Recycling Pricing of Valuable WEEE Based on Hotelling Model

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***Abstract*—Considering the convenience of third-party recycler, this paper uses the Hotelling competition model and the Stackelberg game to study the recycling competition between a producer and a third-party recycler in the waste electronic and electrical equipment (WEEE) recycling market. Stackelberg game theory is used to study the pricing decisions of recycling of WEEE for producer and third-party recycler under the conditions of non-competition, partial coverage competition and complete coverage competition. The results of the study show that the recycling prices of producer and recycler are not negatively related to the unit revenue. The market coverage is positively related to both sides' unit revenue. The producer's profit under full coverage with competition is always higher than the partial coverage with competition, and the profit of recycler depends on unit revenue.**

Keywords—WEEE, Competition, Hotelling model, Stackelberg game, recovery of e-waste, Pricing

I. Introduction

With the rapid development of science and technology, a series of new products with multifunctional, more innovative design enter the market to meet the different needs of customers. New version products receive the consumer's favorite, as the quality is better and the price is cheaper. Which makes the product, especially electrical and electronic products, with a shorter life cycle, and the number of WEEE increased dramatically. According to the statistics, the global total number of WEEE up to 41.8 million tons in 2014, and is expected up to 50 million tons in 2018[1].

To address resource and environmental issues, Thomas proposed the concept of Extended Producer Responsibility (EPR), which gives producers the responsibility for the product throughout the entire life cycle, especially collection , recycling and the ultimate disposition of the end of life(EOL) products.

On the one hand, waste electrical and electronic equipment (WEEE) contain a variety of harmful substances, such as mercury, chromium, bromide, etc. If not effectively deal with, not only will cause a great waste of resources, but also pollution of land and water resources, and ultimately will seriously affect the human survival of the ecological environment. As a result, many countries enforce take-back regulation, which holds producers are responsible for the collection and proper disposal of products. On the other hand, WEEE such as cell phones or computers, which contain precious metals (e.g., gold or platinum), which attracts other entities participate in recycling e-waste. Supporting this observation, The European Commission reports that nearly 2/3 of e-waste has end-up outside the reach of producers,most of which are handed by informal, third-part recyclers.

While remanufacturing is considered an effective way to make full advantage of the residual value of used products, not all products are suitable for the use of remanufacturing for the recovery of surplus value. However, many WEEE not only need a lot of resources in the manufacturing process, but also in the use of the process still need to consume a lot of energy, such as refrigerators, air conditioners, water heaters and so on. As the new versions of products are usually more environmental friendly and energy efficient than older products, therefore from the perspective of environmental protection, such products are not suitable for remanufacturing. In recent years, with the increase in the price of precious metals in the market and the development of recycling technology, the dismantling and recycling of WEEE also has a net worth, so that producers are willing to dismantle and recycle the WEEE. In this article, research on the recovery competition between a producer and a third part recycler without considering remanufacturing.

II. Literatures

Under the regulation of government policies, manufacturers have began to get involved in the field of recycling of used electrical and electronic products. In general, recycled WEEE can be dismantled and remanufactured[2]. Further, from an economic and environmental perspective, remanufacturing is an ideal recovery model for used products that can achieve the overall re-commercialization of life-end products[3]-[4]. Domestic and foreign scholars focus on re-manufacturing closed-loop supply chain research focused on competition in the new products and remanufacturing products pricing and production decision-making, of which new products and remanufacturing products in the internal competition, the existing study major explore how the manufacturer adjust the proportion of new and remanufactured products in the hybrid manufacturing system according to the optimal pricing strategy. For example, [5] studies the competitive relationship between a single manufacturer's new product and remanufacture product in two periods. References

[6] and [7] study the joint pricing and product remanufacturing technical options for remanufacture product under heterogeneous consumer demand. Reference [8] analyzes the relationship between market size, manufacturing strategy, pricing and recovery, assuming that the new product and remanufactured product have the same quality.

In the external competition for new products and remanufacture product, [9] analyzed the impact of product interchangeability on manufacturer and remanufacturer pricing decisions. Reference [10] points out that OEM can provide high-quality products for high-end consumers, to prevent remanufacturers to remanufacture. Reference [11] studied the third-party enterprises to refurbish IT products and remanufacturing, and considers the original manufacturer's optimal re-licensing fees and pricing decisions. For the competition system of three products, [12] studies the pricing decisions of manufacturers and remanufacturers in two-cycle, multi-cycle and infinite cycles. Reference [13] explores the pricing strategies of manufacturers to adopt remanufacturing strategies to compete with another manufacturer, and explores the impact of remanufacturing profitability and competitors' entry into the market. Reference [14] studied how the manufacturer and remanufacturer could determine the scale of production of new and remanufactured products based on the size of the remanufacturing cost. Through the study of the above related literatures, it is found that the current research on waste electrical and electronic products mostly on the pricing or production decisions of new and remanufactured products, and there is little literature on the waste electrical and electronic products in the recovery process of recycling pricing.

The profitability of the dismantling and recycling of used electrical and electronic products has also attracted third party recycler to enter the market, leading to the competition with producer in the recycling market of WEEE. At present, there is less research on the recovery competition between producer and third-party recycler. Reference [15] used the stylized model with the original equipment manufacturer (OEM) to study these issues and face competition from independent remanufacturers and examine three key factors of regulatory effectiveness: remanufacturing level, consumer surplus and OEM profit. Reference [16] considered the producer who is subject to the Extended Producer Responsibility (EPR) competing with recycler who is not subject to the EPR and study how take-back regulation affect environment and economic. In their model, the producer and recycler make their decisions simultaneously, however, in our really life, the recycler generally makes his decision after the producer, hence, in our paper we use the Stackelberg game between the producer and recycler, in which the produce is the leader.

We examined a situation where in either party can enter the recovery market as long as it is profitable. Based on the convenience of the third party recycler, this paper uses the Hotelling competition model and the Stackelberg game theory to study the recycling competitive performance of the producer and the third party recycler in the recycling market of WEEE. An important research question here is how the producer and recycler should decide their collection prices in order to maximize their profits, and how their optimal collection prices and profit change with the unit revenue.

III. Problem Description

In this section, we formulate a generic market model for a single product category. There are three stakeholders in our model: a producer (denoted by subscript m), a recycler (denoted by subscript s), a set of consumers who are waste-holders and possess used products. When the electrical products reach their end of life, the producer recovery WEEE from the consumers at the recovery price p_m , and can obtain revenue μ from the disposing of unit WEEE. Recycling of WEEE has a net income, which attracts third-party recycler into the recycling of WEEE. The recycler compete with producer in the recycling market for WEEE at a recycling price p_r , and can obtain revenue σ from the of disposal unit WEEE.

The decision sequence of stakeholders is as follows: As the leader of the Stackelberg game, the producer first determines the unit electronics recovery price p_m , then the recycler as the follower of the Stackelberg game with the advantage of geographical advantage (the competitive advantage) determines his recovery price p_r . Finally, according to the recovery prices given by the producer and the recycler, the consumers determine whether and to whom to return the WEEE by comparing the available utility. Decision order and decision variables are shown in Fig. 1.

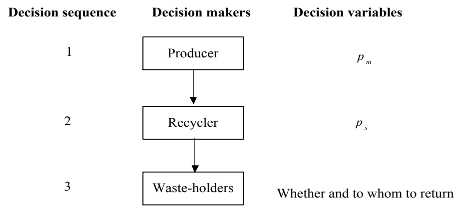


Fig. 1. Decision sequence and decision variables

A. Parameters Description

TABLE I. THE MAJOR PARAMETERS AND NOTATIONS

Notation	***Explanation***
Q	fixed demand in sales markets
β	recovery rate
μ	saving from recovery
v	unit price of the product sold by the producer
σ	unit revenue from recovery of recycler
θ	the location of the waster holders, $\theta \in [0,1]$
i	$i = m$ denotes producer, $i = s$ denotes recycler
ρ_i	the market coverage of player i
p_i	unit recovery price announced by player i
a	recycler's location on the Hotelling line
$U_i(\theta) = \mathrm{p}_i - w_i d(\mathrm{i},\theta)$	the net utility of the waster holder θ
$d(\mathrm{i},\theta)$	distance between consumer and decision maker i
w_i	the inconvenience of return, $w_s = t$ and $w_m = k$

B. Basic assumptions

- The market at steady state, in which there exists Q unit products that have been previously sold and have reached their end of life.
- Both the produce and recycler have a net profit when recycling WEEE.
- The waste volume is uniformly distributed, and waster holders are evenly distributed on [0,1] Hotelling line.
- Assuming that the producer is located at the right end of the Hotelling line (point 1), the recycler is located on point a of Hotelling line, where $a \neq 0$, reflecting the channel advantage of recycler.
- The producer and the recycler give the same recycling price to all waster holders, i.e., there is no price discrimination.

C. Decision model construction

The sales price of the producer's unit product is v, and the fixed demand in the sales market is Q. When these electronic products reach the end of life, β is recovered and $1-\beta$ is not recovered. Unit WEEE profit from recovery (unit cost of raw materials) of producer is μ, and recovery price is p_m. Based on the assumptions in this paper, the distance between the producer and the consumer θ is $d(\mathrm{m},\theta)=1-\theta$. When WEEE is sold to the producer, consumer θ receives the net utility is $U_m(\theta)=\mathrm{p}_m-k(1-\theta)$. Define ρ_m as the proportion of consumers selling WEEE to producer and achieving a higher non-negative utility, i.e., producer's market coverage. The profit of the producer includes the profit obtained from the sale of the new product and the WEEE recycling and recovery profit. The profit function is as follows:

$$\Pi_m(\mathrm{p}_m)=Q[v-\mu+(\mu-\mathrm{p}_m)\beta\rho_m(\mathrm{p}_m,\mathrm{p}_s)] \quad (1)$$

The recycler obtains σ from recycling unit WEEE at recovery price p_s. To describe the competitive advantage of WEEE recycling for recycler, this paper assumes that the recycler is located at point a on the Hotelling line, indicating that the recycler has a position advantage over the producer. Then the distance between the recycler and the consumer θ is $d(\mathrm{m},\theta)=|a-\theta|$, and when WEEE is sold to recycler, the net utility that consumers receive is $U_s(\theta)=\mathrm{p}_s-t|a-\theta|$. We define ρ_s as the proportion of consumers who have sold WEEE to the recycler when they receive higher non-negative utility, i.e., the market coverage of the recycler. The recycler's profit function can be expressed as follows,

$$\Pi_s(\mathrm{p}_s)=Q\beta\rho_s(\mathrm{p}_m,\mathrm{p}_s)(\sigma-\mathrm{p}_s) \quad (2)$$

In the Hotelling model of this article, the location of the consumer determines the size of the utility that can be obtained when the WEEE is sold to the producer or the recycler. Consumers decide whether and to whom to sell WEEE between the manufacturer or the recycler by comparing the acquired utility. It should be noted that consumers will sell WEEE only when consumers get the net utility is greater than 0, otherwise they will continue to hold WEEE. The total surplus of the consumers is calculate as the surplus of customers who return their WEEE to the producer plus the customers who return their WEEE to the recycler. The expression is as follows,

$$\Pi_c=Q\beta\left(\int_{\{\theta|U_m(\theta)\geq\max\{U_s(\theta),0\}\}}U_m(\theta)\mathrm{d}\theta+\int_{\{\theta|U_s(\theta)\geq\max\{U_m(\theta),0\}\}}U_s(\theta)\mathrm{d}\theta\right).(3)$$

IV. Decision Analysis

A. Market coverage

For the given recovery price p_m and p_s, we first analyze the market coverage. According to the previous assumptions, there are three different forms of market coverage.

(a)Market coverage without competition: in this case, the WEEE recycling price p_m and p_s developed by the producer and the recycler are relatively low, and there is no direct competition between them, i.e., the two sides in the [0, 1] range of market coverage does not overlap (as shown in Fig. 2 (a1) and (a2)). The market coverage of producer is $\rho_m=1-\theta_1=\frac{p_m}{k}$, where $\theta_1=(\theta|\mathrm{U_m}(\theta)=0)$. The recycler's coverage to the left is $\rho_s^l=a-\max\{\theta_3,0\}$ and to the right is $\rho_s^r=\theta_2-a=\frac{p_s}{t}$, where $\theta_2=(\theta|\mathrm{U}_s^r(\theta)=0)=a+\frac{p_s}{t}$ and $\theta_3=(\theta|\mathrm{U}_s^l(\theta)=0)=a-\frac{p_s}{t}$.

(b) Partially coverage with competition: in that case, the recovery prices p_m and p_s announced by producer and recycler respectively are relatively high, which lead to direct competition between the two sides. The producer's coverage can be expressed as $\rho_m=1-\theta_{ind}=\frac{p_m-p_s+t-ta}{k+t}$, where $\theta_{ind}=(\theta|\mathrm{U_m}(\theta)-\mathrm{U}_s^r(\theta)=0)$, which means that consumers sell WEEE to producer and recycler with the same utility. At this point, the recycler does not completely cover the market on its left (as shown in Fig.2 (b)), the recycler's market coverage is $\rho_s=\theta_{ind}-\theta_3=\frac{(\mathrm{k}+2\mathrm{t})p_s-tp_m+kt(1-a)}{(k+t)t}$.

(c) Full coverage with competition: In this case, the recovery price determined by the producer and the recycler is so high that all WEEE is recovered, i.e., $\rho_\mathrm{m}+\rho_s=1$ (as shown in Fig. 2 (c) below). The producer's coverage is $\rho_m=1-\theta_{ind}=\frac{p_m-p_s+t-ta}{k+t}$, while the recycler's coverage is $\rho_s=\theta_{ind}=\frac{p_s-p_m+k+ta}{k+t}$.

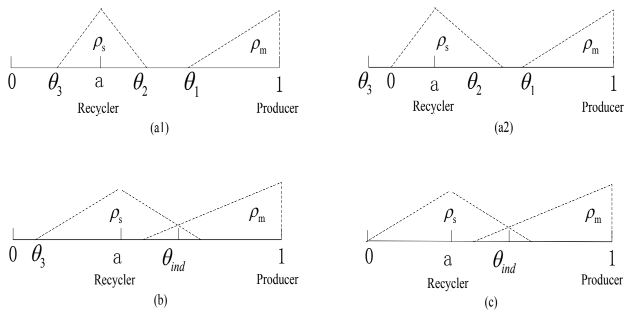


Fig. 2. Maker coverage

B. *Market coverage without competition*

When producer and recycler are relatively low in pricing for WEEE, there is no competition between the two sides. When $\theta_3 > 0$, the market coverage of the recycler is $\rho_s = \theta_2 - \theta_3$, and when $\theta_3 < 0$, the market coverage of the recycler is $\rho_s = \theta_2 - 0$. The producer's market coverage is $\rho_m = 1 - \theta_1$.

Proposition 1: When the market coverage of producer and recycler is not overlapping (i.e. $\theta_2 < \theta_1$), the optimal recycling prices, market coverage and optimal profits of producer and recycler are shown in Table 2 below.

TABLE II. OPTIMAL DECISIONS WHEN MARKET COVERAGE WITHOUT OVERLAP

	p_m^*	p_s^*	ρ_m^*	ρ_s^*	Π_m^*	Π_s^*
$\theta_3 > 0$	$\frac{\mu}{2}$	$\frac{\sigma}{2}$	$\frac{\mu}{2k}$	$\frac{\sigma}{t}$	$Q(\mathrm{v} - \mu + \frac{\beta\mu^2}{4k})$	$\frac{Q\beta\sigma^2}{2t}$
$\theta_3 \le 0$	$\frac{\mu}{2}$	$\frac{\sigma - at}{2}$	$\frac{\mu}{2k}$	$\frac{at+\sigma}{2t}$	$Q(\mathrm{v} - \mu + \frac{\beta\mu^2}{4k})$	$\frac{Q\beta(at+\sigma)^2}{4t}$

Proof: When $\theta_3 > 0$, i.e., $p_s < at$ (as is shown in Fig.2 (a1)), then the recycler's coverage to the left is $\rho_s^l = a - \theta_3 = \frac{p_s}{t}$, and to the right is $\rho_s^r = \frac{p_s}{t}$. Thus the recycler's total coverage is $\rho_s = \frac{2p_s}{t}$ and the manufacturer's coverage is $\rho_m = \frac{p_m}{k}$. Using the backward sequential decision making approach. Substituting $\rho_s = \frac{2p_s}{t}$ into (2), we have the optimal recovery price of the recycler is a $p_s^* = \frac{\sigma}{2}$, and the recycler's optimal coverage is $\rho_s^* = \frac{\sigma}{t}$. Then, substituting $\rho_m = \frac{p_m}{k}$ into the (1), the optimal recycling price of the manufacturer is optimized by $p_m^* = \frac{\mu}{2}$, and the market coverage is $\rho_m^* = \frac{\mu}{2k}$. At this time, the best profits of recycler and producer are $\Pi_s^* = \frac{Q\beta\sigma^2}{2t}$ and $\Pi_m^* = Q(\mathrm{v} - \mu + \frac{\beta\mu^2}{4k})$ respectively.

When $\theta_3 \le 0$, i.e., $p_s \ge at$ (as is shown in Fig.1 (a2)), then the recycler's coverage to the left is $\rho_s^l = a$ and to the right is $\rho_s^r = \frac{p_s}{t}$. According to the backward sequential decision making approach, similar to the above derivation, we have the optimal recovery price of the recycler is $p_s^* = \frac{\sigma - at}{2}$, and the recycler's optimal coverage is $\rho_s^* = \frac{at+\sigma}{2t}$. The optimal recycling price of the manufacturer is optimized by $p_m^* = \frac{\mu}{2}$, and the market coverage is $\rho_m^* = \frac{\mu}{2k}$. At this time, the best profits of recycler and producer are $\Pi_s^* = \frac{Q\beta(at+\sigma)^2}{4t}$ and $\Pi_m^* = Q(\mathrm{v} - \mu + \frac{\beta\mu^2}{4k})$ respectively.

When $\theta_3 > 0$ and $p_s < at$, i.e., $\sigma < 2at$, the total coverage of producer and recycler is $\rho^* = \rho_m^* + \rho_s^* = \frac{\mu}{2k} + \frac{\sigma}{t} = \frac{t\mu + 2k\sigma}{2kt}$. When $\theta_3 \ge 0$ and $p_s \ge at$, i.e., $\sigma \ge 3at$, the total market coverage of both sides is $\rho^* = \rho_m^* + \rho_s^* = \frac{\mu}{2k} + \frac{at+\sigma}{2t} = \frac{t\mu + atk + k\sigma}{4kt}$. Thus, the market coverage is a monotonically increasing function with respect to μ and σ, that is, the market coverage increase as the revenue of the recycling increase.

C. *Partially coverage with competition*

When the recycling price of producer and recycler are relatively high, there will be direct competition between two sides. Under partially coverage, $\theta_3 > 0$ and $\theta_1 \ge \theta_2$ should be satisfied. Furthermore, the recycler's coverage is $\rho_s = \frac{(k+2t)p_s - tp_m + kt(1-a)}{(k+t)t}$ and producer's coverage is $\rho_m = \frac{p_m - p_s + t - ta}{k+t}$.

Proposition 2: When the market coverage is partially covered, the unique optimal recovery prices, market coverage and optimal profits of producer and recycler are shown in Table 3.

TABLE III. OPTIMAL DECISIONS UNDER PARTIAL COVERAGE

	Price	***Market coverage***
Producer	$p_m^* = \frac{2\mu+\sigma-3t+3ta}{4} + \frac{t(\sigma+t-ta)}{4(2k+3t)}$	$\rho_m^* = \frac{\mu-\sigma+t-ta}{4(k+t)} + \frac{\mu+2t-2ta}{4(k+2t)}$
Recycler	$p_s^* = \frac{\sigma-t+ta}{2} + \frac{t(\mu+2t-2ta)}{4(k+2t)} + \frac{t(\sigma+t-ta)}{4(2k+3t)}$	$\rho_s^* = \frac{\sigma+t-ta}{2t} - \frac{\mu-\sigma+t-ta}{4(k+t)} + \frac{\sigma+t-ta}{4(2k+3t)}$
	Profit	
Producer	$\Pi_m^* = Q(\mathrm{v} - \mu) + \frac{Q\beta[4t^2(1-a)+t(3\mu-2\sigma+3k-3ak)+k(2\mu-\sigma)]^2}{4(4k+6t)(k^2+3kt+2t^2)}$	
Recycler	$\Pi_s^* = \frac{Q\beta[4k^2(\sigma+t-at)+kt(9t-2\mu+13\sigma-9at)+t^2(4ta-3\mu+10\sigma+4t)]^2}{16t(k+t)(k+2t)(2k+3t)^2}$	

Proof: As the Stackelberg game is considered in this system, we can use the backward sequential decision making approach to analyze the problem.

Firstly, substituting the market coverage of recycler $\rho_s = \frac{(k+2t)p_s - tp_m + kt(1-a)}{(k+t)t}$ into (2), we can obtain the profit function of recycler is

$$\Pi_s(\mathrm{p}_s) = Q\beta(\sigma - \mathrm{p}_s)\frac{(k+2t)p_s - tp_m + kt(1-a)}{(k+t)t} \tag{4}$$

For the given producer's recovery price p_m, we take the first partial derivative of $\Pi_s(\mathrm{p}_s)$ with p_s, and have $\frac{\partial \Pi_s(\mathrm{p}_s)}{\partial \mathrm{p}_s} = Q\beta\frac{(k+2t)\sigma + tp_m - kt(1-a) - 2(k+2t)p_s}{(k+t)t}$. From $\frac{\partial \Pi_s(\mathrm{p}_s)}{\partial \mathrm{p}_s} = 0$, we can obtain

$$p_s = \frac{tp_m - kt(1-a)}{2(k+2t)} + \frac{\sigma}{2} \tag{5}$$

Taking the second partial derivative of $\Pi_s(\mathrm{p}_s)$, we have $\frac{\partial^2 \Pi_s(\mathrm{p}_s)}{\partial \mathrm{p}_s^2} = \frac{-2(k+2t)Q\beta}{(k+t)t} < 0$, which implies that the profit of the recycler $\Pi_s(\mathrm{p}_s)$ is a joint concave function, i.e., the optimal recovery price of recycler is unique.

Substituting (5) into (1), according to $\rho_m = \frac{p_m - p_s + t - ta}{k+t}$, we have

$$\Pi_m(\mathrm{p}_m) = Q[v - \mu + (\mu - \mathrm{p}_m)\beta\frac{2(2t+k)(\mathrm{p}_m + t - ta) - k\sigma - t(\mathrm{p}_m - k + 2\sigma + ak)}{2(2t+k)(k+t)}] \tag{6}$$

Taking the first partial derivative of producer's profit function $\Pi_m(\mathrm{p}_m)$, from $\frac{\partial \Pi_m(\mathrm{p}_m)}{\partial \mathrm{p}_m} = 0$, we can get the optimal recovery of producer is

$$p_m^* = \frac{2\mu+\sigma-3t+3ta}{4} + \frac{t(\sigma+t-ta)}{4(2k+3t)} \tag{7}$$

Taking the second partial derivative of $\Pi_m(\mathrm{p}_m)$, we have $\frac{\partial^2 \Pi_m(\mathrm{p}_m)}{\partial \mathrm{p}_m^2} = \frac{-4Q\beta}{(k+t)t} < 0$, which implies that the profit of the producer $\Pi_m(\mathrm{p}_m)$ is a joint concave function of p_m, i.e., the optimal recovery price of producer is unique.

Substituting (7) into (5), the optimal recovery price of recycler is $p_s^* = \frac{\sigma-t+ta}{2} + \frac{t(\mu+2t-2ta)}{4(\mathrm{k}+2\mathrm{t})} + \frac{t(\sigma+t-ta)}{4(2\mathrm{k}+3\mathrm{t})}$. The optimal market coverage of produce and recycler is $\rho_m^* = \frac{\mu-\sigma+t-ta}{4(\mathrm{k}+\mathrm{t})} + \frac{\mu+2t-2ta}{4(\mathrm{k}+2\mathrm{t})}$ and $\rho_s^* = \frac{\sigma+t-ta}{2t} - \frac{\mu-\sigma+t-ta}{4(\mathrm{k}+\mathrm{t})} + \frac{\sigma+t-ta}{4(2\mathrm{k}+3\mathrm{t})}$ respectively. Substituting them into the objective function, we can obtain the profits in proposition 2.

D. Full coverage with competition

If the recovery price of producer and recycler is high, the two sides have direct competition, what's more, market recovery coverage of 1, at this time, $\theta_3 \le 0$ and $\theta_1 \ge \theta_2$, i.e., the WEEE market is fully covered. Furthermore, $\rho_s = \frac{p_s - p_m + k + ta}{k+t}$ and $\rho_m = \frac{p_m - p_s + t - ta}{k+t}$.

Proposition 3: When the market coverage is full covered, the unique optimal recovery prices, market coverage and optimal profits for the producer and the recycler are shown in Table 4.

TABLE IV. OPTIMAL DECISIONS UNDER FULL COVERAGE

	Price	***Market coverage***	***Profit***
Producer	$p_m^* = \frac{\mu+\sigma-k-2t+ta}{2}$	$\rho_m^* = \frac{k+\mu-\sigma+2t-ta}{4(k+t)}$	$\Pi_m^* = Q(\mathrm{v}-\mu) + \frac{Q\beta(\mathrm{K}+\mu-\sigma+2\mathrm{t}-\mathrm{ta})^2}{8(\mathrm{k}+\mathrm{t})}$
Recycler	$p_s^* = \frac{\mu+3\sigma-3k-2t-ta}{4}$	$\rho_s^* = \frac{3k-\mu+\sigma+2t+ta}{4(k+t)}$	$\Pi_s^* = \frac{Q\beta(3\mathrm{k}-\mu+\sigma+2\mathrm{t}+\mathrm{ta})^2}{16(\mathrm{k}+\mathrm{t})}$

Proof: Similarly, using the backward sequential decision making approach to analyze the problem. Substituting $\rho_s = \frac{p_s - p_m + k + ta}{k+t}$ into (2), we have

$$\Pi_s(\mathrm{p}_s) = Q\beta(\sigma - \mathrm{p}_s)\frac{p_s - p_m + k + ta}{k+t} \tag{8}$$

For the given recovery price of producer p_m, take the first partial derivative of $\Pi_s(\mathrm{p}_s)$ with p_s, and we can obtain $\frac{\partial \Pi_s(\mathrm{p}_s)}{\partial \mathrm{p}_s} = Q\beta \frac{p_m - 2p_s - k - ta + \sigma}{(k+t)t}$. From $\frac{\partial \Pi_s(\mathrm{p}_s)}{\partial \mathrm{p}_s} = 0$, we can obtain

$$p_s = \frac{p_m - k - ta + \sigma}{2} \tag{9}$$

Taking the second partial derivative of $\Pi_s(\mathrm{p}_s)$, we have $\frac{\partial^2 \Pi_s(\mathrm{p}_s)}{\partial \mathrm{p}_s^2} = \frac{-2Q\beta}{(k+t)t} < 0$, which implies the profit of the recycler $\Pi_s(\mathrm{p}_s)$ is a joint concave function of p_s, i.e., the optimal recovery price of recycler is unique.

Substituting (9) into (1), according to $\rho_m = \frac{p_m - p_s + t - ta}{k+t}$, we can obtain

$$\Pi_m(\mathrm{p}_m) = Q[v - \mu + (\mu - \mathrm{p}_m)\beta \frac{p_m + k - \sigma + 2t - ta}{2(k+t)}] \tag{10}$$

Taking the first partial derivative of $\Pi_m(\mathrm{p}_m)$ with p_m, from $\frac{\partial \Pi_m(\mathrm{p}_m)}{\partial \mathrm{p}_m} = 0$, we can obtain the optimal recovery price of manufacturer is

$$p_m^* = \frac{\mu+\sigma-k-2t+ta}{2} \tag{11}$$

Taking the second partial derivative of $\Pi_m(\mathrm{p}_m)$, we have $\frac{\partial^2 \Pi_m(\mathrm{p}_m)}{\partial \mathrm{p}_m^2} = -\frac{Q\beta}{k+t} < 0$, which implies that the profit of the producer $\Pi_m(\mathrm{p}_m)$ is a joint concave function of p_m, i.e., the optimal recovery price of producer is unique.

Substituting (11) into (9), we can get the optimal recovery price of recycler is $p_s^* = \frac{\mu+3\sigma-3k-2t-ta}{4}$. Further, the optimal market coverage of produce is $\rho_m^* = \frac{k+\mu-\sigma+2t-ta}{4(k+t)}$, and the optimal market coverage of recycler is $\rho_s^* = \frac{3k-\mu+\sigma+2t+ta}{4(k+t)}$. Substituting them into the objective function, we can obtain the profits as in proposition 3.

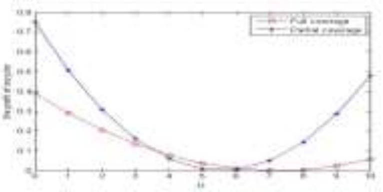
Corollary 1: $\frac{\partial p_m}{\partial \mu} > 0$, $\frac{\partial p_m}{\partial \sigma} \ge 0$, $\frac{\partial p_s}{\partial \mu} \ge 0$, $\frac{\partial p_s}{\partial \sigma} > 0$, $\frac{\partial \rho_m}{\partial \mu} > 0$, $\frac{\partial \rho_m}{\partial \sigma} \le 0$, $\frac{\partial \rho_s}{\partial \mu} \le 0$, $\frac{\partial \rho_s}{\partial \sigma} > 0$.

The recovery prices of the producer and the recycler are always negatively correlated with the unit revenue of WEEE. The market coverage of the producer is always positively related to the unit revenue and is not positively related to unit revenue of recycler. The market coverage of the recycler is always positively related to unit revenue and non-positive correlation with the manufacturer's unit revenue.

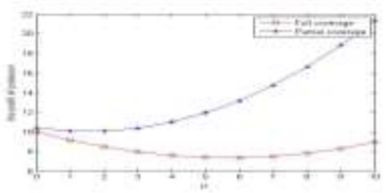
V. NUMERICAL EXAMPLES

In order to better understand the impact of parameters on the profits of producer and recycler, the following numerical simulation is conducted. Take the parameters as follows: $\mathrm{a} = 0$, $k = 1$, $k = 0.8$, $Q = 1$, $v = 10$, $\beta = 0.7$, μ=0.6, σ=0.8.

In the case of partial coverage and full coverage with competition, the effect of μ on the profit of producer and recycler are shown in Fig. 3. It can be seen from Fig. 3 (a) that profit of the recycler decreases first and then increases with the increase of μ, what's more, when $\mu \in (3.57, 6.12)$, the profit of the recycler under partial coverage is higher than that under full coverage. From Fig. 3 (b) we can conclude that the profit of the producer in full coverage is always higher than the profit of the partial coverage.



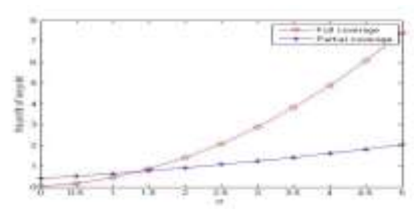
(a) Recycler



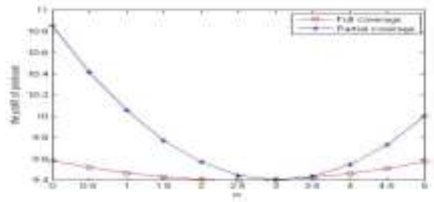
(b) Producer

Fig. 3. Effect of μ on the profits of producer and recycler

In the case of partial coverage competition and full coverage with competition, the effect of σ on the profit of producer and recycler are shown in Fig. 4. As can be seen from Fig. 4 (a), the profit of the recycler increases with σ, and the increase rate of the partial coverage is greater than that under full coverage. Further, when $\sigma < 1.34$, the profit under full coverage is higher than that under partial coverage, and when $\sigma > 1.34$, the profit under partial coverage is higher than that under full coverage. It can be seen from Fig.4 (b) that profit of the producer increases first and then decreases with σ. In the vast majority of case, the profit under full coverage is higher than that under partial coverage, but only when $\sigma \in (2.94, 3.23)$, the profit under partial coverage is higher than that under full coverage.



(a) Recycler



(b) Producer

Fig. 4. Effect of σ on the profits of producer and recycler

VI. CONCLUTION

In order to solve the problem of price decision under the competition of recycling WEEE with the competition of a producer and a third party recycler, the Hotelling competition model and Stackelberg game theory are used to determine the market coverage of producer and recycler. We identify the equilibrium solution for the recycling price of WEEE of the producer and the third party recycler in three cases of no competition, partial coverage competition and full coverage competition. A numerical simulation is also conducted, and the result indicates,

- The recovery prices of the producer and the recycler are always negatively correlated with the revenue of the unit WEEE recycling .
- The market coverage of the producer is always positively related to the unit revenue from the disposing of WEEE and is not positively related to unit revenue of recycler. The market coverage of the recycler is always related to unit revenue, and non-positive correlation with the manufacturer's unit revenue.
- The profit of the producer under full coverage is always higher than that of the partial coverage, while the profit of the recycler depends on the unit revenue of producer and recycler.

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