Probablity Homework 1

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March 17, 2024

Problem 1

We want to know the Probablit of sum 5.
The first we should do is to create the sample space for the problem.
We stop throwing only if the sum of two dices is 5 or 7 it means that if we draw the desicion tree for this problem it will stop in some position(when sum 5 or 7 happens) or it will continue infinitly.
So the smaple space is:
$\Omega = \{(1, 4), (4, 1), (2, 3), (3, 2), (3, 4), (4, 3)(1, 6), (6, 1), (5, 2), (2, 5)\}$
We know that the sample space will happen because if we throw dices infinite times, the Probablity of not showing sum 5 or 7 is that every time the sum happens something else and then the Probablity is $\liminf \frac{1}{n}$ and is zero.
So the Probablity of the sample space we wrote is 1. So we can limit or whole problem's sample space to Ω (Because the 3 conditions for a Probablity sample space that is in book is correct for this Ω so we can accept it).
Now that we have chosen our limited sample space, for answering to problem we can easily count Probablity of showing sum 5, that is 4.
And our Ω had 10 members so the final Probablity is $\frac{4}{10}$.

Problem 2

I claim that the Probablity is $\frac{1}{4}$.The prove is:
First thing we should notice is that the three pieces can form a triangle if and only if the triangle inequalities hold:

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"The sum of the lengths of any two pieces must be
greater than the length of the third piece."
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We can write a desicion three and calculate the Probablity at each point. But the efficeint way is to use a geometary theorem that says:

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" If we have a  Equilateral triangle ,
the distance of each point in the triangle to the triangle's sides
is equal to height of the triangle "
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Because when we cut our first position(suppose i n) on the piece of wood, the first side of triangle we want to make will be showed and the sum of two others sides is $1 - n$.
So we can simulate the probelm to choosing a point in a Equilateral triangle with height 1 (wood height) and use the geometary theorem said earlier.
So we create a Equilateral triangle with height 1 and mark its sides's middle points and connect them.
Base on another geometary theorem 4 equal triangles will be created.
If we pay attention to these triangles we see that each point we choose in middle triangle, it matches the condition we wanted to create a triangle with piece of wood(other 3 triangles don't satisfie the condition) and if we suppose that the point has distances a,b,c to sides, as the sum of a,b,c is 1(base on earlier theorem) and it satisfies the condition we wanted so we can choose the a,b,c as our disered points on the peice of wood and create a triangle with it.
So our sample space is $\Omega = \{4$ triangles with equal area$\}$,and we has the answer only if we choose a point in the middle triangle and because all triangles has equal area that is 1 out of 4, so the final Probablity is $\frac{1}{4}$ as claimed.

Problem 3

For this problem we use the theorem that says $\Pr[A] = 1 - \Pr[A']$. And for calculate Probablity that none of S divides 3, we calculate Probablity that each S does divide 3, and then use the theorem to calculate the Probablity that each S doesn't divide 3, and because we want all these situations happen together, we multipy all Probablities .
We set $\Pr[S_n] = \{S_n divides 3\ \}$.
So base on the theorem $\Pr[S'_n]$ = 1 - $\Pr[S_n]$.

$\Rightarrow 1 - \Pr[S_1] = 1 - \frac{\lfloor \frac{n}{3} \rfloor}{n}$
$\Rightarrow 1 - \Pr[S_2] = 1 - \frac{\lfloor \frac{n-1}{3} \rfloor}{n-1}$
$\Rightarrow \cdots$
$\Rightarrow \cdots$
$\Rightarrow 1 - \Pr[S_n] = 1 - 0 = 1$

If we continue the calculation we can find each $\Pr[S'_n]$ and for final answer we multipy them together.

Problem 4

The last passenger wouldn't able to sit on his own sit if and only if it is not empty. It means that someone else has already sitten there. So we try to simulate the conditions by using a desicion tree.
For first passenger there is two conditions:

1- sits on his own sit with Probablity $\frac{1}{200}$.
2- sits on someone else's sit with Probablity $\frac{199}{200}$.

If the first condition happens, all passengers will sit on their own sit as well as the last one.
But if the second condition happens, and suppose the first passenger sits on jth sit then all 2 to j-1th passengers will sit on their own sit and the jth passengers again has two situations:

1- Sit on first sit with Probablity $\frac{1}{200-(j-1)}$.
2 Sit on another sit.

Just like before if the first situation happens the last passenger will sit on his own sit and if not the desicion tree continues.
If we continue this method and sum Probablities(because there is "or" between situations) then we get to $\frac{1}{2}$ for the final answer.
It means the last passenger has $\frac{1}{2}$ to sit on his own sit.

Problem 5

Like we have in context of the question, we set A_m as area with distance between mr_0 and $(m-1)r_0$.
Therefore we have:
$A_m = \{\text{points with } (m-1)r_0 < \text{d} < mr_0\}$
Now we try to calculate each A_m:

$\Rightarrow A_1 = \{\text{points with } 0 < \text{d} < r_0\}$
$\Rightarrow A_2 = \{\text{points with } r_0 < \text{d} < 2r_0\}$
$\Rightarrow \cdots$

Because we had a relationship between Probablities in context of question therefore we can set $\Pr[A_1] = \alpha$ and write other Probablities base on α.
so we have:

$\Rightarrow A_1 = \alpha, A_2 = \frac{\alpha}{2}, \ldots$
$\Rightarrow A_1 + A_2 + \ldots = 1$
$\Rightarrow \alpha = \frac{1}{2}$

Now we are going to calculate the Probablity of square:
The square will be inside a circle with radius $2r_0$ (call it S_2).
And also there is a circle with radius r_0 in the square(call it S_1).
If we pay attention we see that these S_1, S_2 are A_1, A_2 that we calculated earlier.
So base on these explainations if a shot wants to be inside the square it is inside the S_1 'or' inside the area between S_1 and the square.
As we calculated S_1 before (it is eqaul to α) So for calculating the Probablity of shooting in square, all we need to do is calculating Probablity of area between the S_1 and the square itself(by subtracting areas) and then sum it

with α.
After calculating every area(S_1, S_2, the square) the subtracted area is: $\frac{4-\pi}{3\pi}$
So the Probablity that shot be inside area between S_1 and the square is: $\frac{1}{4} * \frac{4-\pi}{3\pi}$

Therefore the fianl Probablity is: $\frac{1}{2} + \frac{1}{4} * \frac{4-\pi}{3\pi}$