SHAID SMARAK COLLEGE

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Assignment No. 3 of Digital logics

Submitted By:-

1st semester

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Submitted To:-

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1) Express the following function in SOP & POS.

↳ a) $F(A,B,C,D) = B'D + \cancel{A'B}\ \cancel{A'B}\ A'D + BD$

→ The function has A, B, C, D as variables, so, it has 16 terms.

- 1st term $B'D$ is missing two variables A & C.

$\therefore B'D = B'D(A + A')$
$= AB'D + A'B'D$

Again

$B'D = AB'D + A'B'D$
$= AB'D(C+C') + A'B'D(C+C')$
$= AB'CD + AB'C'D + A'B'CD + A'B'C'D$

- 2nd term $A'D$ is also missing two variables B & C

$\therefore A'D = A'D(B + B')$
$= A'BD + A'B'D$
$= A'BD(C+C') + A'B'D(C+C')$
$= A'BCD + A'BC'D + A'B'CD + A'B'C'D$

- 3rd term BD is missing A & C.

$\therefore BD = BD(A + A')$
$= ABD + A'BD$
$= ABD(C+C') + A'BD(C+C')$
$= ABCD + ABC'D + A'BCD + A'BC'D$

Now, combining all terms of the function, we g[et]

$F = AB'CD + AB'C'D + A'B'CD + A'B'C'D + A'BCD + A'BC'D +$
$A'B'CD + A'B'C'D + ABCD + ABC'D + A'BCD + A'BC'D$

$= AB'CD + AB'C'D + A'B'CD + A'B'C'D + A'BCD + A'BC'D +$
$ABCD + ABC'D$

$[\because x + x = x]$

constructing truth table, we got:

A	B	C	D	Minterms	Designation
0	0	0	0	$A'B'C'D'$	m_0
0	0	0	1	$A'B'C'D$	m_1
0	0	1	0	$A'B'CD'$	m_2
0	0	1	1	$A'B'CD$	m_3
0	1	0	0	$A'BC'D'$	m_4
0	1	0	1	$A'BC'D$	m_5
0	1	1	0	$A'BCD'$	m_6
0	1	1	1	$A'BCD$	m_7
1	0	0	0	$AB'C'D'$	m_8
1	0	0	1	$AB'C'D$	m_9
1	0	1	0	$AB'CD'$	m_{10}
1	0	1	1	$AB'CD$	m_{11}
1	1	0	0	$ABC'D'$	m_{12}
1	1	0	1	$ABC'D$	m_{13}
1	1	1	0	$ABCD'$	m_{14}
1	1	1	1	$ABCD$	m_{15}

In ascending order we got:

$F = m_{11} + m_9 + m_3 + m_1 + m_7 + m_5 + m_{15} + m_{13}$

$= m_1 + m_3 + m_5 + m_7 + m_9 + m_{11} + m_{13} + m_{15}$

$\therefore F(A,B,C,D) = \Sigma(1,3,5,7,9,11,13,15)$

Now, In POS.

$F(A,B,C,D) = B'D + A'D + BD$

$= D(B' + A' + B)$

$= D(1 + A')$ $[\because B + B' = 1]$

$= D \cdot 1$ $[\because 1 + A' = 1]$

$= D$

$\therefore$ The function is missing 3 variables A, B & C

So, ~~B=~~ $F = D + AA'$

$= (D+A)(D+A')$

Here, Both terms are missing B & C. So,

~~A+D~~ $F = (D+A) + BB'$

$= (A+B+D)(A+B'+D)$

$= (A+B+D+CC')(A+B'+D+CC')$

$= (A+B+C+D)(A+B+C'+D)(A+B'+C+D)$
$(A+B'+C'+D)$

Again,

$F = (D+A') + BB'$

$= (A'+B+D)(A'+B'+D)$

$= (A'+B+D+CC')(A'+B'+D+CC')$

$= (A'+B+C+D)(A'+B+C'+D)(A'+B'+C+D)$
$(A'+B'+C'+D)$

combining all term & removing the repeated ones, we get:

$F = (A+B+C+D)(A+B+C'+D)(A+B'+C+D)(A+B'+C'+D$
$\cdot (A'+B+C+D)(A'+B+C'+D)(A'+B'+C+D)(A'+B'+C'+$

$= M_0 M_2 M_4 M_6 M_8 M_{10} M_{12} M_{14}$

$\therefore F(A,B,C,D) = \pi(0,2,4,6,8,10,12,14)$

b) $F(x,y,z) = (xy+z)(xz+y)$

$\hookrightarrow$ $F = (xy+z)(xz+y)$

$= (x+z)(y+z)(x+y)(y+z)$

$= (x+y)(x+z)(y+z)$ $\quad [\because x+x = x]$

$= (x+yz)(y+z)$

$= xz + xy + yzz + zyy$

$= xz + xy + yz + yz$ $\quad [\because x \cdot x = x]$

$= xy + xz + yz$

- 1st xy is missing z, so.

  $F = xy(z+z')$

  $= xyz + xyz'$

- 2nd term xz is missing y, so.

  $F = xz(y+y')$

  $= xyz + xy'z$

- 3rd term yz is missing x, so

  $F = yz(x+x')$

  $= xyz + x'yz$

combining all terms & removing repeated ones, we get:

$F = xyz + xyz' + xy'z + x'yz$

Now, In Truth table:-

x	y	z	Minterms	Designation
0	0	0	$x'y'z'$	m_0
0	0	1	$x'y'z$	m_1
0	1	0	$x'yz'$	m_2
0	1	1	$x'yz$	m_3
1	0	0	$xy'z'$	m_4
1	0	1	$xy'z$	m_5
1	1	0	xyz'	m_6
1	1	1	xyz	m_7

From the above truth table, we get

$F = m_3 + m_5 + m_6 + m_7$

$\therefore F(x,y,z) = \Sigma(3,5,6,7)$

Again, we have:

$F(x,y,z) = (xy+z)(xz+y)$

$= (x+z)(y+z)(x+y)(y+z)$

$= (x+y)(x+z)(y+z)$

Each term in the function is missing one variable, we have:-

- $x+y = x+y+\cancel{z}zz' = (x+y+z)(x+y+z')$
- $x+z = x+z+yy' = (x+y+z)(x+y'+z)$
- $y+z = y+z+xx' = (x+y+z)(x'+y+z)$

combining & removing the repeated ones:

$F = (x+y+z)(x+y+z')(x+y'+z)(x'+y+z)$

Now, In truth table:

x	y	z	Maxterms	Designation
0	0	0	$x+y+z$	M_0
0	0	1	$x+y+z'$	M_1
0	1	0	$x+y'+z$	M_2
0	1	1	$x+y'+z'$	M_3
1	0	0	$x'+y+z$	M_4
1	0	1	$x'+y+z'$	M_5
1	1	0	$x'+y'+z$	M_6
1	1	1	$x'+y'+z'$	M_7

From the truth table, we get.

$F(x,y,z) = M_0, M_1, M_2, M_4$

$\therefore F(x,y,z) = \Pi(0,1,2,4)$

② Express the complement of the following SOP.

a) $F(A,B,C,D) = \Sigma(0,2,6,11,13,14)$

↳ $F'(A,B,C,D) = \Sigma(1,3,4,5,\cancel{6}7,8,9,10,12,15)$

$= m_1 + m_3 + m_4 + m_5 + m_7 + m_8 + m_9 + m_{10} + m_{12} + m_{15}$

$= A'B'C'D + A'B'CD + ABC'D' + A'BC'D + A'BCD + AB'C'D' + AB'C'D + AB'CD' + ABC'D + ABCD.$

b) $F(x,y,z) = \Pi(0,3,6,7)$

↳ $F'(x,y,z) = M_1 \cdot M_2, M_4 \cdot M_5$

$= (x+y+z')(x+y'+z)(x'+y+z)(z'+y+z')$

③ convert the following into other canonical form.

a) $F(x,y,z) = \Sigma(1,3,7)$

↳ constructing a truth table for the function.

x	y	z	Minterms	Designation
0	0	0	$x'y'z'$	m_0
0	0	1	$x'y'z$	m_1
0	1	0	$x'yz'$	m_2
0	1	1	$x'yz$	m_3
1	0	0	$xy'z'$	m_4
1	0	1	$xy'z$	m_5
1	1	0	xyz'	m_6
1	1	1	xyz	m_7

From the truth table:-

$F(x,y,z) = \Sigma(1,3,7)$

$= m_1 + m_3 + m_7$

$= x'y'z + x'yz + xyZ$

Now,

complement of $F(x, y, z)$,

$$F'(x, y, z) = \{\Sigma(1, 3, 7)\}'$$
$$= \Sigma(0, 2, 4, 5, 6)$$
$$= m_0 + m_2 + m_4 + m_5 + m_6$$

Now,

$$F'(x, y, z) = (m_0 + m_2 + m_4 + m_5 + m_6)'$$
$$= m_0' \cdot m_2' \cdot m_4' \cdot m_5' \cdot m_6'$$
$$= (x'y'z')' \cdot (x'yz)' \cdot (xy'z')' \cdot (xy'z)'(xyz')'$$
$$= (x+y+z)(x+y'+z')(x'+y+z)$$
$$(x'+y+z')(x'+y'+z)$$
$$= M_0 M_2 M_4 M_5 M_6$$

(b) $F(A, B, C, D) = \Pi(0, 1, 2, 3, 4, 6, 12)$

↳ constructing a truth table for the function

A	B	C	D	Min	Dign	Max	Dign
0	0	0	0	$A'B'C'D'$	m_0	$A+B+C+D$	M_0
0	0	0	1	$A'B'C'D$	m_1	$A+B+C+D'$	M_1
0	0	1	0	$A'B'CD'$	m_2	$A+B+C'+D$	M_2
0	0	1	1	$A'B'CD$	m_3	$A+B+C'+D'$	M_3
0	1	0	0	$A'BC'D'$	m_4	$A+B'+C+D$	M_4
0	1	0	1	$A'BC'D$	m_5	$A+B'+C+D'$	M_5
0	1	1	0	$A'BCD'$	m_6	$A+B'+C'+D$	M_6
0	1	1	1	$A'BCD$	m_7	$A+B'+C'+D'$	M_7
1	0	0	0	$AB'C'D'$	m_8	$A'+B+C+D$	M_8
1	0	0	1	$AB'C'D$	m_9	$A'+B+C+D'$	M_9
1	0	1	0	$AB'CD'$	m_{10}	$A'+B+C'+D$	M_{10}
1	0	1	1	$AB'CD$	m_{11}	$A'+B+C'+D'$	M_{11}
1	1	0	0	$ABC'D'$	m_{12}	$A'+B'+C+D$	M_{12}
1	1	0	1	$ABC'D$	m_{13}	$A'+B'+C+d$	M_{13}
1	1	1	0	$ABCD'$	m_{14}	$A'+B'+C'+D$	M_{14}
1	1	1	1	$ABCD$	m_{15}	$A'+B'+C'+D'$	

From the truth table:-

$F(A,B,C,D) = \pi(0,1,2,3,4,6,12)$

$= M_0 M_1 M_2 M_3 M_4 M_6 M_{12}$

$\therefore F'(A,B,C,D) = \pi(5,7,8,9,10,11,13,14,15)$

$= M_5 M_7 M_8 M_9 M_{10} M_{11} M_{13} M_{14} M_{15}$

Now,

$F(A,B,C,D) = (M_5 M_7 M_8 M_9 M_{10} M_{11} M_{13} M_{14} M_{15})'$

$= m_5 + m_7 + m_8 + m_9 + m_{10} + m_{11} + m_{13} + m_{14} + m_{15}$

$\therefore F(A,B,C,D) = \Sigma(5,7,8,9,10,11,13,14,15)$

4) Convert the following into SOP & POS.

a) $(AB+C)(B+C'D)$

↳ let $F' = (AB+C)(B+C'D)$

$= AB.B + ABC'D + BC + CC'D$

$= AB + ABC'D + BC$

• 1st term AB is missing C & D

$F = AB(C+C') = ABC + ABC'$

$= ABC(D+D') + ABC'(D+D')$

$= ABCD + ABCD' + ABC'D + ABC'D'$

• 3rd term BC is missing A & D

$F = BC(A+A') = ABC + A'BC$

$= ABC(D+D') + A'BC(D+D')$

$= ABCD + ABCD' + A'BCD + A'BCD'$

combining & removing repeated terms.

$F = ABCD + ABC'D + ABCD' + ABC'D' + A'BCD + A'BCD'$

$= m_{15} + m_{13} + m_{14} + m_{12} + m_7 + m_6$

$= \Sigma(6,7,12,13,14,15)$

Again,

$$F = (AB+C)(B+C'D)$$
$$= (A+C)(C+B)(C'+B)(B+D)$$
$$= (A+C)(B+D)(C'+B)(C+B)$$

All the terms are missing two variable,

- 1st term $(A+C)$ is missing B & D

$$F = A+C+BB' = (A+B+C)(A+B'+C)$$
$$= (A+B+C+DD')(A+B'+C+DD')$$
$$= (A+B+C+D)(A+B+C+D')(A+B'+C+D)(A'+B+C+D')$$

- 2nd term $(B+D)$ is missing A & C

$$F = B+D+AA' = (A+B+D)(A'+B+D)$$
$$= (A+B+D+CC')(A'+B+D+CC')$$
$$= (A+B+C+D)(A+B+C'+D)(A'+B+C+D)(A'+B+C'+D)$$

- 3rd term $(C'+B)$ is missing A & D

$$F = C'+B+AA' = (A+B+C')(A'+B+C')$$
$$= (A+B+C'+DD')(A'+B+C'+DD')$$
$$= (A+B+C'+D)(A+B+C'+D')(A'+B+C'+D)(A'+B+C'+D')$$

- 4th term $(C+B)$ is also missing A & D

$$F = C+B+AA' = (A+B+C)(A'+B+C)$$
$$= (A+B+C+DD')(A'+B+C+DD')$$
$$= (A+B+C+D)(A+B+C+D')(A'+B+C+D)(A'+B+C+D')$$

Now, combining & removing repeated terms,

$$F = (A+B+C+D)(A+B+C+D')(A+B'+C+D)(A+B'+C+D')$$
$$(A'+B+C+D)(A'+B+C+D')(A'+B+C'+D)(A'+B+C'+D')$$
$$(A+B+C'+D)(A+B+C'+D').$$

$$= M_0 M_1 M_4 M_5 M_8 M_9 M_{10} M_{11} M_2 M_3$$
$$= M_0 M_1 M_2 M_3 M_4 M_5 M_8 M_9 M_{10} M_{11}$$
$$= \Pi(0, 1, 2, 3, 4, 5, 8, 9, 10, 11)$$

(b) $x' + x(x+y')(y+z')$

↳ let $F = x' + x(x+y')(y+z')$

$= x' + (xx + xy')(y+z')$

$= x' + x(1+y')(y+z')$

$= x' + xy + xz'$

All term are missing atleast one variable.

- 1st term x' is missing y & z

$F = x'(y+y') = x'y + x'y'$

$= x'y(z+z') + x'y'(z+z')$

$= x'yz + x'yz' + x'y'z + x'y'z'$.

- 2nd term is missing z

$F = xy(z+z')$

$= xyz + xyz'$

- 3rd term is missing y

$F = xz'(y+y') = xyz' + xy'z'$

Now, combining & removing repeated ones.

$F = x'yz + x'yz' + x'y'z + x'y'z' + xyz + xyz' + xy'z'$

We know,

$F = m_3 + m_2 + m_1 + m_0 + m_7 + m_6 + m_4$

$\therefore F = \Sigma(0, 1, 2, 3, 4, 6, 7)$ #

Again,

$F = x' + x(x+y')(y+z')$

$= x' + (xx + xy')(y+z')$

$= x' + x(1+y')(y+z')$

$= x' + \cancel{xy} \; x(y+z')$

$= x' + y + z'$

$= x' + y + z'$

We know that

$F = (x' + y + z')$

$= M_5$

$\therefore F = \pi(5)$ #

(5) Find the complement of the following.

a) $xy' + x'y$

↳ let $F = xy' + x'y$

$= (xy')' . (x'y)'$ [∵ Demorgans]

$= (x'+y'')(x''+y')$ [∵ "]

$= (x'+y)(x+y')$

$= (x'x + x'y' + xy + yy')$

$= xy + x'y'$

b) $(AB'+C)D' + E$

↳ let $F = (AB'+C)D' + E$

$= AB'D' + CD' + E$

$= (AB'D' + CD' + E)'$

$= A'B''D'' + C'D'' + E'$

$= \cancel{A'BD} + \cancel{CD} + \cancel{E'}$

$= (A'B'' + D'').(C'+D'') \cdot E'$

$= (A' + B'' + D'').(C'+D'') \cdot E'$

$= (A' + B + D)..(C'+D) \cdot E'$

c) $AB(C'D + CD') + A'B'(C'+D)(C+D')$

↳ let $F = AB(C'D+CD') + A'B'(C'+D)(C+D')$

$= \{AB(C'D+CD') + A'B'(C'+D)(C+D')\}'$

$= \{AB(C'D+CD')\}' \cdot \{A'B'(C'+D)(C+D')\}'$

$= [(AB)' + (C'D+CD')'] \cdot (A'B')' + (C'+D)' + (C+D')'$

$= (A'+B').(C'D)'.(CD)'.[(AB) + C.D' + C'D]$

$= [(A'+B')(C+D')(C'+D)][(A+B) + CD' + C'D]$

(d) $(x+y'+z)(x'+z')(x+y)$

$\hookrightarrow$ let $F = (x+y'+z)(x'+z')(x+y)$

$= \{(x+y'+z)(x'+z')(x+y)\}'$

$= (x+y'+z)' + (x'+z')' + (x+y)'$

$= x'yz' + xz + x'y'$ #