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Robust optimisation approach to the design of service networks for reverse logistics

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In this paper we discuss a typical service network of a company providing repair and refurbishment services for its products. To optimise the network we propose a mixed-integer linear programming (MILP) model that minimises the total cost subject to flow balance and logical constraints. The solution of the model determines which facilities are to be used in the network and the associated flows. The model is validated against data obtained from a computer manufacturer and real-life sources. The optimal network configuration depends on several factors such as the number of products returned, percent of faulty products and warranty fraction of modules, which are uncertain in nature. We also address the issue of identifying these uncertain parameters, and their levels to be considered. A min-max robust optimisation model is then developed and solved to address these uncertainties.

Keywords: reverse logistics; network design; robust optimisation

1. Background

There are many ways a product may come back into the supply chain. Electronic consumer products are sometimes sold with a 30-day return policy under which the product may be returned to the retailer if the customer is not satisfied with it. Also, when the product is sold online or delivered by the retailer to the customer's address, the customer may refuse to accept the product due to damaged packaging or finding the product to be different from the one ordered. The products returned to retailers in these ways are termed 'commercial returns'.

Toktay (2003) reports the amount of commercial returns to be 12% of the total products sales; it can be as high as 15–20% for hi-tech products. Commercial returns are usually as good as new, as they have only been lightly used or not used at all. Companies such as Dell and Apple sell these products as refurbished products after the refurbishment process, as the cost of refurbishing is more than compensated for by the profit generated by the sale of the products. The refurbishment process involves cleaning, dismantling, assembling, testing and packaging so as to bring the product back to its original specifications.

A product may also be returned to the company if it malfunctions. In such cases the company repairs the faulty product to restore it to working condition. The repair operation may be carried out either by original equipment manufacturers (OEMs) or repair vendors (RVs) designated by the OEM. According to Guide *et al*. (2006), a product may be returned due to imperfection, dissatisfaction or remorse. Commercial returns are the products that are returned due to dissatisfaction or remorse, while faulty products are those returned due to malfunction.

In order to deal with commercial returns and faulty products, a company may set up a separate reverse logistics network or use the existing forward supply chain, hire a third-party service provider or use a combination of these. Savaskan *et al*. (2004) studied the impact of self-collection, existing retailer and third-party outsourcing for the collection of used products and found the option of using the existing retailer the best one.

Jayaraman *et al*. (1999) developed a MILP for a network with both reverse flow of used products and forward flow of remanufactured goods. The authors show that the optimal solution of the network is dependent on the reverse flow of used products and also on the demand for the remanufactured products. The network presented in this paper is similar to that of Jayaraman *et al*. (1999) as both consider forward and reverse flow of products. Fleischmann *et al*. (2001) investigated the options of the sequential and integrated design of recovery networks by presenting two case studies: copier remanufacturing and paper recycling. The authors concluded that the options

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investigated may lead to different solutions, but cost differences are negligible, except when there are major structural cost differences between the forward and the reverse channels, along with high return volumes.

Sarkis and Sundarraj (2002) conducted a case study at Digital Equipment Corporation to locate a repair parts warehouse. The authors combined an analytical network process (ANP) with an optimisation model to find a suitable location while considering the qualitative and quantitative factors involved. Kusumastuti *et al.* (2008) developed a MILP to recommend an optimal repair network design for a computer manufacturer. Min and Ko (2008) also considered a MILP and genetic algorithms to design a network of collection centres for returned products from online sales.

An important consideration in setting up reverse logistics networks is selecting a network design that functions reasonably well under uncertainty. Uncertainty in reverse logistics networks can be due to variance in the number of products returned, the volume of demand, the price of refurbished products, etc. Under these circumstances, what is required is a network design that is 'robust' to such uncertainties. Mulvey *et al.* (1995) compared robust optimisation with stochastic optimisation and stated that stochastic optimisation is suitable for problems where solutions can be adjusted easily in response to changing conditions, while robust optimisation is for problems where once a solution is implemented, it is not changed for a considerable length of time. Also, if the problem has high uncertainty in data, then stochastic optimisation, with expected value minimisation as the objective, may not yield appropriate solutions.

Realff *et al.* (2004) presented a min-max robust optimisation approach to address the uncertainty in reverse network design for carpet recycling. The uncertain parameters considered are the volume of carpet collected for recycling and the price of recycled material. Chen and Lee (2004) presented a mixed-integer nonlinear programming model for a multi-echelon supply chain network. The authors used a number of discrete scenarios with associated probabilities to model the uncertainty in demand, while fuzzy sets were used to model the uncertainty in product prices. Listes and Dekker (2005) used a stochastic-programming-based approach to design a sand recovery network in The Netherlands. Hong *et al.* (2006) used a min-max robust-optimisation-based approach to determine the configuration for a recovery network for used electronic products in the state of Georgia (USA). Listes (2007) presented a two-stage stochastic model for networks with both forward and reverse flows. Due to the computationally intensive nature of the problem the author resorted to the integer L-shaped method to solve it. Kara *et al.* (2007) presented a simulation modelling approach to carry out a what-if analysis on a service network. More recently, Du and Evans (2008) presented a reverse logistics network design approach with two objective functions.

In this research, we consider the service network of a company providing after-sale service in the Asia-Pacific region. The network consists of repair and refurbishment flows in both the forward and reverse direction. The company is interested in performing a strategic analysis of its current network and determining the optimal structure of the network for the future (5–10 years). We use a MILP model to represent the network; the model determines the locations of regional distribution centres (DCs) and country DCs and optimal flows between the facilities, based on input parameters such as supply of products, transportation cost, fraction of modules under warranty, etc. As the service network design is a strategic problem, we also identify the uncertain parameters affecting the network. The uncertainty in data affecting the network is addressed using the robust optimisation approach. The approach provides a single solution that is 'good' for all the possible scenarios under which the network is likely to operate. This approach is suitable for our problem as the network configuration, once designed, will not change for a few years, and we do not have enough quality information to determine the distribution for various model parameters in order to use simulation or stochastic programming techniques.

As very few researchers have focused on the design of repair and refurbishment networks, this paper contributes to the literature by presenting a repair and refurbishment network model with both forward and reverse flows of modular products. Moreover, we are not aware of any research where uncertainty in a repair and refurbishment network has been addressed; we fill this gap by identifying uncertain parameters affecting the network and addressing the uncertainty by applying the min-max robust optimisation approach. Finally, we validate the model against real-life data obtained from a company and conduct an analysis to show the practical relevance of the model proposed.

The remainder of the paper is organised as follows. In Section 2 we describe the problem along with the details of the Asia-Pacific service network of the company, which is the subject of the case study. Section 3 presents the mathematical model. Problem data and analysis and the robust optimisation methodology are presented in Section 4. Section 5 concludes the paper.

Figure 1. After-market service network in Asia and Australia.

2. Problem description

The company in our case study provides after-sale service for its products, namely laptops and desktops, in the Asia-Pacific region including Australia and Japan (Figure 1). This is achieved through a network of retailers, resellers and service centres. The retailers not only sell new products but also serve as collection points for commercial returns. We consider commercial returns as new products that do not have major defects and are returned for reasons such as customer dissatisfaction, faulty packaging, etc. The commercial returns collected at the retailers are forwarded to service centres within the same country for minor refurbishment, which includes cleaning, repackaging and testing. The commercial returns after minor refurbishment are sold as refurbished products at reduced prices.

In addition to carrying out minor refurbishment, service centres also perform the repair operations. When a customer brings a faulty product to a service centre, the service personnel identify the faulty module and replace it with a repaired or a new module to complete the repair process. Later, the faulty module is checked to determine its warranty status. If the module is out-of-warranty, further inspection is carried out to determine if the part is worth repairing. The reasons for not sending the part for repair can be varied, including, but not limited to, outdated module, cost considerations and irreparability. The under warranty or repairable out-of-warranty faulty modules are then forwarded to a country distribution centre or a regional DC. The country DCs serve as first level consolidation points for emerging markets and serve only the markets of the country they are situated in. The country DCs also cannot send/receive modules directly to/from OEMs and RVs. However, they have the potential to be upgraded to regional DCs, the major consolidation points. The country DC forwards the faulty modules to one or more regional DCs, which then send the modules to the repair vendors or the OEMs.

The faulty modules after repair re-enter the network and travel towards the service centres. As not all faulty modules are repairable, some new modules are also purchased from OEMs to replenish the inventory and to meet the repair demand. The faulty modules that are not repairable are disposed off. In our case the company has hired the services of a third party for this purpose; the decision of whether to dispose of a module or not is made at the service centres. Figure 2 shows the flow of faulty, repaired and new modules through the network.

3. Mathematical model

We model the problem described in the previous section as a cost minimisation, MILP. The problem could very well have profit maximisation as the objective, but the network presented here is basically a repair and refurbishment (after-sale service) network. In service networks, cost incurred is usually much more than the profit generated, so we chose to use cost minimisation as the objective. The input data for the model include: location of various facilities, incoming volume of returned/faulty products, costs associated with various processes in the network, profit

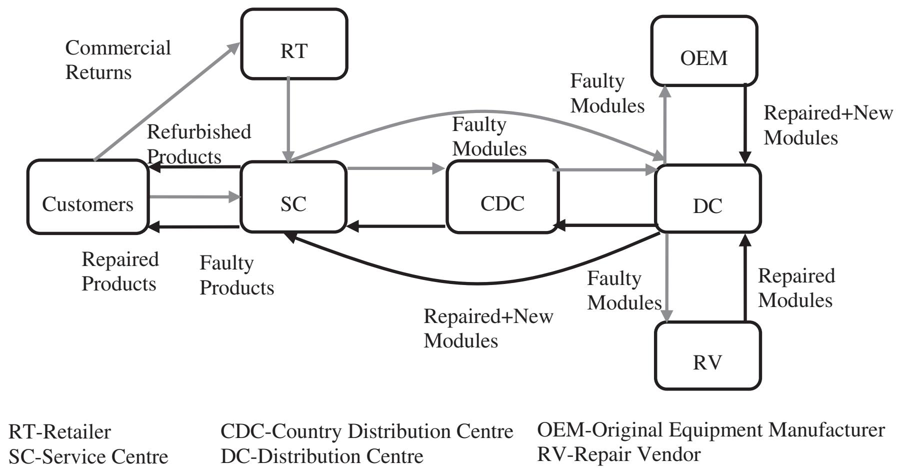

Figure 2. Reverse logistics network for electronics products.

generated by repairing a faulty product and selling refurbished products, etc. With this input data, the model determines which DCs and country DCs to use and the flow of products/modules on different links, and calculates various costs incurred in the network.

Sets

- Z Countries
- Q Regions
- M_u Faulty and new modules
- N Nodes in the network
- N_c Feasible locations for retailers, $N_c \subset N$
- N_c^z Feasible locations of retailers in country $z \in Z$, $N_c^z \subset N_c$
- N_c^q Feasible locations of retailers in region $q \in Q$, $N_c^q \subset N_c^z$, $z \in \mathrm{Z}$
- N_d Feasible locations for distribution centres, $N_d \subset N$
- N_d^q Feasible locations of distribution centres in region $q \in Q$, $N_d^q \subset N_d$
- N_d^z Feasible locations of distribution centres in country $z \in Z$, $N_d^z \subset N_d$
- N_l Feasible locations for country distribution centres, $N_l \subset N$
- N_l^z Feasible locations of country distribution centres in country $z \in Z$, $N_l^z \subset N_1$
- N_l^q Feasible locations of country distribution centres in region $q \in Q$, $N_l^q \subset N_1$
- N_o Original equipment manufacturers, $N_o \subset N$
- N_o^m OEM for module $m \in M_u$, $N_o^m \subset N_o$
- N_r Feasible locations for repair centres, $N_r \subset N$
- N_r^m Repair vendors for module $m \in M_u$ and $N_r^m \subset N_r \cup (N_o - N_o^m)$
- N_s Feasible locations for service centres, $N_s \subset N$
- N_s^z Feasible locations of service centres in country $z \in Z$, $N_s^z \subset N_s$
- N_s^q Feasible locations of service centres in region $q \in Q$, $N_s^q \subset N_s^z$, $z \in Z$
- Γ Arcs in the network, $\Gamma = \{i, j\}$
- P_u Commercial returns
- P_f Faulty products, $P_f \cap P_u = \phi$
- P_r Refurbished products
- P_u^r Used products yielding $r \in P_r$ as refurbished product

Parameters

- BP_m Purchase price of one unit of new module $m \in M_u$
- D_{pi} Demand for refurbished product $p \in P_r$ at retailer $i \in N_c$
- DC_w Disposal cost of one unit of non-repairable module/non-refurbishable product $w \in P_u \cup M_u$
- FC_i Cost of operating facility $i \in N_d \cup N_l$

PC_{pi}	Processing cost of one unit of product $p \in P_r \cup P_f$ at facility $i \in N_s$
PR_p	Selling price of one unit of refurbished product $p \in P_r$
RC_{mi}	Repair cost of one unit of out-of-warranty faulty module $m \in M_u$ at repair facility $i \in N_r^m \cup N_o^m$
RP_m	Repair price for replacing one unit of out-of-warranty faulty module $m \in M_u$
S_{pi}	Supply of commercial return/faulty product $p \in P_u \cup P_f$ at service centre ($i \in N_c$) or retailer ($i \in N_s$)
TC_{ij}	Transportation cost for arc $\{i,j\}$
V_{mi}	Capacity to store module $m \in M_u$ at facility $i \in N_d \cup N_l$
X_{mp}	Number of modules $m \in M_u$ in one unit of faulty product $p \in P_f$
β_p	Fraction of refurbishable product $p \in P_r$
γ_m	Fraction of faulty module $m \in M_u$ under warranty
λ_m	Fraction of faulty module $m \in M_u$
μ_m	Fraction of out-of-warranty faulty module $m \in M_u$ sent for repair

Decision variables

f_{pij}	Reverse flow of product $p \in P_u \cup P_f$ on arc $\{i,j\}$
e_{mij}	Reverse flow of module (under warranty) $m \in M_u$ on arc $\{i,j\}$
e'_{mij}	Forward flow of module (under warranty) $m \in M_u$ on arc $\{i,j\}$
g_{mij}	Reverse flow of module (out-of-warranty) $m \in M_u$ on arc $\{i,j\}$
g'_{mij}	Forward flow of module (out-of-warranty) $m \in M_u$ on arc $\{i,j\}$
h_{mij}	Flow of new module $m \in M_u$ on arc $\{i,j\}$
d_{mi}	Total number of faulty modules $m \in M_u$ yielded from faulty products at service centre $i \in N_s$
r_{pi}	Refurbished product $p \in P_r$ yielded at service centre $i \in N_s$
y_i	Binary variable (1/0) for assigning facility to location $i \in N_d \cup N_l$

3.1 *Objective function*

The objective function (Equation (1)) of the model is to minimise the total cost. The costs in the network include transportation, processing, repair, purchasing, disposal and facility costs. The transportation cost is incurred due to the product-module movement between the facilities. The processing cost for commercial returns is the cost of minor refurbishment at service centres. For faulty products, processing cost is the labour cost to identify faulty modules at service centres. Repair cost is the cost of repairing out-of-warranty faulty modules. The under warranty module repair cost is paid by the OEM and is not included. Purchasing cost is incurred when new modules are bought from OEMs to top-up the inventory. Disposal cost is incurred when the network arranges for the disposal of the non-repairable faulty modules or non-refurbishable commercial returns. Facility cost includes the operating cost of facilities. The network generates revenue from the sale of refurbished products and repair of out-of-warranty faulty modules.

Minimise Total Cost =

Transportation cost:

$$\sum_{m\in M_u}\sum_{i\in N_s}\sum_{j\in N_d}(e_{mij}+g_{mij})TC_{ij}+\sum_{m\in M_u}\sum_{i\in N_d}\sum_{j\in N_o^m\cup N_r^m}(e_{mij}+g_{mij})TC_{ij}+\sum_{m\in M_u}\sum_{z\in Z}\sum_{i\in N_s^z}\sum_{j\in N_l^z}(e_{mij}+g_{mij})TC_{ij}$$
$$+\sum_{m\in M_u}\sum_{i\in N_l}\sum_{j\in N_d}(e_{mij}+g_{mij})TC_{ij}+\sum_{m\in M_u}\sum_{z\in Z}\sum_{i\in N_s^z}\sum_{j\in N_l^z}(e_{mij}+g_{mij})TC_{ij}+\sum_{m\in M_u}\sum_{i\in N_l}\sum_{j\in N_d}(e_{mij}+g_{mij})TC_{ij}$$
$$+\sum_{m\in M_u}\sum_{z\in Z}\sum_{j\in N_s^z}\sum_{i\in N_l^z}(e'_{mij}+g'_{mij})TC_{ij}+\sum_{m\in M_u}\sum_{j\in N_l}\sum_{i\in N_d}(e'_{mij}+g'_{mij})TC_{ij}+\sum_{p\in P_u}\sum_{z\in Z}\sum_{i\in N_c^z}\sum_{j\in N_s^z}f_{pij}TC_{ij}$$
$$+\sum_{m\in M_u}\sum_{j\in N_d}\sum_{i\in N_o^m}h_{mij}TC_{ij}+\sum_{m\in M_u}\sum_{i\in N_d}\sum_{j\in N_s}h_{mij}TC_{ij}+\sum_{m\in M_u}\sum_{i\in N_d}\sum_{j\in N_l}h_{mij}TC_{ij}+\sum_{m\in M_u}\sum_{z\in Z}\sum_{i\in N_l^z}\sum_{j\in N_s^z}h_{mij}TC_{ij}.$$

Processing cost:

$$+\sum_{p\in P_r}\sum_{q\in Q}\sum_{i\in N_s^q} PC_{pi}r_{pi} + \sum_{p\in P_f}\sum_{q\in Q}\sum_{j\in N_s^q} PC_{pj}S_{pj}.$$

Repair cost:

$$+\sum_{m\in M_u}\sum_{j\in N_d}\sum_{i\in N_o^m\cup N_r^m} RC_{mi}g'_{mij}.$$

Purchasing cost:

$$+\sum_{m\in M_u}\sum_{j\in N_d}\sum_{i\in N_o^m} h_{mij}\,BP_m.$$

Disposal cost:

$$+\sum_{p\in P_u}\sum_{z\in Z}\sum_{i\in N_c^z}\sum_{j\in N_s^z}(1-\beta_p)\,DC_p f_{pij} + \sum_{m\in M_u}\sum_{i\in N_s} DC_m(1-\mu_m)(1-\gamma_m)d_{mi}.$$

Facility cost:

$$+\sum_{i\in N_d\cup N_l} FC_i y_i.$$

Revenue:

$$-\sum_{p\in P_r}\sum_{q\in Q}\sum_{i\in N_s^q} PR_p\, r_{pi} - \sum_{m\in M_u}\sum_{i\in N_s}(1-\gamma_m)d_{mi}RP_m. \tag{1}$$

3.2 *Constraints*

Constraint (2) models the identification of faulty modules at service centres. We set the total number of faulty modules m obtained at a service centre equal to a fraction of the total modules m received:

$$d_{mj} = \sum_{p\in P_f} \lambda_m X_{mp} S_{pj} \quad (\forall m \in M_u)(\forall q \in Q)(\forall j \in N_s^q). \tag{2}$$

The flow of commercial returns from retailers to service centres is modelled in constraint (3), which equates the total outflow from a retailer to the service centres, both in the same country, to the total supply of commercial returns at the retailer:

$$\sum_{i\in N_s^z} f_{pji} = S_{pj} \quad (\forall p \in P_u)(\forall z \in Z)(\forall j \in N_c^z). \tag{3}$$

The outflow of faulty modules under warranty (or out-of-warranty) from a service centre to country DCs or DCs is equal to the total number of under-warranty (or out-of-warranty) faulty modules collected at the service centre. Constraints (4) and (5) model this flow:

$$\sum_{k\in N_d} e_{mjk} + \sum_{k\in N_l^z} e_{mjk} = \gamma_m d_{mj} \quad (\forall m \in M_u)(\forall z \in Z)(\forall j \in N_s^z), \tag{4}$$

$$\sum_{k\in N_d} g_{mjk} + \sum_{k\in N_l^z} g_{mjk} = \mu_m(1-\gamma_m)d_{mj} \quad (\forall m \in M_u)(\forall z \in Z)(\forall j \in N_s^z). \tag{5}$$

Constraints (6) and (7) model the outflow of faulty modules from country DCs to DCs as equal to the total inflow from the service centres. Similarly, constraints (8) and (9) equate the flow of faulty modules from DCs to OEMs and

RVs to the total inflow from service centres and country DCs:

$$\sum_{k \in N_d} e_{mjk} = \sum_{i \in N_s^z} e_{mij} \quad (\forall m \in M_u)(\forall z \in Z)(\forall j \in N_l^z), \tag{6}$$

$$\sum_{k \in N_d} g_{mjk} = \sum_{i \in N_s^z} g_{mij} \quad (\forall m \in M_u)(\forall z \in Z)(\forall j \in N_l^z), \tag{7}$$

$$\sum_{k \in N_o^m \cup N_r^m} e_{mjk} = \sum_{i \in N_s} e_{mij} + \sum_{i \in N_l} e_{mij} \quad (\forall m \in M_u)(\forall j \in N_d), \tag{8}$$

$$\sum_{k \in N_o^m \cup N_r^m} g_{mjk} = \sum_{i \in N_s} g_{mij} + \sum_{i \in N_l} g_{mij} \quad (\forall m \in M_u)(\forall j \in N_d). \tag{9}$$

The refurbishment process is modelled in constraint (10). We obtain the total number of refurbished units at a service centre as a fraction of the total inflow of commercial returns that can be transformed into the refurbished product at the service centre

$$\sum_{p' \in P_u^p} \sum_{i \in N_c^z} \beta_{p'} f_{p'ij} = r_{pj} \quad (\forall p \in P_r)(\forall z \in Z)(\forall j \in N_s^z). \tag{10}$$

Constraints (11) and (12) ensure that the refurbished product and repair demands are satisfied. The refurbished product demands at the retailers are satisfied through the refurbishment of the commercial returns. Repair demands at the service centres are satisfied by the inflow of repaired and new modules from DCs and country DCs:

$$D_{pj} = r_{pj} \quad (\forall p \in P_r)(\forall q \in Q)(\forall j \in N_s^q), \tag{11}$$

$$d_{mj} = \sum_{i \in N_d \cup N_l^z} (e'_{mij} + g'_{mij} + h_{mij}) \quad (\forall m \in M_u)(\forall z \in Z)(\forall j \in N_s^z). \tag{12}$$

The flows (forward) of repaired modules from DCs to service centres (constraints (13) and (17)), DCs to country DCs (constraints (14) and (18)), country DCs to service centres (constraints (15) and (19)) and OEMs/RVs to DCs (constraints (16) and (20)) are balanced by the flows (reverse) of faulty modules:

$$e_{mji} = e'_{mij} \quad (\forall m \in M_u)(\forall i \in Nd)(\forall j \in N_s), \tag{13}$$

$$e_{mji} = e'_{mij} \quad (\forall m \in M_u)(\forall i \in N_d)(\forall j \in N_l), \tag{14}$$

$$e_{mji} = e'_{mij} \quad (\forall m \in M_u)(\forall z \in Z)(\forall i \in N_l^z)(\forall j \in N_s^z), \tag{15}$$

$$e'_{mij} = e_{mji} \quad (\forall m \in M_u)(\forall i \in N_r^m \cup N_o^m)(\forall j \in N_d), \tag{16}$$

$$g_{mji} = g'_{mij} \quad (\forall m \in M_u)(\forall i \in N_d)(\forall j \in N_s), \tag{17}$$

$$g_{mji} = g'_{mij} \quad (\forall m \in M_u)(\forall i \in N_d)(\forall j \in N_l), \tag{18}$$

$$g_{mji} = g'_{mij} \quad (\forall m \in M_u)(\forall z \in Z)(\forall i \in N_l^z)(\forall j \in N_s^z), \tag{19}$$

$$g'_{mij} = g_{mji} \quad (\forall m \in M_u)(\forall i \in N_r^m \cup N_o^m)(\forall j \in N_d). \tag{20}$$

Constraint (21) sets the outflow of new modules from a country DC to service centres equal to the inflow of new modules from the DCs. Similarly, the outflow of new modules from a DC to service centres and country DCs is set equal to the inflow from the OEMs using constraint (22):

$$\sum_{i\in N_d} h_{mij} = \sum_{i\in N_s^z} h_{mji} \quad (\forall m \in M_u)(\forall z \in Z)(\forall j \in N_l^z), \tag{21}$$

$$\sum_{i\in N_o^m} h_{mij} = \sum_{i\in N_s} h_{mji} + \sum_{i\in N_l} h_{mji} \quad (\forall m \in M_u)(\forall j \in N_d). \tag{22}$$

The total flow of modules through a facility cannot be more than the capacity of the facility. This is modelled for country DCs and DCs using constraints (23) and (24), respectively:

$$\sum_{i\in N_s^z} (e_{mij} + g_{mij} + h_{mji}) \le V_{mj} y_j \quad (\forall m \in M_u)(\forall z \in Z)(\forall j \in N_l^z), \tag{23}$$

$$\sum_{i\in N_s \cup N_l} (e_{mij} + g_{mij} + h_{mji}) \le V_{mj} y_j \quad (\forall m \in M_u)(\forall j \in N_d). \tag{24}$$

We also want to ensure that each country can, at most, have one DC; this is enforced using constraint (25). Also, there can only be one DC or country DC in a particular region; we enforce this using constraint (26):

$$\sum_{i\in N_d^z} y_i \le 1 \quad (\forall z \in Z), \tag{25}$$

$$\sum_{i\in N_d^q \cup N_l^q} y_i \le 1 \quad (\forall q \in Q). \tag{26}$$

If there is a DC in a country then there should be no country DCs in that country; we capture this using constraint (27):

$$\sum_{i\in N_l^z} y_i \le |N_l^z| \left(1 - \sum_{j\in N_d^z} y_j\right) \quad (\forall z \in Z). \tag{27}$$

Finally, we specify the binary and positive restrictions on decision variables:

$$y_i \in \{0, 1\} \quad (\forall i \in N_d \cup N_l), \tag{28}$$

$$\text{All variables} \ge 0. \tag{29}$$

4. Problem data and analysis

In this case study, we considered the company's network in the Asia-Pacific region spread over 16 countries (see Figure 1). The 16 countries were divided into 78 regions and each region was assigned one service centre and one retailer. It was assumed that the customers in a particular region go to retailers/service centres in that region to return/repair the products. The retailers were allowed to forward the commercial returns to any of the service centres within the same country.

We consider commercial returns of two types of products: laptop and desktop computers. After refurbishment, these are sold as refurbished computers. A total of 126 modules were considered in the repair part of the network. The faulty modules are collected at the service centres, who can then forward the modules to either a country DC, if present in the country, or to any of the DCs. Currently, the company has DCs in three locations: Sydney, Singapore and Tokyo. The DCs in Tokyo and Sydney serve the Japanese and Australian markets, respectively; the Singapore DC serves the rest of Asia. The company also has country DCs in Beijing, Shanghai, Bangalore and Seoul.

We developed two optimisation models. The first model represents the existing network. It is a linear programming model as the locations of DCs and country DCs are fixed and there are no binary variables. The second mathematical model is based on Section 2 where any DC can serve any market and a service centre can

Table 1. Regional DC and country DC locations considered (P = potential, E = existing).

Region	Country	DC	Country DC
Bangalore	India	P	E
Bangkok	Thailand	P	P
Beijing	China	P	E
Ho chin minh City	Vietnam	P	P
Hong Kong	Hong Kong	P	P
Jakarta	Indonesia	P	P
Kuala Lumpur	Malaysia	P	P
Manila	Philippines	P	P
Seoul	South Korea	P	E
Shanghai	China	P	E
Singapore	Singapore	E	
Sydney	Australia	E	
Taipei	Taiwan	P	P
Tokyo	Japan	E	

Table 2. Summary of problem data.

Data	No.
Countries	16
Regions	78
Retailers	78, one per region
Service centres	78, one per region
Country DCs	11
DCs	14
Types of modules	126
Types of commercial returns	2 (computer and laptop)
Types of refurbished products	2 (refurbished – computer and laptop)
Arcs in the network	417,712
Binary variables	25

directly ship to the DC bypassing the country DC. For this model, we considered a total of 11 (potential) country DC locations (four existing) in 10 countries and 14 regional DC locations (three existing) in 13 countries. Eleven of the country DC locations were the same as the 11 DC locations, thus it was left up to the model to decide where to open a DC or a country DC. Also, the model is allowed to shut down a facility if it leads to an improved solution. Shutdown costs were added to each of the existing facilities for this purpose. The DC and country DC locations considered in this paper are presented in Table 1.

The data for the problem was provided by the company; the missing data was filled in from freely available sources (logistics company websites). The data is summarised in Table 2. The models were developed in ILOG OPL Studio 6.0 and solved to optimality using ILOG CPLEX 11.1. The first mathematical model was solved in under a minute, while the second mathematical model was solved in less than 10 minutes.

The optimised network (Figure 3) provides a saving of 11.1% over the existing network (Figure 4) and uses four DCs, Sydney, Tokyo, Singapore and Shanghai, and two country DCs, Bangalore and Seoul. On observing the various network costs, we found that the optimised network has lower transportation costs and higher facility and repair costs than the existing network. However, the savings in transportation costs more than compensate for the increased facility and repair costs. The upgrade of the Shanghai country DC resulted in the closure of the Beijing country DC, as we had constraint (27), which does not allow a country DC to be located in a country with a regional DC.

Observing the flow variables in the optimised network we find that the flows through country DCs and DCs are different from the existing network (Table 3). For example, the Seoul country DC now receives 55% fewer modules than the existing network. We also noticed changes in flows from service centres to DCs (Table 4). For example, the Guam service centre no longer forwards the modules to the Singapore DC, but instead sends them to the Shanghai DC. The change in inter-facility flows is due to the fact that the optimisation process chooses the lower cost paths in the network. From the problem data we observed that, in general, the transportation cost of modules on the Guam–Singapore DC-OEM + RV path was significantly higher than on the Guam–Shanghai DC-OEM + RV path.

The company also wanted to investigate the possibility of performing repairs in China. For this purpose, we considered two scenarios. The first involved moving all the OEMs and RVs, excluding those in Taiwan, to China. This resulted in savings of about 5.1% over the optimal solution. The network configuration was found to be the same as the optimal solution. The second scenario located all the OEMs and RVs in China. A saving of 4.74% over the optimal solution was obtained in this case; the network configuration was again unchanged. The savings were due to the reduction in transportation costs in the network. The first option provided slightly better savings, mainly due to the fact that Taiwan OEMs/RVs are cheaper to reach from some of the DCs. For example, the cost of shipping from the Singapore DC to an OEM/RV in China is greater than shipping from a Singapore DC to an OEM/RV in Taiwan. We considered the above possibilities without taking into account the costs that may be incurred when moving the OEMs/RVs to new locations and the fact that it may not be possible to get modules repaired in a country other than the one where the vendor is currently located. We also assumed that repair and purchase costs will stay the same. Thus, the above possibilities should only be considered if the costs incurred on the

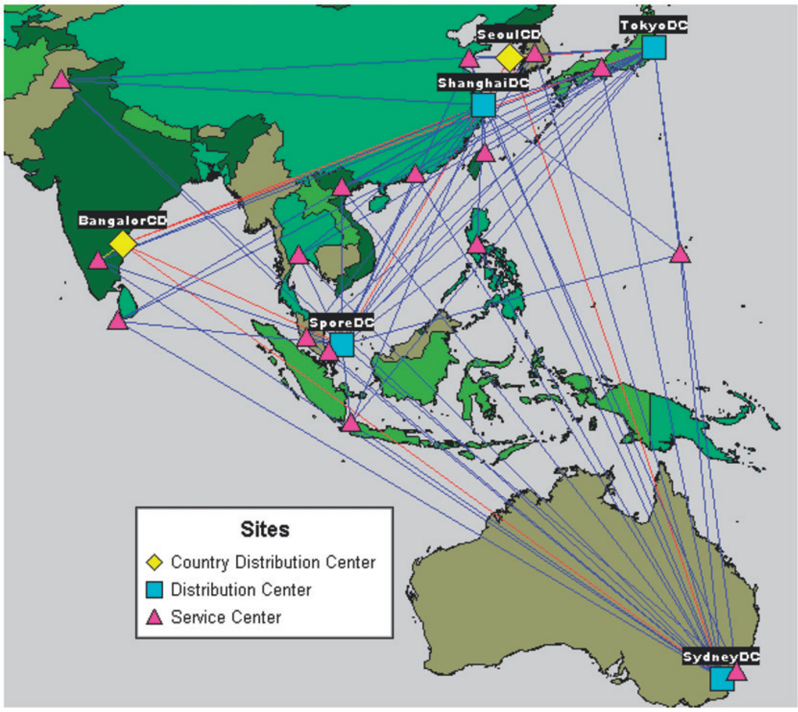

Figure 3. Optimised after-sale service network for the company.

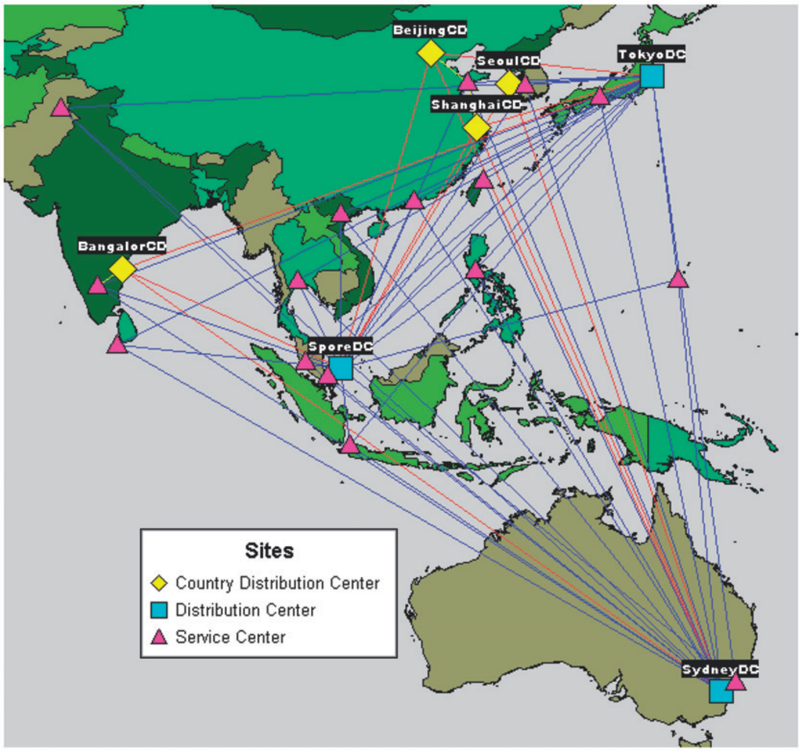

Figure 4. Existing after-sale service network for the company.

Table 3. Flow of modules through country DCs (CDC) and DCs.

Facility	Existing network	Optimised network	% Change
Shanghai DC	401,261	1,238,400	209%
Beijing CDC	441,908	0	Shutdown
Bangalore CDC	175,720	175,720	0%
Seoul CDC	248,730	112,020	−55%
Singapore DC	1,757,095	605,408	−66%
Tokyo DC	1,076,763	1,046,655	−3%
Sydney DC	257,420	200,814	−22%

Table 4. Flows from service centres in existing and optimised networks.

Origin facility	Existing network	Optimised network
Guam SC	Singapore DC: 100%	Shanghai DC: 100%
Sri Lanka SC	Singapore DC: 100%	Shanghai DC: 100%
Hong Kong SC	Singapore DC: 100%	Singapore DC: 58%, Shanghai DC: 42%
Philippines SC	Singapore DC: 100%	Singapore DC: 56%, Shanghai DC: 44%
Taiwan SC	Singapore DC: 100%	Singapore DC: 58%, Shanghai DC: 42%
South Korea SC	Seoul CDC: 100%	Seoul CDC: 52%, Singapore DC: 48%
Australia SC	Sydney DC: 100%	Sydney DC: 78%, Shanghai DC: 22%

network are no more than the cumulative savings over the next few years and it is possible to carry out the repairs in other countries.

4.1 *Robust optimisation of service network*

Setting up a reverse logistics network for the company is a strategic planning problem. In the deterministic model presented above, all the parameters are assumed to be known exactly, which is not true in real life as some of the parameters change over time. For instance, as the demand grows faster in China over the next few years, the supply of returns to the country DCs located there would grow as well. However, as it is not clear how much the demand would grow (the management can only venture educated guesses), the designed network should be robust to a range of growth.

We use min-max robust criteria (Kouvelis and Yu 1997) with the scenario-based optimisation approach to handle the uncertainty in the network. The approach used is conservative as it minimises the maximal deviation of the robust solution from the optimal solution (regret) of each scenario, over all the scenarios. The application of min-max robust optimisation requires the generation of equally probable scenarios. The scenarios are defined as an occurrence of particular values of the problem parameters. Thus, the probability information is not required as all the scenarios are considered to be equally likely. This makes the approach suitable for problems where only limited information is available as is the case with the problem presented in this paper. The robust solution obtained may coincide with the optimal solution of one of the scenarios. At the same time, it is possible for the robust solution to be sub-optimal for all the scenarios.

Let, S be the set of scenarios. The model presented in Section 3 is then solved to obtain the optimal solution for each of the scenarios. Let, O^s be the optimal solution value for scenario $s \in S$. Let, R^s be the value of the robust solution for scenario $s \in S$. The robust optimisation model can then be written as

$$\text{Minimise: } \delta' \tag{30}$$

$$\text{subject to } \delta \geq R^s - O^s, \quad \forall s \in S, \tag{31}$$

$$\text{flow balance constraints (Equations (2) − (22))}, \quad \forall s \in S, \tag{32}$$

$$\text{logical constraints (Equations (23) and (24))}, \quad \forall s \in S, \tag{33}$$

$$\text{Equations (25) − (28)}, \tag{34}$$

$$\text{Equation (29)}, \quad \forall s \in S. \tag{35}$$

The objective function term (Equation (30)) minimises the deviation of the robust solution from the optimal solutions for different scenarios, over all the scenarios. The first constraint (Equation (31)) calculates the maximum deviation of the robust solution from the optimal solutions. Constraints (32) and (33) ensure that flow balance and logical constraints are satisfied for all scenarios. Constraint (34) is the set of constraints (25)–(28) from the mathematical model of Section 3. Constraint (35) restricts the value of decision variables to the positive domain.

To solve the robust optimisation model, a series of steps need to be followed. First, an expert decision maker is asked to specify possible scenarios under which the network is likely to operate. It is important that due care is taken when specifying the scenarios as the approach is highly sensitive to the input data. Scenarios that are very unlikely to occur but can significantly affect the network structure should be avoided as they can easily skew the robust solution. Once the scenario set has been specified, the mathematical model in Section 3 is solved to obtain the optimal solution for each of the scenarios. Once all the scenarios have been solved the robust model presented above is solved to obtain the robust solution for the service network.

It is possible that different scenarios may have a similar network structure so we can use the 'warm start' feature of ILOG CPLEX to reduce the computation times. The warm start feature of CPLEX allows the user to set the initial solution for the mathematical model. If a full feasible solution is provided for an MILP, CPLEX makes it the incumbent solution. However, the user can also provide the values of integer variables only and CPLEX will try to find the values of continuous variables and make it the incumbent solution. When only few of the integer variables are provided, CPLEX tries to extend the initial solution to obtain a full solution. Finally, if the full solution provided is infeasible or if the values of integer variables provided result in an infeasible solution, CPLEX tries to repair the solution to obtain an incumbent solution, failing which it behaves as if no solution was provided. For our case we can specify the starting values of the binary variables, i.e. the country DC and DC binary values of the optimal base case solution.

4.2 *Application of robust optimisation*

We conducted a thorough sensitivity analysis on three parameters of interest: supply of faulty modules, warranty fraction (γ) and fraction of out-of-warranty modules sent for repair (μ). The sensitivity analysis revealed that the supply of faulty modules is the most important parameter affecting the network configuration; the other two parameters are implicitly covered by varying the supply. Next, we specified the range over which supply can vary in different countries. The company sees strong growth in China, so we chose three levels of supply growth for China: low (+30%), medium (+60%) and high (90%). The markets in India and Japan are expected to show moderate growth, which resulted in two levels of supply for India and Japan: low (+30%) and high (+60%). The remaining countries were grouped together and were assigned two levels of supply growth: low (+30%) and high (+60%). Thus, in total, 24 scenarios were considered.

We wrote a program in C++ that calls the model implementation in ILOG OPL and solves one scenario at a time (using CPLEX) by making the required changes in the problem data. We then tried to solve the robust optimisation model. However, we were not able to do so due to memory issues. To address this problem, we divided the scenarios into two sets: scenarios with the same network configuration (S1) and scenarios with a different configuration than those in S1; we called this set S2. In our case, S1 and S2 had the cardinality of 20 and 4, respectively. We then performed 20 runs of the robust optimisation model, with each run consisting of scenarios from S2 and one scenario from S1. We recorded robust solutions for each of the runs and found that all of the runs had the same robust solution. We also found that the robust solution coincided with the optimal solution of the scenarios in S1.

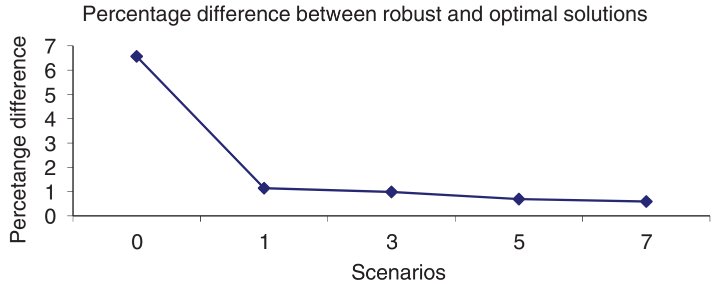

Figure 5. Comparison of robust and optimal solutions for scenarios 0, 1, 3, 5 and 7.

Table 5. Flows from service centres in existing and robust networks.

Facility origin	Existing network	Robust network
Guam SC	Singapore DC: 100%	Shanghai DC: 34% Taipei DC: 66%
Sri Lanka SC	Singapore DC: 100%	Shanghai DC: 24% Taipei DC: 76%
Hong Kong SC	Singapore DC: 100%	Shanghai DC: 24% Taipei DC: 76%
Philippines SC	Singapore DC: 100%	Shanghai DC: 25% Taipei DC: 75%
Taiwan SC	Singapore DC: 100%	Taipei DC: 100%
South Korea SC	Seoul CDC: 100%	Seoul CDC: 35% Shanghai DC: 3% Taipei DC: 62%
Australia SC	Sydney DC: 100%	Sydney DC: 28% Shanghai DC: 34% Taipei DC: 38%

Let R^s and O^s be the robust and optimal solution value for scenario $s \in \mathrm{S1} \cup \mathrm{S2}$. We know that R^s–O^s is equal to zero for $s \in \mathrm{S1}$, so we can safely say that the robust optimisation model in every run is effectively minimising the maximal deviation of the robust solution (optimal solution of $s \in \mathrm{S1}$) from the optimal solutions of $s \in \mathrm{S2}$. Thus, we can conclude that the robust solution obtained by running all 24 scenarios together (S1∪S2) will coincide with the one obtained above.

The robust solution obtained has one additional DC (Taipei DC) compared with the optimal solution and is 6.56% more expensive. However, the robust solution costs 5.27% less than the existing network to set up and operate. Figure 5 shows the performance of the robust solution for the optimised network (scenario 0) and scenarios 1, 3, 5 and 7. From the figure we can see that the robust solution, although not optimal, performs reasonably well over all the scenarios. The robust solution coincided with the optimal solution of the remaining scenarios (not shown in Figure 5).

Table 5 shows the flows from service centres for the existing and robust networks. From the table we can see that a lot of module flow is now directed to the Taipei DC. For instance, service centres in South Korea forward about 53% of the faulty modules to Taipei DC. This increase in flow is due to the presence of a number of OEMs and RVs in Taiwan; if modules are forwarded to Taipei DC, considerable savings in transportation are achieved due to the presence of OEMs and RVs there.

5. Conclusion

In this research, we developed a deterministic model in the form of a MILP for designing a service network for electronic products. We also collected all the relevant data from a company's service network in the Asia-Pacific region and used it to validate the model. The optimal network configuration requires the use of one extra DC but results in a reduction in network cost. From an analysis of the results we found that the reduction in network cost is mainly due to the decrease in transportation costs, which more than compensates for the increase in repair and facility costs. We also found that locating DCs closer to the OEMs and RVs or finding OEMs and RVs closer to DCs is worth exploring, as it may result in significant cost savings.

As setting up a reverse logistics network for the company is a strategic planning problem, we identified the parameters that are likely to vary over time. We also developed the scenarios using the levels and range for the uncertain parameters for which the proposed network should be robust. A min-max robust optimisation model was developed to obtain a robust solution for the scenarios considered. This approach was selected as not enough information was available to use stochastic optimisation. The robust solution obtained resulted in one extra DC location compared with the optimal network configuration. The robust solution performs reasonably well under all the scenarios considered with only a slight increase in cost.

Future research can focus on different product types, including non-modular ones. Also, the model can be extended to represent the disposal of non-repairable modules, thereby introducing end-of-life product management into the model. Finally, decomposition and heuristic approaches can be developed to facilitate the application of this model to more complex data sets.

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References

Chen, C.-L. and Lee, W.-C., 2004. Multi-objective optimization of multi-echelon supply chain networks with uncertain product demands and prices. *Computers & Chemical Engineering*, 28 (6/7), 1131–1144.

Du, F. and Evans, G.W., 2008. A bi-objective reverse logistics network analysis for post-sale service. *Computers & Operations Research*, 35 (8), 2617–2634.

Fleischmann, M., *et al*., 2001. The impact of product recovery on logistics network design. *Production and Operations Management*, 10 (2), 156–173.

Guide Jr, V.D.R., *et al*., 2006. Time value of commercial product returns. *Management Science*, 52 (8), 1200–1214.

Hong, I.-H., *et al*., 2006. Planning the e-scrap reverse production system under uncertainty in the state of Georgia: a case study. *IEEE Transactions on Electronics Packaging Manufacturing*, 29 (3), 150–162.

Jayaraman, V., Guide, V.D.R., and Srivastava, R., 1999. Closed-loop logistics model for remanufacturing. *Journal of the Operational Research Society*, 50 (5), 497–508.

Kara, S., Rugrungruang, F., and Kaebernick, H., 2007. Simulation modelling of reverse logistics networks. *International Journal of Production Economics*, 106 (1), 61–69.

Kouvelis, P. and Yu, G., 1997. *Robust discrete optimization and its applications*. Boston, MA: Kluwer.

Kusumastuti, R.D., Piplani, R., and Lim, G.H., 2008. Redesigning closed-loop service network at a computer manufacturer: a case study. *International Journal of Production Economics*, 111 (2), 244–260.

Listes, O., 2007. A generic stochastic model for supply-and-return network design. *Computers & Operations Research*, 34 (2), 417–442.

Listes, O. and Dekker, R., 2005. A stochastic approach to a case study for product recovery network design. *European Journal of Operational Research*, 160 (1), 268–287.

Min, H. and Ko, H.-J., 2008. The dynamic design of a reverse logistics network from the perspective of third party logistics service providers. *International Journal of Production Economics*, 113 (1), 176–192.

Mulvey, J.M., Vanderbei, R.J., and Zenios, S.A., 1995. Robust optimization of large-scale systems. *Operations Research*, 43 (2), 264–281.

Realff, M.J., Ammons, J.C., and Newton, D.J., 2004. Robust reverse production system design for carpet recycling. *IIE Transactions (Institute of Industrial Engineers)*, 36 (8), 767–776.

Sarkis, J. and Sundarraj, R.P., 2002. Hub location at Digital Equipment Corporation: a comprehensive analysis of qualitative and quantitative factors. *European Journal of Operational Research*, 137 (2), 336–347.

Savaskan, R.C., Bhattacharya, S., and Van Wassenhove, L.N., 2004. Closed-loop supply chain models with product remanufacturing. *Management Science*, 50 (2), 239–252.

Toktay, L.B., 2003. Forecasting product returns. *In*: V.D.R. Guide and L.V. Wassenhove, eds. *Business aspects of closed-loop supply chains*. Pittsburgh, PA: Carnegie Mellon University Press.