Part 1

Question 1

1. Compute the table of the joint distribution P (X, Y, Z). Show the rule(s) you used, and the steps of calculating each joint probability.

1. Compute the table of joint distribution $P(x, y, z)$.

$$P(x,y,z) = P(z|x,y) \times P(x,y) = P(z|x,y) \times ((P(y|x) \times P(x))$$
$$= P(z|y) \times ((P(y|x) \times P(x))$$

x	y	z	$P(z\|y) \times ((P(y\|x) \times P(x))$
0	0	0	$P(z=0\|y=0) \times ((P(y=0\|x=0) \times P(x=0)) = 0.70 \times ((0.10) \times 0.35) = 0.0245$
0	0	1	$P(z=1\|y=0) \times ((P(y=0\|x=0) \times P(x=0)) = 0.30 \times ((0.10) \times 0.35) = 0.0105$
0	1	0	$P(z=0\|y=1) \times ((P(y=1\|x=0) \times P(x=0)) = 0.20 \times ((0.90) \times 0.35) = 0.063$
1	0	0	$P(z=0\|y=0) \times ((P(y=0\|x=1) \times P(x=1)) = 0.70 \times (0.60 \times 0.65) = \cancel{0.0455}$ 0.273
0	1	1	$P(z=1\|y=1) \times ((P(y=1\|x=0) \times P(x=0)) = 0.80 \times (0.90 \times 0.35) = 0.252$
1	1	0	$P(z=0\|y=1) \times ((P(y=1\|x=1) \times P(x=1)) = 0.20 \times (0.40 \times 0.65) = \cancel{0.052}$ 0.052
1	0	1	$P(z=1\|y=0) \times P((y=0\|x=1) \times P(x=1)) = 0.30 \times (0.60 \times 0.65) = \cancel{0.0195}$ 0.117
1	1	1	$P(z=1\|y=1) \times P((y=1\|x=1) \times P(x=1)) = 0.80 \times (0.40 \times 0.65) = \cancel{0.234}$ $\cancel{104}$ 0.208

2. Create the full joint probability table of X and Y , i.e., the table containing the following four jointprobabilitiesP(X=0,Y =0),P(X=0,Y =1),P(X=1,Y =0),P(X=1,Y =1). Show the rule(s) used, and the steps of calculating each joint probability.

2. Create the full joint probability table of x and y.

$$P(A,B) = P(B) \times P(A|B) = P(A) \times P(B|A)$$
$$\hookrightarrow P(x,y) = P(y) \times P(x|B) = P(x) \times P(y|x)$$

x	y	$P(B) \times P(A\|B)$
0	0	$P(x=0) \times P(y=0\|x=0) = 0.35 \times 0.10 = 0.035$
0	1	$P(x=0) \times P(y=1\|x=0) = 0.35 \times 0.90 = 0.315$
1	0	$P(x=1) \times P(y=0\|x=1) = 0.65 \times 0.60 = 0.39$
1	1	$P(x=1) \times P(y=1\|x=1) = 0.65 \times 0.40 = 0.26$

3. From the above joint probability table of X, Y, and Z, calculate the following probabilities. Show your working.

3. From the above joint probability ~~of~~ table of x, y, z, calculate the following probabilities.

a) $P(z=0) = P(x=0, y=0, z=0) + P(x=0, y=1, z=0) + P(x=1, y=0, z=0)$
$+ P(x=1, y=1, z=0) = 0.0245 + 0.063 + 0.273 + 0.052 = \underline{0.4125}$

b) $P(x=0, z=0) = P(x=0, z=0, y=0) + P(x=0, z=0, y=1)$
$= 0.0245 + 0.063 = \underline{0.0875}$

c) $P(x=1, y=0 | z=1) \Rightarrow P(A,B) = P(B) \times P(A|B)$

$\star$ $A = (x,y)$, $B = z$ $\rightarrow P(x,y,z) = P(z) \times P(x,y|z)$

rearrange : $P(x=1, y=0 | z=1) = \dfrac{P(x=1, y=0 | z=0)}{P(z=0)} = \dfrac{0.273}{0.4125} = \underline{0.661}$

d) $P(x=0 | y=0, z=0) \rightarrow A = x, \ B = y, z$

$P(A,B) = P(B) \times P(A|B)$

$\rightarrow P(x,y,z) = P(y,z) \times P(x|y,z)$

rearrange : $P(x|y,z) = \dfrac{P(x,y,z)}{P(y,z)} \rightarrow P(x=0 | y=0, z=0) = \dfrac{P(x=0, y=0, z=0)}{P(y=0, z=0)} = \dfrac{0.0295}{0.2975} =$ 0.08235

Question 2

Consider three Boolean variables A, B, and C (can take t or f). We have the following probabilities:

- $P(B=t)=0.7$
- $P(C=t)=0.4$
- $P(A=t|B=t)=0.3$ • $P(A=t|C=t)=0.5$ • $P(B=t|C=t)=0.2$

We also know that A and B are conditionally independent given C. Calculate the following prob- abilities. Show your working.

1. $P(B=t, C=t) = P(C=t) \times P(B=t | C=t) = 0.4 \times 0.2 = 0.08$

2. $P(A=f | B=t) = P(B=t | A=f) \times P(A=f) / P(B=t)$
$= (- P(B=t | A=f) = 1 - 0.3 = 0.7$
$\rightarrow P(A=f) = 1 - P(A=t | C=t) = 1 - 0.5 = 0.5$

$P(B=t | A=f) = P(B=t, A=f) / P(A=f)$
$= (P(A=f | B=t) \times P(B=t)) / P(A=f)$
$= ((1 - P(A=t | B=t)) \times P(B=t) / 1 - P(A=t|C=t)$
$= ((1 - 0.3) \times 0.7) / 0.5 = 0.56$

Therefore, $P(B=t | A=f) \times P(A=f) / P(B=t)$
$= (0.56 \times 0.5) / 0.7 = \underline{0.4}$

3. $P(A=t, B=t \mid C=t) = P(A=t \mid C=t) \times P(B=t \mid C=t)$

$= 0.5 \times 0.2 = \underline{0.1}$

Because A and B are conditionally dependent

4. $P(A=t \mid B=t, C=t) = P(A, B \mid C) / P(B \mid C)$

$= P(A=t, B=t \mid C=t) / P(B=t \mid C=t)$

$= 0.01 \times 0.2 = \underline{0.002}$

5. $P(A=t, B=t, C=t) = P(A=t \mid B=t, C=t) \times P(B=t \mid C=t) \times P(C=t)$

$= 0.002 \times 0.2 \times 0.4 = \underline{0.00016}$

Part 2

1. The conditional probabilities P(Xi = xi|Y = y) for each feature Xi (e.g., age), its possible value xi (e.g., 10-19), and each class label Y = y (y can be no-recurrence-events or recurrence-events).

```
**  recurrence-events  **

P(age)
10-19: 0.0115
20-29: 0.0115
30-39: 0.1839
40-49: 0.3103
50-59: 0.2529
60-69: 0.1954
70-79: 0.0115
80-89: 0.0115
90-99: 0.0115
==================
P(menopause)
ge40: 0.3827
lt40: 0.0123
premeno: 0.6049
==================
```

```
**  no-recurrence-events **

P(age)
10-19: 0.0051
20-29: 0.0101
30-39: 0.1111
40-49: 0.3131
50-59: 0.3283
60-69: 0.1919
70-79: 0.0303
80-89: 0.0051
90-99: 0.0051
==================
P(menopause)
ge40: 0.4583
lt40: 0.0313
premeno: 0.5104
==================
```

```
P(tumor-size)
0-4: 0.0222
10-14: 0.0222
15-19: 0.0778
20-24: 0.1556
25-29: 0.2111
30-34: 0.2556
35-39: 0.0889
40-44: 0.0778
45-49: 0.0222
5-9: 0.0111
50-54: 0.0444
55-59: 0.0111
==================
P(inv-nodes)
0-2: 0.4725
12-14: 0.0330
15-17: 0.0440
18-20: 0.0110
21-23: 0.0110
24-26: 0.0220
27-29: 0.0110
3-5: 0.1758
30-32: 0.0110
33-35: 0.0110
36-39: 0.0110
6-8: 0.1209
9-11: 0.0659
==================
P(node-caps)
no: 0.6000
yes: 0.4000
==================
P(deg-malig)
1: 0.1111
2: 0.3580
3: 0.5309
==================
P(breast)
left: 0.5500
right: 0.4500
==================
P(breast-quad)
central: 0.0602
left_low: 0.3855
left_up: 0.3012
right_low: 0.0843
right_up: 0.1687
==================
P(irradiat)
no: 0.6125
yes: 0.3875
==================
```

```
P(tumor-size)
0-4: 0.0398
10-14: 0.1294
15-19: 0.1144
20-24: 0.1741
25-29: 0.1592
30-34: 0.1692
35-39: 0.0597
40-44: 0.0846
45-49: 0.0149
5-9: 0.0249
50-54: 0.0249
55-59: 0.0050
==================
P(inv-nodes)
0-2: 0.7970
12-14: 0.0099
15-17: 0.0198
18-20: 0.0050
21-23: 0.0050
24-26: 0.0050
27-29: 0.0050
3-5: 0.0842
30-32: 0.0050
33-35: 0.0050
36-39: 0.0050
6-8: 0.0396
9-11: 0.0149
==================
P(node-caps)
no: 0.8743
yes: 0.1257
==================
P(deg-malig)
1: 0.2917
2: 0.5104
3: 0.1979
==================
P(breast)
left: 0.5079
right: 0.4921
==================
P(breast-quad)
central: 0.0876
left_low: 0.3660
left_up: 0.3454
right_low: 0.0928
right_up: 0.1082
==================
P(irradiat)
no: 0.8429
yes: 0.1571
==================
```

2. *The class probabilities P (Y = y) for each class label Y = y.*

```
Class probabilities:
P(No-recurrence-events) = 0.7063197026022305
P(Recurrence-events) = 0.2936802973977695
```

3. For each test instance, given the input vector X = calculated

– score(Y =no-recurrence-events,X), – score(Y =recurrence-events,X),
– predicted class of the input vector.

```
>> Below are the results after testing <<

no-recurrence-events 4.017731924138002E-6
recurrence-events 7.096642912782162E-6
Predicted class label for 0th instance was recurrence-events
Predicted class label was no-recurrence-events and prediction was wrong!
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
no-recurrence-events 3.336587553818684E-4
recurrence-events 2.60210240135346E-5
Predicted class label for 1th instance was no-recurrence-events
Actual class label was no-recurrence-events and prediction was correct!
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
no-recurrence-events 4.70737879759242E-5
recurrence-events 9.715041077797865E-7
Predicted class label for 2th instance was no-recurrence-events
Actual class label was no-recurrence-events and prediction was correct!
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
no-recurrence-events 1.5163354504655905E-4
recurrence-events 1.0219165794406312E-5
Predicted class label for 3th instance was no-recurrence-events
Actual class label was no-recurrence-events and prediction was correct!
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
no-recurrence-events 4.345236143782871E-6
recurrence-events 1.9205993914751727E-6
Predicted class label for 4th instance was no-recurrence-events
Actual class label was no-recurrence-events and prediction was correct!
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
no-recurrence-events 6.000123072810081E-4
recurrence-events 3.8804032962741955E-5
Predicted class label for 5th instance was no-recurrence-events
Actual class label was no-recurrence-events and prediction was correct!
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
no-recurrence-events 2.071279390811688E-4
recurrence-events 7.415569159796725E-5
Predicted class label for 6th instance was no-recurrence-events
Actual class label was no-recurrence-events and prediction was correct!
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
no-recurrence-events 3.150486582669929E-4
recurrence-events 8.934762413310772E-6
Predicted class label for 7th instance was no-recurrence-events
Predicted class label was recurrence-events and prediction was wrong!
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
no-recurrence-events 3.912394516967168E-5
recurrence-events 6.483884504037111E-5
Predicted class label for 8th instance was recurrence-events
Actual class label was recurrence-events and prediction was correct!
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^
no-recurrence-events 4.101703929078483E-5
recurrence-events 5.283165151437644E-5
Predicted class label for 9th instance was recurrence-events
Actual class label was recurrence-events and prediction was correct!
~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^^

Accuracy: 80.0%
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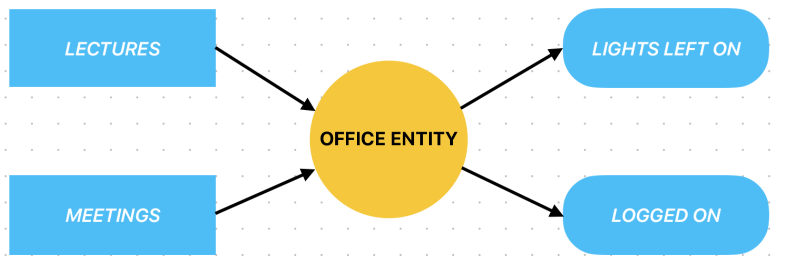
Part 3

This part is to build a Bayesian Network for the problem described below.

1. Construct a Bayesian network to represent the above scenario. (Hint: First decide what your domain variables are; these will be your network nodes. Then decide what the causal relationships are between the domain variables and add directed arcs in the network from cause to effect. Finally, you have to add the prior probabilities for nodes without parents, and the conditional probabilities for nodes that have parents.)



LECTURES	P(LECTUES)
T	0.6

MEETINGS	P(MEETINGS)
T	0.7

LIGHTS LEFT ON	LIGHTS LEFT OFF	P(LIGHTS ON | OFF)
T	F	0.02
T	T	0.5

LOGGED ON	LOGGED OFF	P(LOGGED ON | OFF)
T	F	0.2
T	T	0.8

OFFICE	LECTURES	MEETINGS	P(OFFICE | LECTURES, MEETINGS)
T	T	T	0.95
T	F	T	0.75
T	T	F	0.80
T	F	F	0.06

2. Calculate the number of free parameters in your Bayesian network.

The free parameters can be found using different probability rules such as indirect rule, direct rule, common effect and common cause.
The results are given below :

1. P(Meetings = T)
2. P(Lectures = T)
3. P(Lights on = T | off = F)
4. P(Lights on = T | off = T)
5. P(Logged on = T | off = F)
6. P(Logged on = T | off = T)
7. P(Office = T | lectures = T, meetings = T)
8. P(Office = T | lectures = F, meetings = T)
9. P(Office = T | lectures = T, meetings = F)
10. P(Office = T | lectures = F, meetings = F)

3. What is the joint probability that Rachel has lectures, has no meetings, she is in her office and logged on her computer but with lights off.

ANSWER ON THE NEXT PAGE

3. P(meetings, lectures, office, Lights on, logged on) =

↳ = P(logged on | office) × P(lights on | office) × P(office | meetings, lectures) × P(meetings) × P(lectures)

Therefore,

P(meetings = F, lectures = T, office = T, lights on = F, logged on = T) =

↳ P(logged on = T | office = T) × P(lights on = F | office = T) × P(office = T | meetings = F, lectures = T) × P(meeting = F) × P(lectures = T)

↳ = 0.80 × 0.50 × 0.80 × 0.30 × 0.60 = 0.0576

4. Calculate the probability that Rachel is in the office.

4. meeting = m, lectures = L, Lights on = Lio, office = O, logged on = LO

$$P(O=T) = (P(O=T \mid L=T, M=T) \times P(L=T, M=T)) +$$
$$(P(O=T \mid L=T, M=F) \times P(L=T, M=F)) +$$
$$(P(O=T \mid L=F, M=T) \times P(L=F, M=T)) +$$
$$(P(O=T \mid L=F, M=F) \times P(L=F, M=F))$$

$$= 0.95 \times \underbrace{(0.7 \times 0.6)}_{P(L=T,\, M=T)} + 0.80 \times \underbrace{0.18}_{\substack{(0.3 \times 0.6) \\ P(L=T,\, M=F)}} + 0.75 \times \underbrace{0.28}_{\substack{(0.7 \times 0.4) \\ P(L=F,\, M=T)}} + 0.06 \times \underbrace{0.12}_{\substack{(0.3 \times 0.4) \\ P(L=F,\, M=F)}} = \underline{\underline{0.7602}}$$

5. If we know that Rachel is in the office, what is the conditional probability that she is logged on, but her light is off.

5. P(logged on = T, lights on = F | office = T) =

↳ = P(logged on = T | office) × P(lights on = F | office = T) =

↳ 0.80 × 0.50 = <u>0.40</u>