Operation research

Replacement theory

- Introduction
- Replacement of capital equipment which depreciated with time
- Group and individual replacement policy

Introduction

- The replacement problems are concerned with the situations that arise when some items such as machines, men, electric appliance etc. need replacement due to their decreased efficiency, failure or breakdown.
- In case of items whose efficiency go on decreasing according to their age, we have to spend more and more money on account of increased operating cost, increased repair cost, increased scrap, etc. In such cases the replacement of an old item with a new one is the only alternative to prevent such increased expenses. Thus, it becomes necessary to determine the age at which replacement is more economical rather than continuing with the same.

Types of Failure

There are two types of failure: i) Gradual failure ii) Sudden failure

- **Gradual failure**

It means slow or progressive failure as the life of the item increases, its efficiency decreases resulting in decreased productivity, increased operating cost and decrease in the value of the item, e.g. machines/equipment etc.

- **Sudden failure**

In this type of failure the items do not deteriorate with service but which ultimately fail after some period of usage, thus precipitating cost of failure. Sometimes sudden failure of an item may cause loss of production or may also account for damaged or faulty products.

O.R. Methodology of Solving Replacement Problem

OR provides a methodology for tackling replacement problem which is discussed below:

i) Identify the items to be replaced and also their failure mechanism.

ii) Collect the data relating to the depreciation cost and the maintenance cost for the items which follow gradual failure mechanism. In case of sudden failure of items, collect the data for replacement cost of the failed items.

iii) Select a suitable replacement model

Types of Replacement Problems

i) Replacement policy for items, efficiency of which declines gradually with time without change in money value.

ii) Replacement policy for items, efficiency of which declines gradually with time but with change in money value.

iii) Replacement policy of items breaking down suddenly

a) **Individual replacement policy**

b) Group replacement policy

Individual Replacement Policy:

Replace the equipment at the end of n years, if the maintenance cost in the $(n+1)^{th}$ year is more than the average total cost in the $(n)^{th}$ year and the $(n)^{th}$ years maintenance cost is less than the previous years average total cost.

Example 1 A milk plant is considering replacement of a machine whose cost price is Rs. 12,200 and the scrap value Rs. 200. The running (maintenance and operating) costs in Rs. are found from experience to be as follows:
When should the machine be replaced?

Year:	1	2	3	4	5	6	7	8
Running Cost:	200	500	800	1200	1800	2500	3200	4000

Solution : Calculations for average cost of machine(In Rupees)

Year (1)	Running Cost (2)	Cumulative Running Cost (3)	Depreciation Cost (4)	Total Cost TC (5) = (3) + (4)	Average Cost (6) = (5)/(1)
1	200				
2	500				
3	800				
4	1200				
5	1800				
6	2500				
7	3200				
8	4000				

Calculations for average cost of machine(In Rupees)					
Year (1)	Running Cost (2)	Cumulative Running Cost (3)	Depreciation Cost (4)	Total Cost TC (5) = (3) + (4)	Average Cost (6) = (5)/(1)
1	200	200			
2	500	700			
3	800	1500			
4	1200	2700			
5	1800	4500			
6	2500	7000			
7	3200	10200			
8	4000	14200			

Calculations for average cost of machine(In Rupees)					
Year (1)	Running Cost (2)	Cumulative Running Cost (3)	Depreciation Cost (4)	Total Cost TC (5) = (3) + (4)	Average Cost (6) = (5)/(1)
1	200	200	12000		
2	500	700	12000		
3	800	1500	12000		
4	1200	2700	12000		
5	1800	4500	12000		
6	2500	7000	12000		
7	3200	10200	12000		
8	4000	14200	12000		

Calculations for average cost of machine(In Rupees)					
Year (1)	Running Cost (2)	Cumulative Running Cost (3)	Depreciation Cost (4)	Total Cost TC (5) = (3) + (4)	Average Cost (6) = (5)/(1)
1	200	200	12000	12200	
2	500	700	12000	12700	
3	800	1500	12000	13500	
4	1200	2700	12000	14700	
5	1800	4500	12000	16500	
6	2500	7000	12000	19000	
7	3200	10200	12000	22200	
8	4000	14200	12000	26200	

The computations can be summarized in the following tabular form

Calculations for average cost of machine(In Rupees)					
Year (1)	Running Cost (2)	Cumulative Running Cost (3)	Depreciation Cost (4)	Total Cost TC (5) = (3) + (4)	Average Cost (6) = (5)/(1)
1	200	200	12000	12200	12200
2	500	700	12000	12700	6350
3	800	1500	12000	13500	4500
4	1200	2700	12000	14700	3675
5	1800	4500	12000	16500	3300
6	2500	7000	12000	19000	**3167**
7	3200	10200	12000	22200	3171
8	4000	14200	12000	26200	3275

From the table it is noted that the average total cost per year is minimum in the 6^{th} year (Rs. 3167).

Also the average cost in 7^{th} year (Rs.3171) is more than the cost in 6^{th} year.

Hence the machine should be replaced after every 6 years.

Answer

Example 2

A Machine owner finds from his past records that the maintenance costs per year of a machine whose purchase price is Rs. 8000 are as given below:

Determine at which time it is profitable to replace the machine.

Year:	1	2	3	4	5	6	7	8
Maintenance Cost:	1000	1300	1700	2200	2900	3800	4800	6000
Resale Price **or** Scrap Value	4000	2000	1200	600	500	400	400	400

Solution:

Table shows the average cost per year during the life of machine. Here, The computations can be summarized in the following tabular form:

Year (1)	Maintenance cost (2)	Cumulative maintenance cost (3)	Resale Price(4)	Depreciation Cost (5)	Total cost (6)=(3)+(5)	Average Cost (7)=(6)/(1)
1	1000					
2	1300					
3	1700					
4	2200					
5	2900					
6	3800					
7	4800					
8	6000					

Year (1)	Maintenance cost (2)	Cumulative maintenance cost (3)	Resale Price(4)	Depreciation Cost (5)	Total cost (6)=(3)+(5)	Average Cost (7)=(6)/(1)
1	1000	1000				
2	1300	2300				
3	1700	4000				
4	2200	6200				
5	2900	9100				
6	3800	12900				
7	4800	17700				
8	6000	23700				

Year (1)	Maintenance cost (2)	Cumulative maintenance cost (3)	Resale Price(4)	Depreciation Cost (5)	Total cost (6)=(3)+(5)	Average Cost (7)=(6)/(1)
1	1000	1000	4000			
2	1300	2300	2000			
3	1700	4000	1200			
4	2200	6200	600			
5	2900	9100	500			
6	3800	12900	400			
7	4800	17700	400			
8	6000	23700	400			

Year (1)	Maintenance cost (2)	Cumulative maintenance cost (3)	Resale Price(4)	Depreciation Cost (5)	Total cost (6)=(3)+(5)	Average Cost (7)=(6)/(1)
1	1000	1000	4000	4000		
2	1300	2300	2000	6000		
3	1700	4000	1200	6800		
4	2200	6200	600	7400		
5	2900	9100	500	7500		
6	3800	12900	400	7600		
7	4800	17700	400	7600		
8	6000	23700	400	7600		

Year (1)	Maintenance cost (2)	Cumulative maintenance cost (3)	Resale Price(4)	Depreciation Cost (5)	Total cost (6)=(3)+(5)	Average Cost (7)=(6)/(1)
1	1000	1000	4000	4000	5000	
2	1300	2300	2000	6000	8300	
3	1700	4000	1200	6800	10800	
4	2200	6200	600	7400	13600	
5	2900	9100	500	7500	16600	
6	3800	12900	400	7600	20500	
7	4800	17700	400	7600	25300	
8	6000	23700	400	7600	31300	

Year (1)	Maintenance cost (2)	Cumulative maintenance cost (3)	Resale Price(4)	Depreciation Cost (5)	Total cost (6)=(3)+(5)	Average Cost (7)=(6)/(1)
1	1000	1000	4000	4000	5000	5000
2	1300	2300	2000	6000	8300	4150
3	1700	4000	1200	6800	10800	3600
4	2200	6200	600	7400	13600	3400
5	2900	9100	500	7500	16600	**3200**
6	3800	12900	400	7600	20500	3417
7	4800	17700	400	7600	25300	3614
8	6000	23700	400	7600	31300	3913

The above table shows that the value of Average cost during fifth year is minimum.

Hence the machine should be replaced after every fifth year.

Answer

Group Replacement Policy:

It is proposed to,
(i) replace all items in group simultaneously at fixed interval 't',
whether they have failed or not, and
(ii) continue replacing failed items immediately as and when they fail.

it is explained by numerical example.

Example:

For a certain type of light bulbs(1000 Nos.), following mortality rates have been observed:

Weeks	1	2	3	4	5
Percentage of items failing by at the end of each month	10	25	50	80	100

Each bulb costs Rs.10 to replace an individual bulb on failure. If all bulbs were replaced at the same time in group it would cost Rs. 4 per bulb. It is under proposal to replace all bulbs at fixed intervals of time, whether or not the bulbs have burnt out. And also it is to continue replacing immediately burnt out bulbs.

Determine the time interval at which all the bulbs should be replaced?

SOLUTION:

Let *pi = the probability that a new light bulb fails during the ith week* of its life.

Thus

p1 = the probability of failure in 1st week = 10/100 = 0.10

p2 = the probability of failure in 2nd week = (25-10)/100 =0.15

p3 = the probability of failure in 3rd Week = (50-25)/100 = 0.25

p4 = the probability of failure in 4th week = (80-50)/100 = 0.3

p5 = the probability of failure in 5th week = (100-80)/100 =0.2

Since the sum of all the above probabilities is unity, the further probabilities $p6$, $p7$, $p8$ and so on, will be zero.
Thus, all light bulbs are sure to burnout by the 5th week.

Furthermore, it is assumed that bulbs that fail during a week are replaced just before the end of that week.

Let **Ni = the number of replacements made at the end of the ith week.**
And let all 1000 bulbs are new initially.
Thus,
N0 = the number of replacements made in the beginning.
N1 = the number of replacements made at the end of the 1st week
N2 = the number of replacements made at the end of the 2nd week

$$N_0 = N_0 \qquad =1000$$

$$N_1 = N_0 p_1 = 1000 \times 0.10 \qquad =100$$

$$N_2 = N_0 p_2 + N_1 p_1 = 1000 \times 0.15 + 100 \times 0.10 \qquad =160$$

$$N_3 = N_0 p_3 + N_1 p_2 + N_2 p_1 = 1000 \times 0.25 + 100 \times 0.15 + 160 + 0.10 \qquad =281$$

$$N_4 = N_0 p_4 + N_1 p_3 + N_2 p_2 + N_3 p_1 \qquad =377$$

$$N_5 = N_0 p_5 + N_1 p_4 + N_2 p_3 + N_3 p_2 + N_4 p_1 \qquad =350$$

$$N_6 = 0 + N_1 p_5 + N_2 p_4 + N_3 p_3 + N_4 p_2 + N_5 p_1 \qquad =230$$

$$N_7 = 0 + 0 + N_2 p_5 + N_3 p_4 + N_4 p_3 + N_5 p_2 + N_6 p_1 \qquad =286$$

Individual replacement:

The mean age of bulbs $= 1 \times p_1 + 2 \times p_2 + 3 \times p_3 + 4 \times p_4 + 5 \times p_5$

= 1 x 0.1 + 2 x 0.15 +3 x 0.25 +5 x 0.30 +5 x 0.20 = 3.35 Weeks,

The number of failures in each week in steady state =1000/3.35 = 299

And the cost of replacing bulbs individually on failure

=10x299 (at the rate of Rs. 10 per bulb)

= Rs. 2990

Group replacement calculation:

The replacement of all 1000 bulbs at the same time in bulk costs Rs. 4 per bulb and replacement of an individual bulb on failure costs Rs. 10. Costs of replacement of all bulbs simultaneously are calculated in the Table

End of week	Cost of individual replacement	Total cost of group replacement (Rs.)	Average cost per week (Rs.)
1	100 x 10=1000	1000 x 4 + 100 x 10=5000	5000.00
2	160 x 10=1600	5000 + 160 x 10=6600	3300.00
3	281 x 10=2810	6600 + 281 x 10=9410	3136.67
4	377 x 10=3770	9410 + 377 x 10=13180	3295.00

Table Cost of replacement of bulbs for Bulbs example

The cost of individual replacement in the 4th week is greater than the average cost for 3 weeks.

Therefore the **optimal replacement decision is to replace all the bulbs at the end of every 3 weeks**. **Answer**