientific quantitative technique which can be fruitfully used by players to arrive at an optimal strategy, given firm's objectives.
- **Gives insight into situation of conflicting interests**: game theory gives insight into several less-known aspects which arise in situations of conflicting interests. For example, it describes and explains the phenomena of bargaining and coalition-formation.

LIMITATIONS

1.The assumption that the players have the knowledge about their own pay-offs and pay-offs of others is rather unrealistic. He can only make a guess of his own and his rivals' strategies.

2.As the number of maximum and minimax show that the gaming strategies becomes increasingly complex and difficult. In practice, there are many firms in an oligopoly situation and game theory cannot be very helpful in such situation

3.The assumptions of maximum and minimax show that the players are risk-averse and have complete knowledge the strategies. These do not seen practical.

4. Rather than each player in an oligopoly situation working under uncertain conditions, the players will allow each other to share the secrets of business in order to work out a collusion. Thus, the mixed strategy are also not very useful.


Thank you