

steel content, which can be recovered through recycling; there is a healthy secondary market for remanufactured washing machines and tumble dryers, i.e., coin laundries and coin-op machines in apartment buildings. For such product groups, the configuration of the reverse logistics network is a complex problem. It comprises determining the optimal sites and capacities of collection centers, inspection centers, remanufacturing facilities, and/or recycling plants. There is an increasing interest in this problem and a recent review of the literature can be found in [Aras et al. \(2010\)](#).

In a reverse network, the product holders are at the upstream echelon, which we call generation points. The collection centers accumulate the returns, either dropped off by the product holders or picked up by the collectors. Municipalities, OEMs, retailers, or a combination of these organizations can be responsible for establishing and operating the collection centers ([Sander et al., 2007](#)). After collection, the returns can be sent to recycling and proper disposal, or to inspection and disassembly centers. The inspected products can be disassembled into components in these centers or sold to an external remanufacturer. The returns that are deemed non-remanufacturable through inspection are recycled or disposed. In the event that the OEM decides to establish remanufacturing facilities, then suitable components can be re-used in such facilities to obtain new products that can be sold to secondary markets. As noted earlier, extended producer responsibility required that the OEM bears the responsibility of proper reuse, recycling and disposal of its products. The dotted box in [Fig. 1](#) represents the configurational decisions that need to be made by the OEM in designing the reverse logistics network.

The prevailing studies on reverse logistics network design are driven by an application-oriented approach. Majority of the papers focus on recycling-only networks, such as [Louwers et al. \(1999\)](#) and [Realff et al. \(2004\)](#) on carpet recycling, [Schultmann et al. \(2003\)](#) on battery recycling, [Figueiredo and Mayerle \(2008\)](#) on tire recycling and [Pati et al. \(2008\)](#) on paper recycling. Notable exceptions with a remanufacturing focus are [Krikke et al. \(1999\)](#) on copiers and [Srivastava \(2008\)](#) on appliances and personal computers. Although the proposed models are realistic representations of the network design problem concerning the specific application, they are not readily generalizable to a wide range of industries. Recognizing the need for a more solid modeling framework for reverse logistics network design, we propose a new mathematical formulation that is flexible to incorporate most of the reverse network structures plausible in practice. [Fig. 1](#) displays the generic reverse logistics network that underlies our analytical model, which we validate using a realistic case study on the remanufacturing of washing machines and tumble dryers in Germany.

Our primary concern with this work is to propose a model which can be intrinsically simple but giving a strong basis for an easy extension to other settings. Moreover, this paper intends to

contribute to the reverse logistics literature by improving our understanding of how an OEM needs to react to the trends in the return streams and secondary markets by determining the extent of its involvement in reverse logistics so as to maximize its profits. Naturally, the firm would fully outsource reverse logistics if the only motivation for its involvement is the extended producer responsibility regulations.

The remainder of the paper is organized as follows. In the next section, we establish the relation between our work and the existing literature. In [Section 3](#) a new model is proposed for multi-period reverse logistics network design. We also state the underlying assumptions and highlight the flexibility of our model in representing a wide variety of possible applications. In [Section 4](#), we present a realistic case-study in order to highlight the features of the proposed model and to demonstrate that the model is amenable to solution via a commercial solver. Unlike a significant majority of the earlier papers, this new case study is presented at a level of detail that would enable the readers to re-construct the problem instances. In [Section 5](#), we conduct extensive parametric and scenario analysis to illustrate the potential benefits of using a dynamic model as opposed to its static counterpart. These computational experiments also enable us to derive a number of managerial insights concerning the co-location of inspection/disassembly centers and remanufacturing plants, and the robustness of the optimum reverse network configuration with respect to changes in important factors. The paper ends with our concluding remarks.

2. Overview of the literature

In this section we relate our work with the existing literature by discussing the features comprised by the new model we propose in [Section 3](#). In particular, we analyze the extent to which such feature have been addressed in the literature. We present a selective overview of relevant papers that have been published and not a comprehensive review of all proposed models and cases. For such a review the reader can refer to [Aras et al. \(2010\)](#).

The dynamic nature of the product returns in terms of quality and quantity over time is well established in the literature. Nonetheless, for some products, the return streams exhibit noteworthy growth potential. For example, United Nations University (UNU) asserts that “the WEEE ... is the fastest growing waste stream in the EU, producing 8.3–9.1 million tons in 2005, growing to 12.3 million tons of WEEE by 2020” ([UNU, 2008](#)). Given the steady increase in car manufacturing, a similar trend can be expected in end-of-life vehicles. For example, about 160 million cars were in use in the EU in 1995, and in 2001, that number exceeded 180 million units ([Kanari et al., 2003](#)). Almost all of the detailed reverse network design models, however, neglect the dynamic nature of the problem. One exception we are aware of is [Realff et al. \(2004\)](#). Although the authors incorporate the possible changes in the flows through the reverse network over time, the location and capacity decisions remain static in their model. [Srivastava \(2008\)](#) also takes the dynamic nature of the problem into account. However, the author does not fully integrate all the decisions to be made. In fact, two decision making phases are considered. In the first one, the location of the facilities in a single layer is decided. In the second phase, the location of the facilities is set according to the solution of the first phase and the other decisions are made. As pointed out by [Melo et al. \(2009\)](#) in the context of supply chain management, the dynamic nature of the design problem cannot often be discarded. For instance, when large investments are associated with location decisions, stability with respect to the configuration of the network is highly desirable. Moreover, in many cases, it is important to consider the possibility of making future adjustments in the network configuration to allow gradual

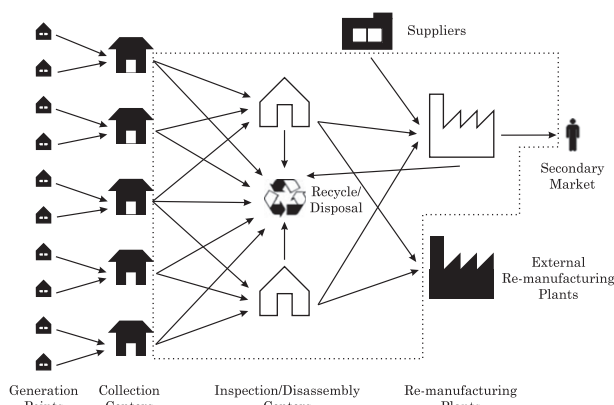


Fig. 1. A reverse logistics network.

changes in the network structure and/or in the capacities of the facilities. To fill this gap in the reverse network design literature, we present a multi-period model of the reverse network design problem.

The OEMs are constantly introducing new products in an effort to sustain/increase their market share. This contributes to the dynamic nature of the return streams, since the new products start appearing among the returned items after a certain time lag, e.g., the average lifespan of a car in use is 12–15 years. Note that the major switch in the display device production technology from CRT monitors to flat LCD panels is expected to impact the stream of returned units in the coming years. In this paper, we use a multi-commodity formulation so as to be able to incorporate these major trends in the reverse network design. We also use a reverse bill of materials in order to capture component commonality among different products and to have the flexibility to incorporate most of the plausible means an OEM can adopt in tackling product returns. By using reverse BOM, the model also addresses the possibility of sending certain components to recycling/disposal and the possibility of purchasing new components for remanufacturing. To the best of our knowledge, Jayaraman et al. (1999) is the first paper that studied a multi-product model of reverse logistics. More recently, Listes and Dekker (2005), Pati et al. (2008), Srivastava (2008), Fonseca et al. (2010), and Gomes et al. (2011) also presented multi-product formulations.

In order to focus on the dynamic nature of the problem, while keeping the mathematical model generic and tractable, we ignore uncertainties that can be important in some contexts. The reader is referred to Realff et al. (2004), Fonseca et al. (2010), Listes and Dekker (2005), and Lieckens and Vandaele (2007) for reverse network design models that incorporate stochastic factors. Our modeling framework is applicable when the OEM has fairly reliable estimates of the amount of returns to be collected during the planning horizon as well as the demand at the secondary market for remanufactured products.

Typically, an OEM is provided with modular capacity options for expanding its inspection/disassembly and remanufacturing capabilities. Different modules options can be driven for instance from different available technologies. In this case, by choosing the capacity of a facility the type of technology employed is also being chosen. The reader can refer to Fonseca et al. (2010) for further details. To the best of our knowledge, the adjustment of capacity decisions over time in a reverse logistics network design problem has been attempted only by Srivastava (2008). Discrete capacity expansions are allowed for the facilities. In the sequential modeling framework adapted by the author, such decisions are made in the second decision making phase, and thus are not taken into account when the location decisions are made. Our model determines if and where to establish the OEM's facilities and the progression of their capacity over time. The amount of returns to be disposed, recycled, and remanufactured by third parties are determined simultaneously.

Although the integration of the OEM's forward and reverse networks is out of the scope of this paper we would like to point out that much work has been done in this area. In particular, several papers can be found in the literature addressing case studies in this context. This is the case with Fleischmann et al. (2001) on paper copiers, Krikke et al. (2003) on refrigerators and Salema et al. (2009) on office document company. The dynamic nature of these network design problems is captured by Lee and Dong (2009) and Salema et al. (2010). For a deeper review of the growing literature on closed-loop supply chains we refer the reader to Aras et al. (2010).

In Table 1 a categorization of the works mentioned above is presented. The first three columns in Table 1 refer to configurational decisions. Columns 4–12 refer to nine features already discussed.

Regarding the inclusion of capacity constraints in the different problems that have been addressed in the literature, we distinguish between the existence of modular capacities (M) and the existence of several capacities from which one must be chosen (ML). The papers are sorted by the publication year which allow us to better capture possible trends. Note that this table is not intended to provide an exhaustive review but rather to illustrate the decisions and features that we have discussed above.

Observing Table 1 we can see the extent to which the model we propose in this paper goes beyond the literature. Some features such as the possibility of changing the capacities over time in order to accommodate changes in the returns, have scarcely been addressed. Moreover, the papers that address location decisions in several layers are quite limited in terms of the features comprised.

3. Mathematical formulation

In this section, we introduce a mixed-integer linear programming (MILP) model for multi-period reverse logistics network design.

The proposed model addresses many features of practical relevance namely, a multi-period setting, modular capacities, capacity expansion of the facilities, reverse bill of materials, minimum throughput at the facilities, variable operational costs, finite demands in the secondary market, and a profit-oriented objective function.

Using our model, an organization responsible for the design and operation of the reverse network can decide on the network structure, while maximizing their profit. The dotted box in Fig. 1 shows the configurational decisions that need to be made by such an organization using the proposed model. The decisions to be made include: When, where, and how many inspection centers to locate with which capacities, if it is profitable to establish remanufacturing facilities, when to invest for the capacity expansion for the facilities, the amounts of products or components to send to recycling facilities and external remanufacturing plants, and the amount of components to purchase for the remanufacturing plants.

In our multi-period setting all decisions are taken over a planning horizon which is assumed to be finite and divided into several time periods. All the network design decisions are implemented in the beginning or end of the time periods. The decisions to be made in each time period comprise the location of inspection centers and remanufacturing facilities, capacity decisions, which include the initial capacities for the new facilities and possibly the expansion of the capacity in the existing ones, inventory to be held and components to purchase at the remanufacturing plants and the network flows.

A single organization is going to manage and operate this reverse logistics network. The revenues are assumed to be obtained from other companies. The organization gains revenues from recycling when the products or components are sold to recycling companies, from external remanufacturing when the products are sold to third party remanufacturers, and from the secondary market when the remanufactured products are sold to the market. Our proposed model determines the amount of products and components to be sent to recycling, external remanufacturing, and the secondary market. The recycling, external remanufacturing and the secondary market are strategic options within the proposed model rather than being geographically located facilities.

We do not explicitly consider the disposal of the products that are not suitable for recycling within the model because the model can already accommodate such situation as recyclable products but with a negative profit. In fact, the unit revenues from recycling for such products can be set negative, accounting for the

Table 1
Reverse logistics features in addition to the location of the reverse facilities.

Article	Location decisions for reverse activities			Multiple products	Reverse BOM	Dynamic returns	Dynamic location	Capacities	Time adjustment capacities	Minimum throughput	Profit oriented	Secondary market
	Inspection disassembly	Recycling	Remanufacturing refurbishing									
Jayaraman et al. (1999)			✓	✓				C				
Krikke et al. (1999)			✓									
Louwers et al. (1999)		✓		✓								✓
Fleischmann et al. (2001)	✓		✓									
Schultmann et al. (2003)	✓				✓			C				
Krikke et al. (2003)	✓	✓	✓	✓	✓							
Realf et al. (2004)	✓	✓	✓	✓	✓	✓		C			✓	✓
Listes and Dekker (2005)	✓	✓		✓				C			✓	✓
Lieckens and Vandaele (2007)		✓	✓					ML			✓	✓
Figueiredo and Mayerle (2008)	✓											
Pati et al. (2008)	✓	✓		✓				C				
Salema et al. (2009)	✓		✓	✓		✓		C		✓	✓	
Srivastava (2008)			✓	✓				C	✓		✓	
Fonseca et al. (2010)	✓	✓		✓	✓			M				
Gomes et al. (2011)	✓	✓		✓	✓	✓		C				
The new model	✓	✓	✓	✓	✓	✓	✓	M	✓	✓	✓	✓

'C': Capacitated
'M': Modular capacities
'ML': Multi-level capacities

cost of disposal, for example, defined by the fee to be paid to send the product to an appropriate disposal facility such as a landfill. In such a case, the products that need to be disposed can be forced to be sent to recycling.

In our model, all the components are assumed to be suitable for remanufacturing. However, in practice this is not the case because some of them can be damaged and thus not re-usable. There is no need to distinguish this situation in the model as the components that are not suitable for remanufacturing can be sent to disposal and the corresponding costs (of both disposal and of buying new components to replace the damaged ones) can be included in the operational costs.

We start by presenting the notation. Afterwards, we introduce the proposed MILP formulation.

3.1. Notation

In order to propose a MILP model for the problem the following notation is introduced.

Sets

P	set of products (disposals)
C	set of components
C_p	set of components for product $p \in P$ ($C_p \subset C$)
T	set of periods in the planning horizon
I^G	set of generation points or collection centers
I^I	set of potential locations for inspection centers
I^R	set of potential locations for remanufacturing plants
R^G	recycling node for collection centers
R^I	recycling node for inspection centers
R^R	recycling node for remanufacturing plants
ER	external remanufacturing plants

SM	secondary market
Q^I	set of capacities of the modules available for inspection centers
Q^R	set of capacities of the modules available for remanufacturing plants

Available channels for the flow:

$$A = \{(i, j) : (i \in I^G \wedge j \in I^I) \text{ or } (i \in I^G \wedge j \in R^G) \cup \{(i, j) : (i \in I^I \wedge j \in I^R) \text{ or } (i \in I^I \wedge j \in R^I) \text{ or } (i \in I^I \wedge j \in ER)\} \cup \{(i, j) : (i \in I^R \wedge j \in R^R)\} \text{ or } (i \in I^R \wedge j \in SM)\}$$

General parameters

S_{ip}^t	supply of product or disposal $p \in P$ from collection center $i \in I^G$ in period $t \in T$
D_p^t	demand of the secondary market for product $p \in P$ in period $t \in T$
α_{pc}	amount of component $c \in C_p$ in one unit of product $p \in P$
$\gamma_{I_p}^I$	unit capacity consumption factor for product $p \in P$ for inspection
$\gamma_{R_p}^R$	unit capacity consumption factor for product $p \in P$ for remanufacturing
γ_c	unit inventory capacity consumption factor for component $c \in C$
MI_i^t	minimum throughput required for an inspection center located at $i \in I^I$ in period $t \in T$
MR_i^t	minimum throughput required for a remanufacturing plant located at $i \in I^R$ in period $t \in T$
KER_i^t	capacity of the external remanufacturing plant located at $i \in ER$ in period $t \in T$

KI_q	capacity of inspection of a module of type $q \in Q^I$
KP_q	production capacity of a module of type $q \in Q^R$
KH_q	inbound handling capacity of a module of type $q \in Q^R$
KIN_q	inventory holding capacity of a module of type $q \in Q^R$

Revenues

PRG_p^t	unit revenue from product $p \in P$ recycled from a collection center in period $t \in T$
PRI_c^t	unit revenue from component $c \in C$ recycled from an inspection center in period $t \in T$
PRR_c^t	unit revenue from component $c \in C$ recycled from a remanufacturing plant in period $t \in T$
PER_{ip}^t	unit revenue from product $p \in P$ sold to an external remanufacturing plant at $i \in ER$ in period $t \in T$
PSM_p^t	unit revenue from product $p \in P$ sold to the secondary market in period $t \in T$

It is assumed that the values above represent discounted values, i.e., the inflation rate has been discounted and thus that all the monetary values report to the current value on money.

Costs

FI_i^t	set-up cost for installing an inspection center at $i \in I^I$ in the beginning of period $t \in T$
FR_i^t	set-up cost for installing a remanufacturing plant at $i \in I^R$ in the beginning of period $t \in T$
FKI_{iq}^t	set-up cost for a module of type $q \in Q^I$ to be added to an inspection center located at $i \in I^I$ in period $t \in T$
FKR_{iq}^t	set-up cost for a module of type $q \in Q^R$ to be added to a remanufacturing plant located at $i \in I^R$ in period $t \in T$
OI_{ip}^t	cost for operating one unit of product $p \in P$ in an inspection center $i \in I^I$ in period $t \in T$
OR_{ip}^t	cost for producing one unit of product $p \in P$ in a remanufacturing plant $i \in I^R$ in period $t \in T$
T_{ijp}^t	unit transportation cost of product $p \in P$ (component $p \in C$) from $i \in I^G$ to $j \in I^I$, or $i \in I^I$ to $j \in I^R$ in period $t \in T$
IC_{ic}^t	unit inventory holding cost for component $c \in C$ in a remanufacturing plant $i \in I^R$ in period $t \in T$
BC_{ic}^t	cost of purchasing one unit of component $c \in C$ for remanufacturing plant $i \in I^R$ in period $t \in T$

Again, it is assumed that the cost parameters represent discounted values.

Decision variables

x_{ijp}^t	amount of product $p \in P$ (component $p \in C$) shipped from site i to site j , (ij) $\in A$, in period $t \in T$
I_{ic}^t	amount of component $c \in C$ hold in inventory in remanufacturing plant $i \in I^R$ in the end of period $t \in T$
b_{ic}^t	amount of component $c \in C$ purchased for remanufacturing plant $i \in I^R$ in the beginning of period $t \in T$

$$y_i^t = \begin{cases} 1 & \text{If an inspection center } i \in I^I \text{ is operating in period } t \in T, \\ 0 & \text{otherwise,} \end{cases}$$

$$z_i^t = \begin{cases} 1 & \text{If a remanufacturing plant } i \in I^R \text{ is operating in period } t \in T, \\ 0 & \text{otherwise.} \end{cases}$$

$$u_{iq}^t = \begin{cases} 1 & \text{If a module of type } q \in Q^I \text{ is added to an inspection center } i \in I^I, \text{ in the beginning of period } t \in T, \\ 0 & \text{otherwise,} \end{cases}$$

$$v_{iq}^t = \begin{cases} 1 & \text{If a module of type } q \in Q^R \text{ is added to a remanufacturing center } i \in I^R, \text{ in the beginning of period } t \in T, \\ 0 & \text{otherwise.} \end{cases}$$

3.2. MIP formulation

Considering the notation introduced above, the multi-product reverse logistics network design problem MPRLND can be formulated as follows:

MPRLND

$$\begin{aligned} \text{Max} \quad & \sum_{t \in T} \left[\sum_{p \in P} \sum_{i \in I^G} PRG_p^t x_{ir^c p}^t + \sum_{c \in C} \sum_{i \in I^I} PRI_c^t x_{ir^c c}^t + \sum_{c \in C} \sum_{i \in I^R} PRR_c^t x_{ir^c c}^t \right. \\ & + \sum_{p \in P} \sum_{i \in I^I} \sum_{j \in ER} PER_{ip}^t x_{ijp}^t + \sum_{p \in P} \sum_{i \in I^R} PSM_p^t x_{ismp}^t \left. \right] \\ & - \sum_{t \in T} \left[\sum_{i \in I^I} FI_i^t (y_i^t - y_i^{t-1}) + \sum_{i \in I^R} FR_i^t (z_i^t - z_i^{t-1}) \right] \\ & - \sum_{t \in T} \left[\sum_{i \in I^I} \sum_{q \in Q^I} FKI_{iq}^t u_{iq}^t + \sum_{i \in I^R} \sum_{q \in Q^R} FKR_{iq}^t v_{iq}^t \right] \\ & - \sum_{t \in T} \sum_{p \in P} \left[\sum_{i \in I^G} \sum_{j \in I^I} OI_{ip}^t x_{ijp}^t + \sum_{i \in I^R} \sum_{j \in I^R} OR_{ip}^t x_{ismp}^t \right] \\ & - \sum_{t \in T} \left[\sum_{p \in P} \sum_{i \in I^G} \sum_{j \in I^I} T_{ijp}^t x_{ijp}^t + \sum_{c \in C} \sum_{i \in I^I} \sum_{j \in I^R} T_{ijc}^t x_{ijc}^t \right] \\ & - \sum_{t \in T} \sum_{c \in C} \sum_{i \in I^R} IC_{ic}^t I_{ic}^t \\ & - \sum_{t \in T} \sum_{c \in C} \sum_{i \in I^R} BC_{ic}^t b_{ic}^t, \end{aligned} \quad (1)$$

$$\text{s.t.} \quad S_{ip}^t = x_{ir^c p}^t + \sum_{j \in I^I} x_{ijp}^t, \quad i \in I^G, p \in P, t \in T, \quad (2)$$

$$\begin{aligned} \sum_{j \in I^G} x_{ijp}^t &= \sum_{j \in ER} x_{ijp}^t + \frac{1}{\alpha_{pc}} x_{ir^c c}^t + \sum_{j \in I^R} \frac{1}{\alpha_{pc}} x_{ijc}^t, \\ i \in I^I, p \in P, c \in C_p, t \in T, \end{aligned} \quad (3)$$

$$\begin{aligned} \sum_{j \in I^I} x_{ijc}^t + I_{ic}^{t-1} + b_{ic}^t &= x_{ir^c c}^t + \alpha_{pc} x_{ismp}^t + I_{ic}^t, \\ i \in I^R, p \in P, c \in C_p, t \in T, \end{aligned} \quad (4)$$

$$\sum_{i \in I^R} x_{ismp}^t \leq D_p^t, \quad p \in P, t \in T, \quad (5)$$

$$\sum_{i \in I^I} \sum_{p \in P} x_{ijp}^t \leq KER_j^t, \quad j \in ER, t \in T, \quad (6)$$

$$\sum_{j \in I^G} \sum_{p \in P} \gamma_{lp} x_{ijp}^t \leq \sum_{\tau=1}^t \sum_{q \in Q^I} KI_q u_{iq}^{\tau}, \quad i \in I^I, t \in T, \quad (7)$$

$$\sum_{p \in P} \gamma_{Rp} x_{ismp}^t \leq \sum_{\tau=1}^t \sum_{q \in Q^R} KP_q v_{iq}^{\tau}, \quad i \in I^R, t \in T, \quad (8)$$

$$\sum_{j \in I^I} \sum_{c \in C} x_{ijc}^t \leq \sum_{\tau=1}^t \sum_{q \in Q^R} KH_q v_{iq}^{\tau}, \quad i \in I^R, t \in T, \quad (9)$$

$$\sum_{c \in C} \gamma_c I_{ic}^t \leq \sum_{\tau=1}^t \sum_{q \in Q^R} KIN_q v_{iq}^{\tau}, \quad i \in I^R, t \in T, \quad (10)$$

$$\sum_{q \in Q^I} u_{iq}^t \leq y_i^t, \quad i \in I^I, t \in T, \quad (11)$$

$$\sum_{q \in Q^R} v_{iq}^t \leq z_i^t, \quad i \in I^R, t \in T, \quad (12)$$

$$\sum_{j \in I^G} \sum_{p \in P} x_{ijp}^t \geq MI_i^t y_i^t, \quad i \in I^I, t \in T, \quad (13)$$

$$\sum_{p \in P} x_{iSMp}^t \geq MR_i^t z_i^t, \quad i \in I^R, t \in T, \quad (14)$$

$$y_i^t \leq y_i^{t+1}, \quad i \in I^I, t \in T \setminus \{T\}, \quad (15)$$

$$z_i^t \leq z_i^{t+1}, \quad i \in I^R, t \in T \setminus \{T\}, \quad (16)$$

$$x_{ijp}^t \geq 0, \quad (i,j) \in A, p \in P, C, t \in T, \quad (17)$$

$$I_{ic}^t \geq 0, \quad i \in I^R, c \in C, t \in T, \quad (18)$$

$$b_{ic}^t \geq 0, \quad i \in I^R, c \in C, t \in T, \quad (19)$$

$$y_i^t \in \{0, 1\}, \quad i \in I^I, t \in T, \quad (20)$$

$$z_i^t \in \{0, 1\}, \quad i \in I^R, t \in T, \quad (21)$$

$$u_{iq}^t \in \{0, 1\}, \quad i \in I^I, q \in Q^I, t \in T, \quad (22)$$

$$v_{iq}^t \in \{0, 1\}, \quad i \in I^R, q \in Q^R, t \in T. \quad (23)$$

Objective function (1) maximizes the profit. We initially sum the revenues from the recycling centers, from the external remanufacturing plants, from the secondary market, and then subtract the costs. The costs are the fixed costs of establishing facilities and capacity modules, operational costs, transportation costs, inventory holding costs, and component purchasing costs.

Constraints (2)–(4) are flow balance constraints. By Constraint (2), products that are collected from the collection centers can be sent to recycling centers or inspection centers which are to be located. At the inspection centers, the inspected products can directly be sent to external remanufacturing facilities or can be disassembled into components. These components can then be recycled or sent to remanufacturing plants via Constraint (3). Constraint (4) is the flow balance constraint for remanufacturing plants. The total inflow, which is composed of components coming from inspection centers, components purchased from suppliers, and components in the inventory, must be equal to the outflow, which is composed of components sent to recycling, products sold to secondary markets, and the components to be held in inventory.

Constraint (5) ensures that the amount of products sold to the secondary market is no more than the demand of each product at each time period.

Constraints (6)–(10) are capacity constraints. Constraint (6) ensures that the amount of products that are sent to external remanufacturing plants do not exceed the capacity of the plants. Constraint (7) is the capacity constraint for operation in the inspection centers and Constraint (8) is the capacity constraint for production in the remanufacturing plants. By Constraint (9), the inbound handling at the remanufacturing plants cannot exceed the capacity, and by Constraint (10) inventory to be held in the remanufacturing plants cannot exceed the inventory holding capacity.

Constraints (11) and (12) assure that not only an expansion can only occur in facilities that have been installed but also that in each expansion at most one module can be chosen for each period.

Constraints (13) and (14) are minimum throughput constraints guaranteeing that an inspection center or a remanufacturing plant can only be established if the operation or production amount exceeds the predefined limits.

Constraints (15) and (16) assure that once a facility is installed it remains operating until the end of the planning horizon.

Lastly, Constraints (17)–(23) are domain constraints.

Model MPRLND is generic in the sense that it includes various possible options present in reverse logistics networks such as inspection, disassembly, recycling, disposal, outsourcing (external remanufacturing), and remanufacturing. Thus, the model is readily applicable for various industries. In the next section, we present an application of the model on a case study considering large household appliances within WEEE.

4. A case study

We applied model MPRLND on a case study inspired by a real-life problem in Germany in the context of reverse logistics network design for washing machines and tumble dryers. We do not solve any specific company's problem. However, all parameters are chosen in a realistic order of magnitude. Our goal is to highlight the features of the model proposed in the previous section and also to show that it can be solved to optimality using a commercial solver for instances of a realistic size.

We start by thoroughly describing the problem and afterwards we present and comment on the results obtained.

4.1. Introduction of the problem

In our study, we assume that washing machines and tumble dryers are to be collected from 40 collection centers located by the municipalities in the 40 most populated cities within Germany. The locations of the collection centers on the map of Germany are depicted in Appendix.

In order to estimate the amount of washing machines and tumble dryers generated at the collection centers, the available data on WEEE is used. According to the information given by UNU (2008), in old members of the European Union, 14–24 kilogram/head/year of WEEE is generated. We took the mean value of 19 kilogram/head/year to estimate the total amount of WEEE generated at the collection centers in the beginning of the planning horizon. We multiplied this number by the population of the cities where each collection center is located to estimate the total amount of WEEE generated at the collection centers in the beginning of the planning horizon.

Washing machines and tumble dryers are both categorized as 'large household appliances' within the WEEE Directive. The percentage of large household appliances within WEEE is estimated by 27.7% by the European Statistics Institute (2010). The percentages of washing machines and tumble dryers within large household appliances are estimated as 30% and 10%, respectively by using the data presented in Basdere and Seliger (2003). These values together with the total amount of WEEE generated approximate the total weight of washing machines and tumble dryers generated at each of the collection centers. In order to convert weight to the total numbers of washing machines and tumble dryers generated, the average weight of a washing machine is estimated as 60 kg and of a tumble dryer as 45 kg. 20% of this total numbers of products are taken to estimate the total number of washing machines and tumble dryers to be collected by a specific company.

Initially, a 5-year planning horizon was considered although, as it will be reported in Section 5.3, in order to put some stress on the model, we also performed some tests with larger planning horizons. In order to estimate the generated amounts in each period, yearly growth rate of EEE was used which is approximated as 2.6% in UNU (2008). For the base case, we used 2.6% of annual increase; however, we also did some tests with alternative growth scenarios, presented in Section 5.5.

The components of a washing machine and their economic benefits are presented in Park et al. (2006). We took the four most profitable components present in a washing machine: frame (steel), motor, ABS parts, and washing tube. All of these components other than the washing tube are also present in a tumble dryer. Thus, frame, motor, and ABS parts are components common to both appliances. Tumble dryers, on the other hand, contain a different valuable component called air blower. As a result, each of the appliances contains four components, three of which are common to both, that are suitable for remanufacturing and for selling

to recycling companies within the model. Apart from the ABS parts, there is exactly one component of each type in the appliances. On the other hand, washing machines and tumble dryers contain two ABS parts.

The revenues from selling these components to recycling companies are estimated by using the values presented in Park et al. (2006). Revenues from collection centers, external remanufacturing facilities, and the secondary market are estimated by considering the revenues of components. Unit revenues for each product and component in the beginning of the planning horizon are presented in Appendix. In Section 5.4, we present some sensitivity analysis with the unit revenues.

It is assumed that there is a single external manufacturer with unlimited capacity and the demand of the secondary market is unlimited for this case study.

All of the 40 cities, where the collection centers are located, are taken as potential location sites for both inspection centers and remanufacturing facilities. Since land prices are higher in the populated cities, a parameter is generated in order to reflect the differences on fixed costs depending on the land prices at the candidate locations. For each candidate location, this parameter is equal to the ratio of the population of the city to the total population multiplied by 100.

A distance matrix is generated between the 40 cities using the highway transportation network within the country. The transportation cost between collection centers and potential inspection centers is taken as 0.005 EUR/kilometer-product, and between inspection centers and potential remanufacturing plants as 0.003 EUR/kilometer-component, which are the values used in the copier remanufacturing case study by Fleischmann et al. (2001). We adapted these values because it is assumed that the transportation costs for transporting washing machines and tumble dryers would be similar for transporting copier machines.

We took two different capacity modules (low and high) for each of the facilities. In the basic setting, we considered that modules with capacities 25,000 and 50,000 units are available for inspection centers and modules with capacities 75,000 and 150,000 units are available for the remanufacturing plants. As we will report in Section 5.2, we also performed some tests with different set-up costs and capacities.

The cost parameters used in the case study for the basic setting in the beginning of the planning horizon are summarized in Appendix.

All the cost values are assumed to increase by the yearly inflation rate (which is reported equal to 0.02% for Germany in 2009 by the European Statistics Institute (2010)), over the planning horizon. The unit revenues, on the other hand, are assumed to increase yearly by 1%.

In the increase we are considering in the costs, we assume that the inflation rate has been discounted and thus that all the monetary values report to the current value of money.

Finally, the unit capacity consumption factors are taken as one, and the minimum throughput requirement to be negligible.

4.2. Computational results

The setting described above resulted in a problem with 1200 binary variables, 58,000 continuous variables, and 5520 constraints.

We took all of our runs on a server with 2.66 GHz Intel Xeon processor and 8 GB of RAM and we used the optimization software CPLEX version 11.2. All the runs are solved to optimality.

Fig. 2 presents the optimal solution of the problem with the initial setting. Fig. 2a demonstrates the resulting reverse logistics network at the end of first period, while Fig. 2b shows the resulting network at the end of the last period. This solution was obtained in approximately 1532 seconds.

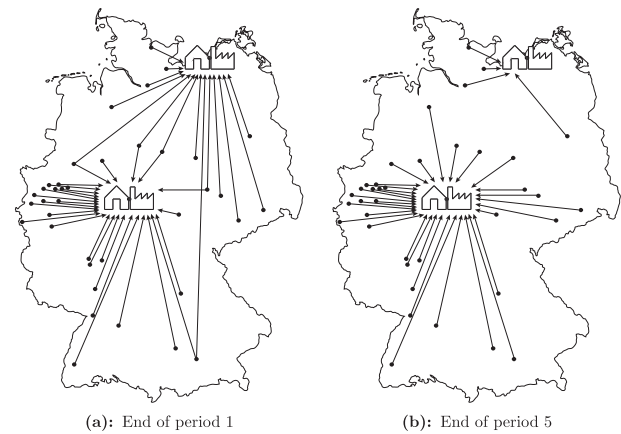


Fig. 2. A solution of the problem.

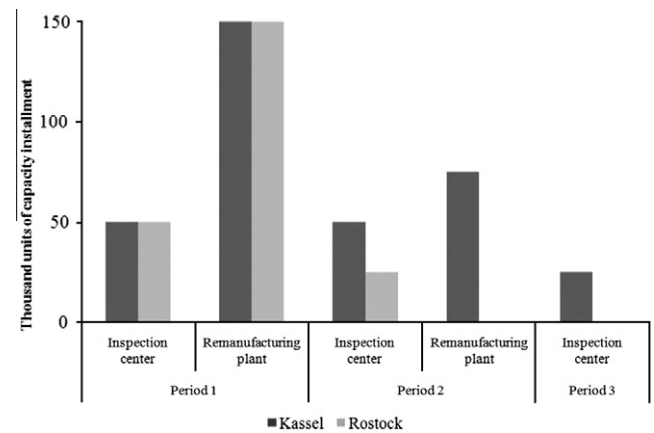


Fig. 3. Capacity installment decisions in the solution.

In the solution presented in Fig. 2, two inspection centers and two remanufacturing facilities are located each at Kassel and Rostock in the beginning of the first period. It is interesting to note that the solution takes advantage of the possibility of co-locating inspection centers and remanufacturing centers although no fixed cost incentive existed for that. This may be explained by the potential savings in the transportation costs among these facilities.

The capacity modules for the facilities are established within the first three time periods. Fig. 3 demonstrates the capacity installment decisions for all of the facilities at the end of each time period. In particular, in the first period a 'high' capacity module is established for all of the located facilities. In the second period, a 'high' capacity module is established for the inspection center located at Kassel, and a 'low' capacity module for the inspection center located at Rostock. In addition, in the second period, a 'low' capacity module is established for the remanufacturing plant located at Kassel. In the third period, only a 'low' capacity module is established for the inspection center at Kassel. The total final capacities of the inspection centers located at Kassel and Rostock are 125,000 and 75,000 units, respectively. On the other hand, the capacities of the remanufacturing plants at the end of the planning horizon are 225,000 and 150,000 units for Kassel and Rostock, respectively.

In Fig. 2a and b we can also observe the allocation of the collection centers to the located inspection centers at the end of the first and last periods. In all periods, most of the collection centers are allocated to the inspection center located in Kassel rather than Rostock. This is due to transportation costs, that is, Kassel is relatively closer to most of the collection centers compared to Rostock. In order to handle the incoming flow at Kassel, more capacity

modules are established for the inspection center and the remanufacturing plant compared to the ones established in Rostock. Although not many collection centers are allocated to Rostock, the two collection centers generating the highest amount of products, which are located in Berlin and Hamburg, accounting for generating approximately 26% of the total supply, are allocated to Rostock in all time periods. Approximately 30% of the total amount of washing machines and tumble dryers generated from the collection centers is allocated to the inspection center at Rostock at the end of the last period.

Fig. 2a shows that the flow from some collection centers, from Bielefeld, Braunschweig, Halle, Hannover, and Munich, are divided between the two inspection centers in the first period. This is due to the capacities of the facilities. At the end of the last period, in Fig. 2b, we no longer observe any multiple allocation of the collection centers to the located facilities. Because of the establishment of new capacity modules in Kassel in the beginning of the second and third periods, flow from some collection centers are redirected from Rostock to Kassel. For example, collection centers located in Bremen, Chemnitz, Dresden, and Magdeburg were allocated to the inspection center located in Rostock in the first period, whereas they are all allocated to Kassel in the last period.

The flows in each of the periods corresponding to the solution demonstrated in Fig. 2 are presented in detail in Fig. 4.

Fig. 4 shows that major flow is sent to the secondary market. This is because it is most profitable to sell remanufactured products to the secondary market compared to revenues either from recycling or external remanufacturing. Even though, secondary market is the most profitable, there is still a significant amount of flow sent to external remanufacturing in each period. Due to fixed costs of establishing capacity modules, it is more profitable to send some flow to external remanufacturers compared to establishing new capacity modules.

As it can be observed from Fig. 4, there is flow to recycling only in the first period. This flow is sent from the collection centers only, that is no flow is sent to recycling both from inspection centers and remanufacturing facilities. The flow sent to recycling is mainly due to the inefficient capacities of the facilities in the first period to handle all the flow. The constraints of the model impose that in each capacity expansion at most one module can be chosen for each period. Thus, enough capacity could not be installed in the first period to avoid recycling. By the establishment of new capacity modules in the second period, no flow is sent to recycling. This is a strong result that can be explained by the financial benefits that can be obtained from using profitable components in remanufacturing and thus it is not surprising.

5. Sensitivity analysis and insights

The computational tests performed with the initial setting presented in the previous section were extended so that we can get a

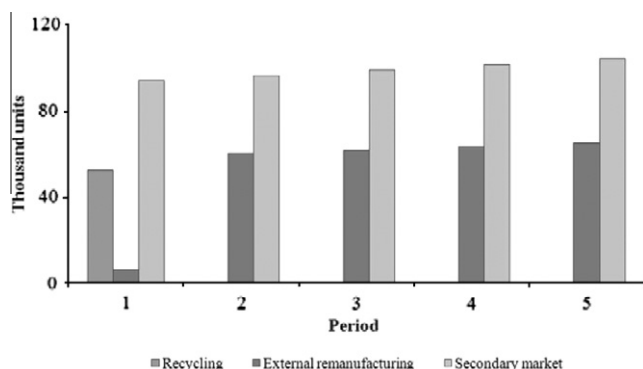


Fig. 4. Flows.

better perception of the potential and value of model MPRLND, and also to see how the changes in the problem parameters can affect the optimal solution.

5.1. Sensitivity to the amount of returns

Initially, we analyze the sensitivity of the solutions to the changes in the amount of products supplied from the collection centers. Remember that according to UNU (2008), in old members of the European Union, 14–24 kilogram/head/year of WEEE is generated. In Section 4, we use the mean value of 19 kilogram/head/year to estimate the total amount of washing machines and tumble dryers generated at the collection centers in the beginning of the planning horizon. In this section, we assume that this number is uniformly distributed between 14 and 24 at each city and generate ten different scenarios of the problem. Each of these scenarios is solved to optimality by using CPLEX on the same computer. Optimum solutions at the end of the planning horizon are summarized in Table 2.

For each scenario, Table 2 lists the locations and the total capacities of the inspection centers and remanufacturing plants at the end of the planning horizon, the CPU time requirement by CPLEX to solve the corresponding instance to optimality, and the optimum objective function value. The numbers presented in parenthesis for each location, in the second and third columns, correspond to the total capacities of the facilities at the end of the planning horizon in thousand units. The first row corresponds to the results with the initial setting discussed in the previous section.

Observe from Table 2 that in all of the scenarios, inspection centers and remanufacturing plants are located at Kassel, Mainz, or Rostock. The capacity installment decisions for the facilities are almost the same. In all of the scenarios, except two, total capacities of the inspection centers are 175,000 units at the end of the planning horizon. On the other hand, in the base case and in scenario 9, total capacities of the inspection centers are 200,000 units because of the establishment of an additional capacity module in the third period. In these two instances, it is more profitable to establish a new capacity module and send products to inspection centers instead of recycling. The total capacities of the remanufacturing plants, on the other hand, are the same in all of the resulting solutions.

Among the solutions listed in Table 2 the highest CPU time requirement is about 26 minutes. On the average, the model is solved in about 17 minutes with all of the scenarios to optimality.

5.2. Sensitivity to changes in the set-up costs and in the capacities

In this section, we look at the sensitivity of the location decisions to changes in the set-up costs and the capacities. To do so, we varied the set-up costs and the capacities of the modules and analyzed the outcomes of the model. The results of the model at the end of the planning horizon are presented in Table 3.

The first column of Table 3 lists the number of the corresponding instances. For each instance, the second and third columns list the set-up costs for inspection centers and remanufacturing plants, respectively. For each set-up cost, we tested the model with three different capacity configurations, tight, medium and loose, each capacity configuration containing two capacity modules for each of the facilities. These settings correspond to a total of 18 instances. The initial setting discussed in Section 4 is also depicted in Table 3 (instance 9).

Observe from Table 3 that the numbers of facilities to be located increase when both the set-up costs and the available capacities are lower. For example, the first instance listed in Table 3 corresponds to the highest set-up costs with loose capacity configurations,

Table 2

Solutions of the model at the end of the planning horizon with different scenarios for the amount of products generated.

	Inspection centers	Remanufacturing plants	CPU time (s)	Obj. Func.
Base case	Kassel (125), Rostock (75)	Kassel (225), Rostock (150)	1531.66	51,041,892.24
Scen. 1	Mainz (100), Rostock (75)	Mainz (225), Rostock (150)	920.50	51,509,241.55
Scen. 2	Mainz (100), Rostock (75)	Mainz (225), Rostock (150)	605.11	53,246,272.18
Scen. 3	Kassel (100), Mainz (75)	Kassel (225), Mainz (150)	1102.71	49,858,989.38
Scen. 4	Mainz (100), Rostock (75)	Mainz (225), Rostock (150)	1108.32	52,647,963.95
Scen. 5	Mainz (100), Rostock (75)	Mainz (225), Rostock (150)	603.63	53,269,410.71
Scen. 6	Mainz (100), Rostock (75)	Mainz (225), Rostock (150)	756.87	52,726,199.14
Scen. 7	Mainz (100), Rostock (75)	Mainz (225), Rostock (150)	1054.72	52,508,169.24
Scen. 8	Kassel (100), Rostock (75)	Kassel (225), Rostock (150)	1568.31	52,535,074.96
Scen. 9	Mainz (100), Rostock (100)	Mainz (225), Rostock (150)	797.34	52,456,893.12
Scen. 10	Mainz (100), Rostock (75)	Mainz (225), Rostock (150)	1243.49	53,712,708.51

Table 3

Solutions of the model at the end of the planning horizon with varying set-up costs and capacity modules.

Instance number	Set-up costs (thousand EUR)		Capacity modules for inspection (thousand units)		Capacity modules for remanu. (thousand units)		Inspection center locations	Remanufacturing plant locations	CPU time (s)
	Ins.	Remanu.	High	Low	High	Low			
1	600	1500	100	50	300	150	Kassel	Kassel	2060.16
2	500	1250	100	50	300	150	Kassel	Kassel	2541.28
3	400	1000	100	50	300	150	Kassel	Kassel	2057.63
4	300	750	100	50	300	150	Kassel	Kassel	2377.80
5	200	500	100	50	300	150	Kassel, Magdeburg	Kassel	4312.60
6	100	250	100	50	300	150	Augsburg, Magdeburg, Oberhausen	Oberhausen	18732.19
7	600	1500	50	25	150	75	Kassel, Rostock	Kassel, Rostock	1523.85
8	500	1250	50	25	150	75	Kassel, Rostock	Kassel, Rostock	2031.89
9	400	1000	50	25	150	75	Kassel, Rostock	Kassel, Rostock	1531.66
10	300	750	50	25	150	75	Mainz, Rostock	Mainz, Rostock	1587.88
11	200	500	50	25	150	75	Magdeburg, Mainz, Oberhausen	Magdeburg, Mainz	1511.87
12	100	250	50	25	150	75	Magdeburg, Mainz, Oberhausen	Magdeburg, Mainz	2237.39
13	600	1500	40	30	100	80	Kassel, Mainz, Rostock	Kassel, Rostock	6604.73
14	500	1250	40	30	100	80	Mainz, Oberhausen, Rostock	Mainz, Rostock	4123.16
15	400	1000	40	30	100	80	Kassel, Mainz, Rostock	Kassel, Mainz, Rostock	6545.02
16	300	750	40	30	100	80	Kassel, Mainz, Rostock	Kassel, Mainz, Rostock	4379.17
17	200	500	40	30	100	80	Magdeburg, Mainz, Oberhausen	Magdeburg, Mainz, Oberhausen	6137.25
18	100	250	40	30	100	80	Lübeck, Magdeburg, Mainz, Oberhausen	Lübeck, Mainz, Oberhausen	5843.05

where only a single inspection center and a remanufacturing plant is located both in Kassel. In the last instance listed in Table 3, on the other hand, with lower set-up costs and tighter capacities, four inspection centers and three remanufacturing plants are located.

In all of the solutions presented in Table 3, remanufacturing plants are co-located with inspection centers. As mentioned before, this can be explained by the potential savings from transportation costs among these facilities.

In general, the model is solved in reasonable CPU time to optimality by CPLEX. The minimum CPU time requirement for the model is about 25 minutes, whereas the maximum is about 5.2 hours. Observe from Table 3 that the last set of instances with tight capacity configuration (instances 13–18) are relatively harder in terms of CPU time requirements.

5.3. The value of the multi-period model and the length of the planning horizon

One important aspect to be analyzed concerns the potential benefits for the company of using the proposed 5-year model, rather than utilizing a static model. Our aim is to find whether the company will achieve additional profits by using the proposed multi-period model.

For this analysis, we compared the profit performance of a single-period model with that of the multi-period model over the 5-year profit horizon on the 18 instances listed in Table 3.

Initially, we solved a single-period model using the average values for the parameters over the entire planning horizon. Then, we fixed the locations and capacities throughout the planning horizon using the solution of the single-period model and calculated the profit that the firm can generate over the five years without altering this network configuration. The results are presented in Table 4 and Fig. 5.

The first column in Table 4 lists the instance numbers corresponding to the instances depicted in Table 3. The second column presents the optimal objective function value of the 5-period model proposed in this study. The third column, on the other hand, presents the total profits that the company will gain if the network configuration of the 1-period solution using the average values for the parameters over the entire planning horizon is fixed for the 5-year planning horizon. Last column lists the percent differences between the total profits.

In most of the solutions presented in Table 4, the total gain in the profits by using the 5-period model are around 2%. Even though this percentage may seem as low, the potential significance is high since overall profits are high. If the total profits are taken approximately as 50 million Euros, 2% corresponds to 1 million Euros.

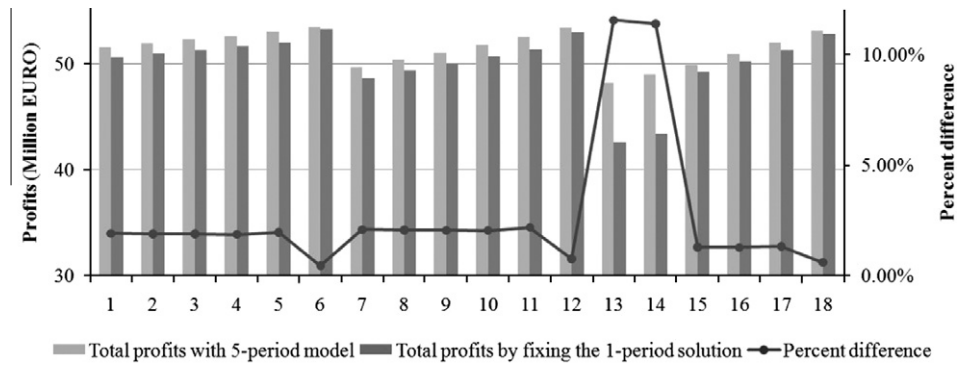


Fig. 5. The value of the multi-period model over the 5-year planning horizon.

The highest percent differences are obtained at instances 13 and 14 as 11.56% and 11.40%, respectively. These percentages correspond to a gain in the profits of about 5.6 million Euros. In both of these instances, the total number of facilities located in the solution of the single-period problem is less than the total number of facilities located in the 5-period solution. This is because the fixed set-up costs are at their highest values. With less number of facilities, since both of these instances correspond to the scenarios with the tightest capacity configurations and there is no possibility of capacity expansion in the single-period solution, the total capacities of the facilities in the single-period solution is insufficient to gain as much profit as the facilities in the 5-period solution. Nevertheless, these instances show that there can be huge gains from using the multi-period model.

The analysis presented in Table 4 demonstrates the amount of benefits a company can make by having a dynamic/long-term perspective in designing its reverse logistics network, hence, the value of the multi-period model proposed in this paper.

The analysis we perform for 5 years can, naturally, be extended to a larger planning horizon. Note, also, that if at some point in the future one realizes that some previous forecasts became totally unrealistic, it is very easy to redesign the optimal plan for the remainder of the planning horizon: one just needs to consider our model and fix the decisions previously made and already implemented. The possibility of future adjustments in the optimal solution is an important feature in any multi-period model and ours is not an exception.

After having quantified the benefits of using a multi-period model namely, the 5-year planning model proposed, we analyzed the solution potential of the model with different lengths for the planning horizon. We considered the base setting presented in Section 4.2 and varied the number of years from 1 to 7.

As expected, the CPU time requirement for the model grows exponentially with the total number of periods. The model is solved to optimality within less than two seconds for a single-period problem, whereas it could be solved in less than four hours for a problem with seven periods.

The additional computational effort of using a 5-year model instead of a single-period model is negligible. However, by using a 5-year model, not only does the company get a more comprehensive and realistic model but also, as shown above, the company may get a clear financial benefit. The extra computational effort for considering larger planning horizons is significant and in this case a trade-off may have to be considered between the comprehensiveness of the model and the effort necessary to solve it. In addition, given the amount of assumptions that may have to be taken regarding the problem data, the computational effort associated with the multi-period model may be reduced by allowing a gap in solving the problem instance.

5.4. Sensitivity to changes in the revenues

In this section, we study the sensitivity of the solutions to the changes in the unit revenues. For this analysis, we varied all the unit revenues by 10% and 20% and test the model for the base setting studied in Section 4.2.

When we increase and decrease unit revenues by 10% the optimal solution does not change. That is, the optimal locations of the inspection centers and remanufacturing plants, allocation and routing strategies, capacity expansions decisions are exactly the same as the solution presented in Section 4.2. The only difference is that total revenue in the objective function increases or decreases in accordance with the change in the unit revenues.

When we increase revenues by 20% again there is no change in the optimal solution. However, when we decrease revenues by 20% we observe some minor changes in the optimal solution. The location and allocation decisions are the same as presented in Fig. 2. However, there are some changes in the capacity installment decisions and flows. Fig. 6 shows the capacity installment decisions and flows in the resulting solution when the revenues are decreased by 20%.

Observe from Figs. 3 and 6a that there is no longer capacity expansion for the inspection facility located at Rostock in the beginning of the second period when the revenues are decreased by 20%. As shown in Fig. 6b, some products are sent to recycling in the second period. These products are sent to recycling directly

Table 4
The value of the multi-period model over the 5-year planning horizon.

Instance number	Total profits by using the 5-period model (EUR)	Total profits by fixing the 1-period solution (EUR)	Percent difference
1	51,584,835	50,600,344	1.91
2	51,923,433	50,938,942	1.90
3	52,262,032	51,277,540	1.88
4	52,600,630	51,616,138	1.87
5	52,992,312	51,954,737	1.96
6	53,479,368	53,245,925	0.44
7	49,663,337	48,628,470	2.08
8	50,352,614	49,311,692	2.07
9	51,041,892	49,994,913	2.05
10	51,735,096	50,678,134	2.04
11	52,507,776	51,361,356	2.18
12	53,362,789	52,957,426	0.76
13	48,184,468	42,613,639	11.56
14	48,982,028	43,398,169	11.40
15	49,856,320	49,208,082	1.30
16	50,890,221	50,241,983	1.27
17	51,963,557	51,275,884	1.32
18	53,122,824	52,802,854	0.60

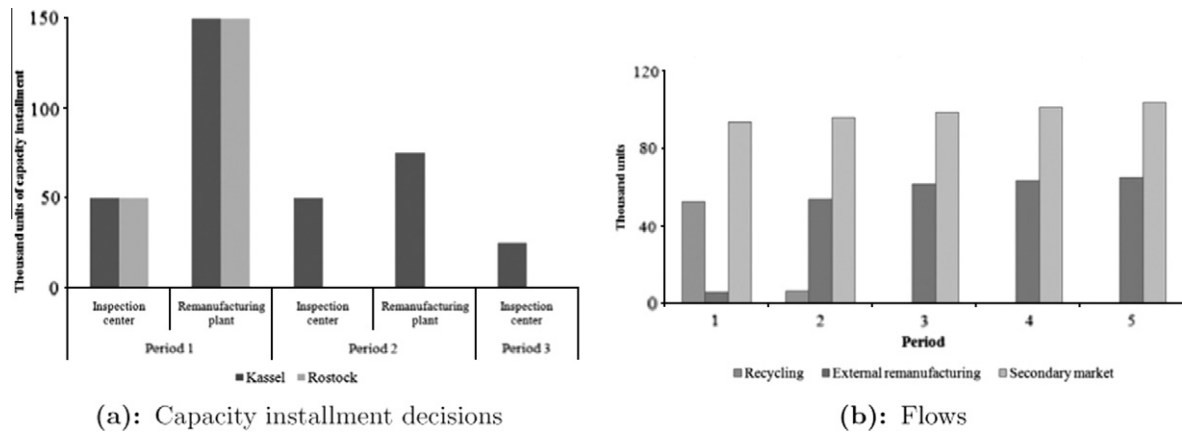


Fig. 6. Resulting solution when the revenues are decreased by 20%.

from the collection centers. Thus, the capacity expansion for inspection is no longer required when more flow is sent to recycling prior to inspection.

This sensitivity analysis with the revenues shows that the optimal solution of the model is robust to some changes in the unit revenues.

5.5. Alternative growth scenarios

We also wanted to analyze the changes in the optimal solution with alternative growth scenarios for the amounts of products generated. Recall from Section 4.1 that in order to estimate the generated amounts of products in each period, we used the yearly growth rate of EEE, which is approximated as 2.6%. We generated a total of four alternative growth scenarios (two optimistic and two pessimistic) in addition to the base case with 2.6% annual increase. In both of the optimistic scenarios (Optimistic-1 and Optimistic-2), the amounts of generated products increases exponentially, where the increase in Optimistic-2 scenario is higher compared to the increase in Optimistic-1 scenario. Similarly, the decreases on the amounts of products generated are exponential in the two pessimistic scenarios, with the decrease being higher in the Pessimistic-2 scenario. In addition, with Pessimistic-2 scenario the amount of products generated starts to decline at the end of the third period.

In all of the scenarios, except with the second pessimistic scenario (Pessimistic-2), the optimal solutions match with the optimal solution of the base case. The locations of the inspection centers and remanufacturing facilities, the allocation and routing decisions, and capacity expansion decisions all match with the results presented in Section 4.2. The only difference is in the amounts of flows and hence the corresponding costs and profits do change.

With scenario Pessimistic-2 we obtain a different optimal solution. The major difference is that the inspection centers and remanufacturing facilities are located at Mainz and Rostock instead of Kassel and Rostock. In addition, no capacity expansion is required for the inspection facilities in the third period and also no capacity expansion is required for the remanufacturing facilities in the second period. This is intuitive since the amount of products generated starts to decline at the end of the third period. Other than that, the recycling and external remanufacturing decisions are the same.

Similar to the results with varying revenues, sensitivity analysis with alternative growth scenarios for the products also demonstrates that the optimal solution is robust to some changes in the problem data.

6. Conclusions

In this paper, we proposed a mathematical programming framework for multi-period reverse logistics network design problems. The paper also contributes to the reverse logistics literature by improving our understanding of how an OEM needs to react to the trends in the return streams and secondary markets by determining the extent of its involvement in reverse logistics so as to maximize its profits.

The proposed model accommodates several features of practical relevance namely, a multi-period setting, modular capacities, capacity expansion of the facilities, reverse bill of materials, minimum throughput at the facilities, variable operational costs, finite demands in the secondary market, and a profit-oriented objective function. The decisions to be made regard the location of the inspection centers and remanufacturing facilities, capacity of the new facilities, capacity expansion of the existing facilities, flow routing through the network, the amount of inventory to hold and the amount of components to purchase from the suppliers in the remanufacturing plants.

A case study considering large household appliances within WEEE was presented and exhaustively analyzed. The results show that utilizing the proposed model, instances with realistic sizes can be solved to optimality by using a commercial solver.

The case study provided important insights on how a reverse logistics network may evolve over time. The results demonstrated that there can be gains in the profit by using a multi-period model compared to using a static one. The study also gave some essential insights such as inspection centers and remanufacturing plants can be co-located due to potential savings in the transportation costs between facilities. Extensive parametric and scenario analysis showed that the optimal solutions are robust to some changes in the problem data.

In this work, for the first time, a measure was proposed for the value of the multi-period model. The new measure was applied to the case study addressed.

A natural extension to the setting considered in this paper regards the inclusion of uncertainty issues (e.g. in the returns). This is a relevant aspect in many practical reverse logistics planning problems. However, the inclusion of such aspects in the multi-period modeling framework proposed is far from straightforward as we are addressing a multi-stage decision making problem with interrelated decisions. Nevertheless, this is certainly a promising and challenging research direction for the future. The inclusion of uncertainty in the problem also calls for the inclusion of risk pooling as this is a feature that under an uncertainty setting may strongly influence the network design.

