Importance of Path Planning Variability:
A Simulation Study

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Abstract

Individuals vary in the way they navigate through space. Some use a mental map, while others rely on known routes to find their way around. We wondered how and why there is so much variation in the population. To address this, we used a variety of path planning algorithms from robotics to simulate the trajectories of 368 human subjects. We found that there is variation on the type of knowledge individuals apply to navigate space, as well as variation within individuals on a trial-by-trial basis. Furthermore, subjects occasionally switched from a route strategy (i.e., follow the route they learned) to a survey strategy (i.e., taking a novel shortcut) halfway through a trajectory. In a second set of simulations, we varied the relative cost of choosing a learned route over alternative paths. When the learned route was relatively costly, the simulated agents tended to take shortcuts. Conversely, when the learned route was less costly, the simulated agents showed preference toward a route strategy. These simulations suggest that variation within and between individuals is the norm. Such variation may be beneficial for robotic swarms or collections of autonomous agents during information gathering.

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1. Introduction

Navigation strategies vary within and between organisms. It is thought that organisms ranging from honeybees to rats to humans utilize mental maps, beacons, and turn sequences to find their way around (Gallistel, 1990, 2017). People differ considerably in their ability to learn and navigate environments (Hegarty et al., 2006; Hegarty & Waller, 2005; Ishikawa & Montello, 2006; Weisberg & Newcombe, 2016, 2018). Some rely more on survey knowledge where a mental representation of the physical environment is represented in an allocentric format, that is a world centered view that allows inferences about distances and directions between locations. Others rely more on an egocentric format where they use a series of movements to follow a set path, or route-based navigation. Using survey knowledge allows more flexible path planning. Through this mentalization, one can take shortcuts or plan paths through areas in the environment that have not previously been experienced. In contrast, familiar routes can be followed out of habit and may require less mental effort.

People use different types of knowledge to navigate (Chrastil & Warren, 2015). *Survey knowledge*, which is a cognitive map that includes distances and directions between locations. Survey knowledge enables flexible path planning resulting in shortcuts or planned trajectories over never experienced paths. *Route knowledge* consists of sequences of actions associated with places or decision points. Typically, the routes are set paths, familiar, and inflexible. Chrastil and Warren suggested that people may not need to acquire a globally consistent survey knowledge but a third type of knowledge, namely labeled graph knowledge (Chrastil & Warren, 2015). *Graph knowledge* consists of a network of nodes linked by edges, where the nodes are locations in the environment and the edges are paths between those nodes. The nodes can be landmarks or decision points along a path. The edges can contain rough metric information or be realized through non-metric topologies, but the metric information does not need to be globally consistent as a whole mental map.

Boone and colleagues showed that there are differences in the human population on which knowledge people tend to use during navigation (Boone et al., 2018; Boone et al., 2019; Hegarty et al., 2020, November). Men tended to demonstrate survey knowledge by taking more shortcuts than women, and women tended to use route knowledge by taking a learned route or at times taking the route in the reverse direction that it was learned (Boone et al., 2018). Although both females and males demonstrated route knowledge when told to find a location, they did have the capacity to use survey knowledge when instructed to take the shortest path (Boone et al., 2019). This suggests that many participants have a mental map, but they might find it easier to take a learned route. It is an open question why this might be the case.

Rather than suggesting that some people are better at finding their way around than others, it might be the case that this population variation has benefits. For example, it has been shown that people and even birds are more adept at problem solving in groups than individually (Laughlin et al., 2006; Liker & Bokony, 2009). The different skills or abilities brought by each individual can be complementary. Certainly, different organisms use different strategies related to their environmental niche. What about variation within organisms? Could this provide an advantage in overall navigation at the population level?

Understanding human variation may have advantages for robot navigation and possibly route planning for self-driving vehicles. In standard robotics navigation, algorithms are designed to find an optimal solution for navigational problems and use that repeatedly without variation. For instance, Simultaneous Localization and Mapping (SLAM) algorithms map novel environments while exploring them by keeping track of the robot's movements (Kohlbrecher et al., 2011; Mur-Artal et al., 2015). These maps are then used to plan paths or routes. Although SLAM maps contain metric information, they rarely contain metadata related to the robot's motivation or behavioral state. Robot navigation and path planning are typically deterministic and strive to reduce variability (LaValle, 2011a, 2011b). In contrast, it has been shown that taking environmental cost and uncertainty into account can result in flexible navigation solutions (Hwu et al., 2018; Xing et al., 2020).

In the present paper, we explore this notion that not only is navigational variability the norm, but also it may provide advantages. Specifically, we use a variety of path planning algorithms to explain the variations in human navigation observed in the Boone, He and colleagues' studies (Boone et al., 2018; Boone et al., 2019; He et al., 2020, November, 2020, October). We show that there is variation between subjects and within subjects. We further suggest that this variation may be due to the environmental or mental costs of traveling through space. This may confer advantages for flexible and adaptive behavior.

2. Methods

Simulations were generated to reproduce human navigation strategies in a virtual reality environment (Boone, 2019; Boone et al., 2019; He et al., 2020, November, 2020, October). The general approach was as follows: 1) Raw positional trajectories from subjects were transformed into a grid map suitable for simulations. 2) Different path planning strategies were simulated to see which strategy best fit an individual subject's behavior on a particular trial. 3) Another set of simulations were carried out on a set of larger maps to investigate whether varying trajectory costs in the environment could explain the observed variation in the human population.

2.1 Converting Subject Data into Simulation Inputs

In the human navigation studies, 368 subjects learned the layout of a virtual maze environment by taking a fixed tour (see Figure 1, Middle) through the environment five times from a first-person perspective using a mouse and keyboard interface. There are 12 landmarks in the environment (see Figure 1, Right) and people were asked to remember the landmark locations. There were 20 trials in the test phase. On each trial, participants were transported to a landmark and were asked to find a goal landmark using the same mouse and keyboard interface. 206 of these subjects were just told to go to the goal, and 162 were told to take the shortest path to the goal. These will be referred to in the remainder of the paper as "Goto Goal", and "Shortcut", respectively. Using the Cartesian coordinates of the environment (see Figure 1 in (Boone et al., 2019)), a 13-by-13 grid world of the environment was created (see Figure 1, Left).

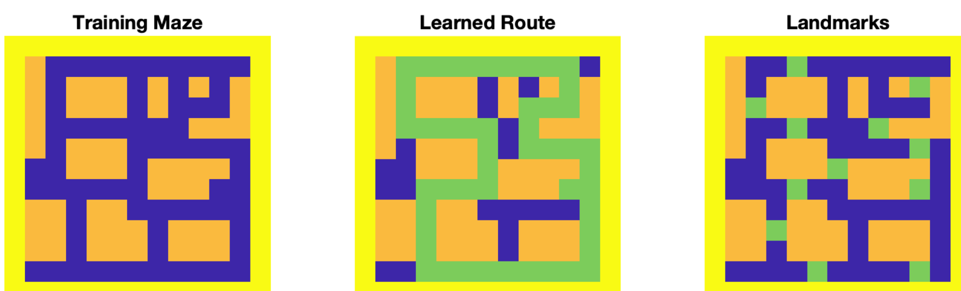

Figure 1. Maps used to simulate human subject data. **Left.** Map is a 13x13 grid. Yellow denotes, the border, orange denotes untraversable areas, and blue denotes traversable areas. **Middle.** Green denotes the learned route. **Right.** Green denotes landmark locations. Map was derived from (Boone et al., 2018; Boone et al., 2019).

Raw trajectories from the subject data were converted into Cartesian coordinates that fit on the map shown in Figure 1. This provided path coordinates for the 368 subjects and their 20 trials. The subject trials were annotated with the instructions Goto Goal or Shortcut.

2.2 Path Planning Algorithms

Subject trajectories were simulated with a suite of path planning algorithms based on the type of knowledge people use to navigate the world (Chrastil & Warren, 2015). 1) *Survey.* Survey knowledge uses map-like knowledge that includes distances and directions between locations in an environment. 2) *Topological*. Topological knowledge consists of a network of nodes linked by edges. In our simulations, the landmarks denote the nodes, and the edges are the Euclidean distances and directions between those nodes. 3) *Route*. Route knowledge consisted of a sequence of locations, in this case, the learned route shown in Figure 1.

To simulate survey knowledge, we used a spiking wavefront propagation path planner (Hwu et al., 2018). The spiking wavefront path planner will calculate the shortest traversable path between two points on a map. Any path planner used in robotics (e.g., A*, Djikstra) would work to simulate survey knowledge in this setup (For a review, see (LaValle, 2011a, 2011b)). Because the spiking wavefront planner is adaptive by taking into account the cost of traversal, it was chosen for these simulated experiments.

Briefly, the spiking wavefront propagation path planner works as follows. A neural network is created with neurons representing each location in the map. The neurons are connected to its eight neighbors, representing movement in the N, NE, E, SE, S, SW, W, and NW directions. The weights between neurons are related to the cost of traversal. In the present study, the map in Figure 1 had a weight value of 120 for the border (denoted in yellow), 100 for the untraversable locations (denoted in orange), and 1 for the open path (denoted in dark blue). These weights represent the cost of traversal, which is realized delaying a signal propagating from one neuron to another according to the cost. The algorithm starts by generating a spike or action potential in the neuron representing the goal location. This action potential is propagated to its neighbors with a delay related to the connection weight. Once the action potential is received by the postsynaptic neuron, the postsynaptic neuron generates a spike and propagates this action potential to its neighbors. This propagation proceeds until the neuron at the start location spikes. At that point, the algorithm is halted. The path can be read out by working from the start neuron to the goal neuron, observing which neuron caused its neighbor to fire a spike. The result is the lowest cost path from start to goal. More details on the spiking wavefront propagation algorithm can be found in (Hwu et al., 2018).

To simulate topological knowledge, the spiking wavefront path planner used landmarks from the starting location to the goal location to create a path. For each sub-path between landmarks, the landmark closest to the agent that was also towards the goal was chosen. The planner iterated through these sub-paths until the goal was reached.

To simulate route knowledge, a path was planned from the starting location to the goal location on the learned route using the sequence of coordinates from the learned route. All 20 trials started and ended on locations situated on the learned route. As in the human studies, a reversed sequence of learned route coordinates could be used to plan paths.

We assumed that subjects might change their strategy midway through a trajectory. Therefore, we simulated a mixture of strategies to simulate the human data. For example, a subject might start on the route, and then halfway through realize they could take a shortcut for the remainder of the trajectory. Because trajectories were relatively short, we only switched strategies halfway through a trajectory.

These assumptions led to 14 path planning algorithms that were used to simulate the human subject data and then used to simulate agent navigation in larger, varied environments (see Table 1). We omitted *Route* then *Route Reversed*, and vice versa, because this would lead to oscillations going forward and backward without reaching the goal.

Table 1. Different Strategies Applied in the Simulation

Strategy	Path Planner Description
Survey	Shortest path between a starting location and a goal. Uses the spiking wavefront path planner
Topological	Series of paths from the starting location to the goal location using landmarks as sub-paths. Uses the spiking wavefront path planner to plan paths between landmarks.
Route	Plans a path using a sequence of coordinates from the learned route (See Figure 1).
Route Reversed	Same as Route strategy, but uses the sequence of coordinates from the learned route in reverse order.
Survey then Route	The first half of the trajectory uses the survey strategy, and the second half of the trajectory uses the route strategy. Note that the survey strategy may be slightly longer than 50% to ensure that the start of the route strategy is on the learned route.
Survey then Reverse Route	The first half of the trajectory uses the survey strategy, and the second half of the trajectory uses the route strategy in reverse.
Topological then Route	The first half of the trajectory uses the topological strategy, and the second half of the trajectory uses the route strategy.
Topological then Reverse Route	The first half of the trajectory uses the topological strategy, and the second half of the trajectory uses the route strategy in reverse.
Route then Survey	The first half of the trajectory uses the route strategy, and the second half of the trajectory uses the survey strategy.
Reverse Route then Survey	The first half of the trajectory uses the route strategy in reverse order, and the second half of the trajectory uses the survey strategy.
Route then Topological	The first half of the trajectory uses the route strategy, and the second half of the trajectory uses the topological strategy.
Reverse Route then Topological	The first half of the trajectory uses the route strategy in reverse order, and the second half of the trajectory uses the topological strategy.
Topological then Survey	The first half of the trajectory uses the topological strategy, and the second half of the trajectory uses the survey strategy.

Survey then Topological	The first half of the trajectory uses the survey strategy, and the second half of the trajectory uses the topological strategy.

2.3 Simulating Subject Data

To simulate human navigation, we applied the 14 strategies given in Table 1 to each subject for each of the 20 trials. We used the Frechet distance metric to calculate how close the strategy resembled the path taken by the subject (Danziger, 2020; Eiter & Mannila, 1994). The Frechet distance between two curves in a metric space is a measure of the similarity between the curves. In our case, it is the distance between two sets of path coordinates. Frechet distance does not require that the two sets of points being compared have the same length.

2.4 Simulating Trajectories on Novel Maps

To test whether these different path planning strategies scale and to provide a possible explanation for the variation in human behavior, we created 5 maps that had 32-by-32 locations (see Figure 2). Each map had a learned route (shown in green in Figure 2) and an open path (shown in dark blue in Figure 2). The cost of traversing the learned route was always equal to one. The untraversable regions, which are shown in yellow, had a cost 120. The cost of traversing the open path varied and was set to 1, 2, 4, 8, and 16. A value of 16 meant it was 16 times more costly to travel over the open path than over the learned route. In the spiking wavefront planner, this cost was applied by delaying the propagation of the wave by the cost. In this way, the best path may not be the shortest if traveling over the learned route is less costly. The learned route was randomly generated but tended to follow the perimeter of the map environment with forays into the center of the environment. Since it was not learned, there was no order associated with the learned route. Therefore, there was no preference for taking the route in the forward or reverse order. Thirteen landmarks were scattered across the environment. These were required to be on the traversable areas of the map. 16 trials were randomly generated using the landmarks as starting and goal locations. To provide complexity to the trajectories, trials could not be straight lines between landmarks.

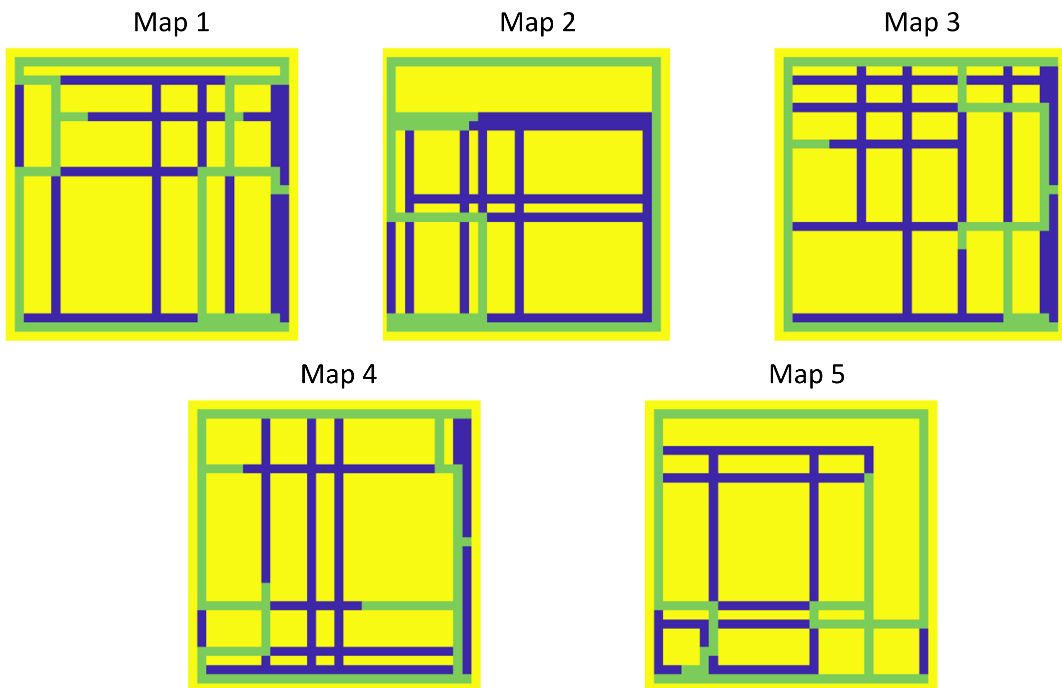

Figure 2. Maps used for simulated environments. The maps contain 32-by-32 grid locations. The yellow areas are untraversable. The green areas denote the learned route. The dark blue areas denote traversable open paths. The cost of the learned route is always one. The cost of the open path can vary from 1, 2, 4, 8, to 16.

2.5 Software and Computation

All simulations and analyses were carried out using MATLAB (MathWorks) scripts on a 3GHz 6-Core Intel i5 iMac with 32GB of memory. The scripts will be made available on GitHub upon publication.

3. Results

Simulations were carried out to emulate the variation in human navigation strategies with path planning algorithms. In the first set of simulations, we replicated the trajectories of human subjects in the human navigation studies using combinations of path planning strategies. In the second set of simulations, we tested whether cost of traversal on different maps could explain the variation observed in humans.

3.1 Replicating Human Navigation Strategies

Each of the 368 subjects ran 20 trials, each with different start and goal locations. We ran all 14 path planning models (see Table 1) with the same start and goal locations and measured which model trajectory was most similar to the subject trajectory using the Frechet distance metric.

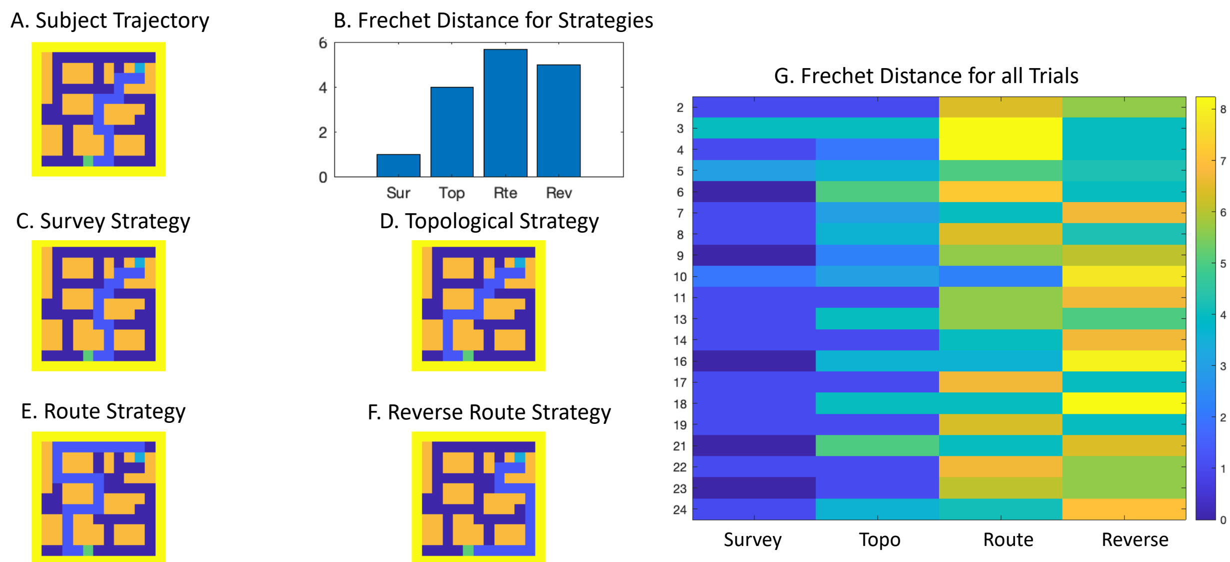

Figure 3. Representative example of a subject that relied on survey knowledge. **A.** Subject trajectory shown in light blue for trial 13. The start is marked in cyan and the goal is marked in green. **B.** Frechet distance for trial 13. The y-axis shows the average Frechet distance for the different path planners. Sur = Survey, Top = Topological, Rte = Route, and Rev = Reverse Route. **C.** Survey path planner model trajectory for trial 13. **D.** Topological path planner model trajectory for trial 13. **E.** Route trajectory, which follows the learned route, for trial 13. **F.** Reverse Route trajectory, which follows the learned route in reverse, for trial 13. **G.** Heatmap showing the Frechet distance for 4 strategies in all 20 trials. The rows are trials, and the columns are strategies. The colorbar to the right of the heatmap denotes the Frechet distance value.

Depending on the subject, different path planners predicted the subject's navigation behavior. For example, Figure 3 shows a representative example of a subject that preferred to use survey knowledge on all trials (see Figures 3B and 3G). Figures 3C through 3F show the different strategies on trial 13. Note how close the survey path planner in Figure 3C resembles the subject trajectory in Figure 3A. The histogram in Figure 3B and the heatmap in Figure 3G show that the trajectories calculated by the survey path planner most resembled this subject's trajectories on all trials. Figure 4 shows a representative example of a subject that preferred to use route knowledge on nearly all of the trials (see Figures 4B and 4G). Note how in this case, the route path planner almost identically matches the trajectory of this subject (compare Figure 4A to 4E).

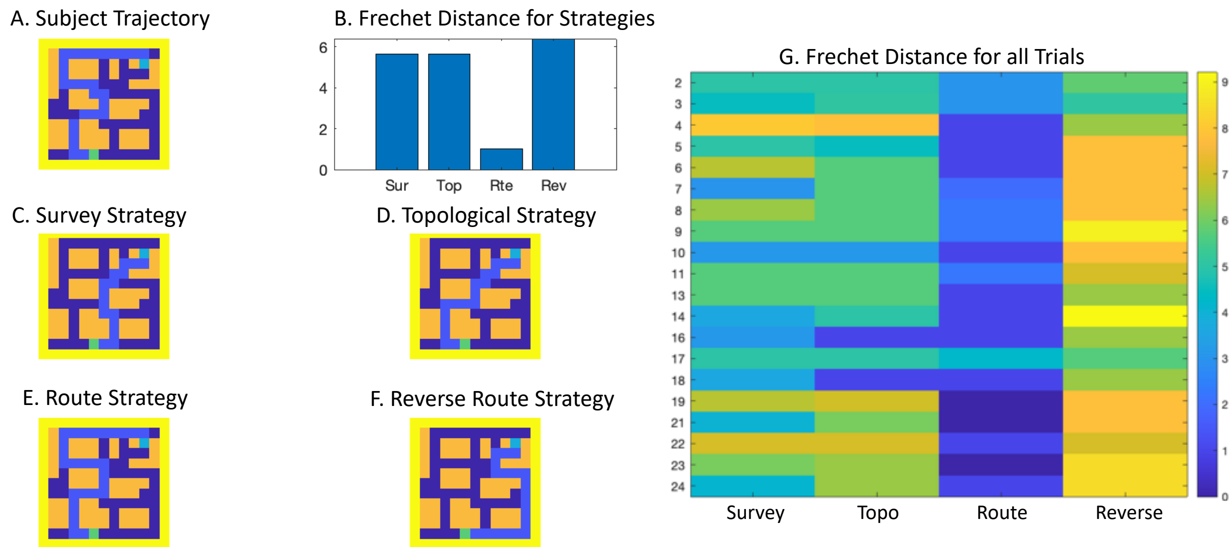

Figure 4. Representative example of a subject that relied on route knowledge. **A.** Subject trajectory shown in light blue for trial 13. The start is marked in cyan and the goal is marked in green. **B.** Frechet distance for trial 13. The y-axis shows the average Frechet distance for the different path planners. Sur = Survey, Top = Topological, Rte = Route, and Rev = Reverse Route. **C.** Survey path planner model trajectory for trial 13. **D.** Topological path planner model trajectory for trial 13. **E.** Route trajectory, which follows the learned route, for trial 13. **F.** Reverse Route trajectory, which follows the learned route in reverse, for trial 13. **G.** Heatmap showing the Frechet distance for 4 strategies in all 20 trials.

We hypothesized that subjects may mix strategies within a given trial. Therefore, we tested which of the 14 strategies given in Table 1 best predicted the subject's navigation behavior. Figure 5 shows the Frechet distances for the strategy that best predicted a trajectory. The minimum Frechet distance when using a pure strategy (i.e., always stick with the same strategy in a given trial) to predict trajectories (*Median* = 2.8) was significantly larger than when including a mixture of strategies (*Median* = 2.2, Wilcoxon signed rank test, V=442270, $p < .001$). This suggests that subjects occasionally mix strategies within a trial.

Subjects would occasionally start on a route, and then switch to a survey strategy. Figure 6 shows a histogram of which of the 14 path planners best predicted the subject's behavior. On 7.99% of the "Goto Goal" trials and 9.2% of the Shortcut trials, subjects started using route knowledge (either forward or reverse) and then switched to using survey knowledge. For example, see the representative subject in Figure 7, who during Trial 3, started on the learned route and then about midway through their trajectory took a shortcut. Interestingly, subjects did not switch to using graph knowledge after starting on the learned route. Moreover, subjects did not tend to start using survey knowledge or graph knowledge and then switch to the learned route.

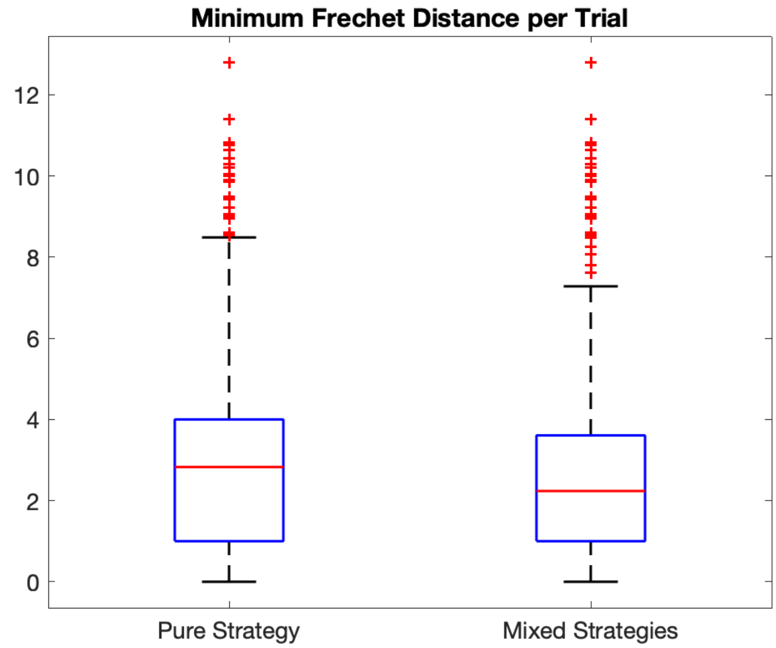

Figure 5. Minimum Frechet distance for the strategy that best predicts the subject's trajectory for all subjects and all trials. The boxplot on the left is the prediction when using the same strategy throughout a given trial. The boxplot on the right is the prediction when subject's switched strategies midway through a trajectory. On each box, the red line is the median, the blue lower and upper edges of the box represent the 25th and 75th percentiles, respectively, the whiskers extend to extreme datapoints that are not considered outliers, and the outliers are denoted with red plus signs.

In (Boone et al., 2019), it was shown that participants had the capacity to use survey knowledge when instructed to take the shortest path to a goal location. We wondered whether we could observe this result in the path planner predictions. Figure 6A shows the best fitting path planner algorithm for those participants who were told to go to the goal location. Each participant conducted 20 trials with different starting and ending locations. Note the large percentage of trials where subjects used the route strategy. Figure 6B shows the best fitting path planner algorithm for participants who were told to take the shortest path. Note the decrease in route strategies in this case.

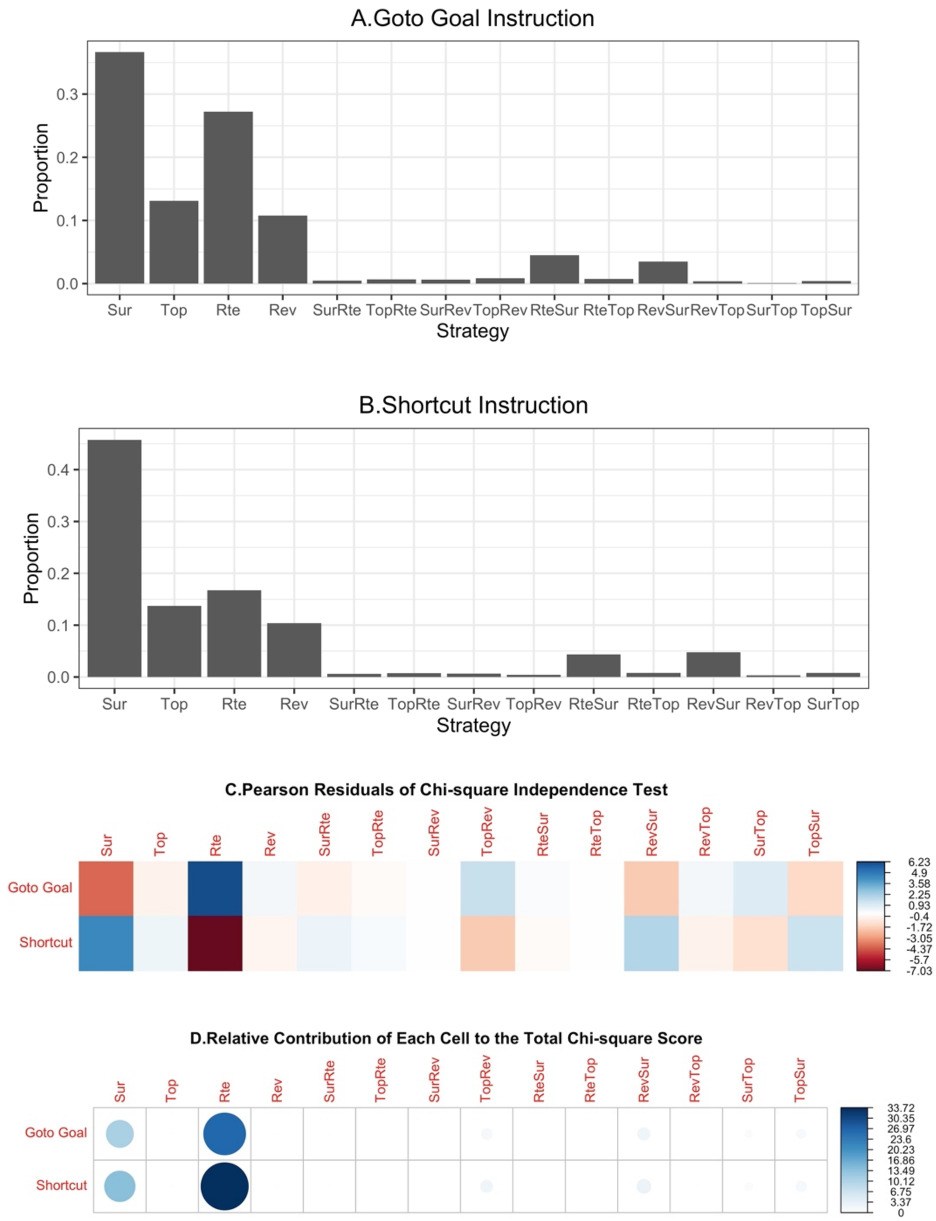

Figure 6. Strategies used under different conditions. **A.** Proportion of the strategy that best predicts a participant's behavior when they were instructed to go to the goal location (n = 206). **B.** Proportion of the strategy that best predicts a participant's behavior when they were instructed to take the shortest path to the goal location (n = 162). Sur=survey, Top=topological, Rte=route, Rev=Reverse Route. The remaining 10 labels represent combined strategies. For example, SurRte represents survey strategy on the first half of the trajectory then route strategy on the second half on the trajectory. C. Pearson residuals for each strategy using the Chi-square independent test. The Positive residuals are in blue. Positive values in cells specify a positive association between the instructions given and the specific strategy choice. D. The relative contribution of each strategy to the total Chi-square score. A larger and darker circle indicates a more contributing cell to the Chi-square score.

A chi-square test of independence showed a significant relationship between the instructions and the strategy used (X^2 (13) = 146.37, p <.001). As shown in Figure 6C, Goto Goal condition was positively associated with using route strategy (Pearson residuals = 6.23) but negatively associated with using survey strategy (Pearson residuals = -4.03). In contrast, Shortcut condition was positively associated with using survey strategy use (Pearson residuals = 4.54) but negatively associated with using the route strategy (Pearson residuals = -7.03). Figure 6D shows that the Shortcut condition/Route strategy (33.72%), Goto Goal/Route strategy (26.51%), Shortcut/Survey (14.10%), and Goto Goal/Survey (11.09%) contributed most to the Chi-square score. These results suggest that the path planning algorithms could also predict that subjects could take shortcuts when instructed to do so.

Given that subjects use a range of strategies for navigating these environments, we considered whether they stick to the same strategy in all trials, similar to the subjects in Figures 3 and 4, or if they alter strategies from trial to trial as depicted in Figure 7. Looking across all subjects and all trials, it appeared that most subjects were similar to that shown in Figure 7, where depending on the trial the same subject might apply a different navigation strategy.

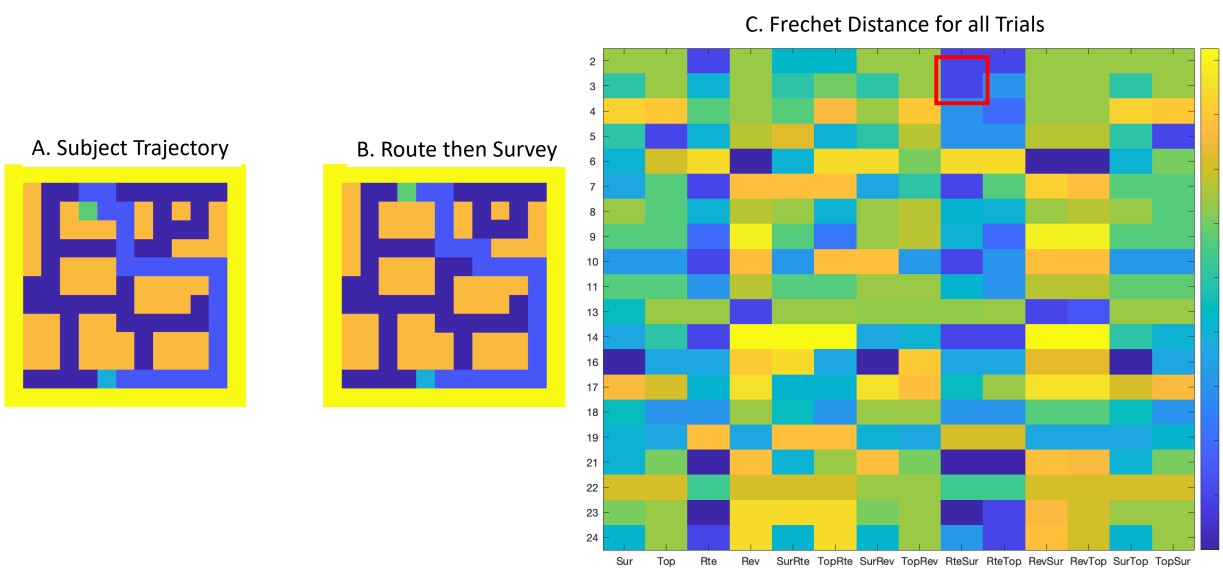

Figure 7. Representative subject that mixes strategies. **A.** Subject's trajectory on trial 3. Cyan denotes the start location and green denotes the goal location. **B.** Subject's behavior was best predicted by starting on the learned route and then taking a shortcut by switching to the survey strategy. **C.** Heatmap of the Frechet distance for all trials and all 14 path planners. The rows are trials, and the columns are strategies. The colorbar to the right of the heatmap denotes the Frechet distance value. The red outline is for trial three, which is shown in **A** and **B**.

To measure how much a subject varied their strategy from trial to trial, we constructed a metric of strategy variation:

$$S_i = \sum_{j=1}^{20} I_{ij} = \begin{cases} 1 \; if \; strategy \; i \; used \; on \; trial \; j; \\ 0 \; otherwise \end{cases} \tag{1}$$

$$C = 1 - \frac{\sum_{i=1}^{14} \begin{cases} 1 \; if \; S_i \neq 0; \\ 0 \; otherwise \end{cases}}{14} \tag{2}$$

$$P = \frac{\max(S)}{20} \tag{3}$$

$$SV = \frac{C+P}{2} \tag{4}$$

Where S_i represents the number of times a participant used strategy *i* across the 20 trials, *I* is one if the Strategy *i* was used on trial *j*. *C* represents to what extent different strategies were used. For example, if survey was used on all trials, the value of *C* would be $(1 - \frac{1}{14})$ or 0.93, whereas if 12 different strategies were used across the 20 trials, the value of *C* would be $(1 - \frac{12}{14})$ or 0.14. *P* represents how often the preferred strategy is used. Where *max(S)* is how often the preferred (i.e., maximum) strategy is used across all 20 trials, which is the total number of trials. For example, if the survey strategy was used on all trials, the value of *P* would be $\left(\frac{20}{20}\right)$ or 1. If the survey strategy was used on two trials, the value of *P* would be $\left(\frac{2}{20}\right)$ or 0.10. *SV* is the overall metric for strategy variation where a value close to 0 means the subject varied their strategy from trial to trial and a value close to 1 means the subject always used the same strategy.

Subjects did tend to vary their strategy from trial to trials. The violin plots in Figure 8 shows the *SV* values (Equation 4) calculated for all subjects by different instructions conditions. Mean of *SV* is 0.54, which is significantly smaller than 0.965 ($t(367) = -74.06$, $p < .001$) which is theoretically largest value of *SV*, indicating no variation across 20 trials. Two sample t-tests shows that strategy variation in Goto Goal condition ($M = 0.54$) is not significantly different from that in Shorctut condition ($M = 0.54$), $t(366) = .49$, $p = .63$, two-tailed. In general, subjects tended to vary their strategy somewhat from trial to trial, regardless of the instructions. This suggests that environmental differences or different difficulty of a given trial may impact the preferred navigation strategy.

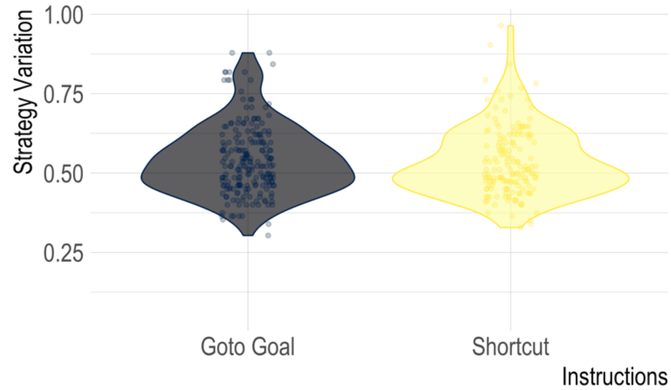

Figure 8. Strategy variation between trials for all subjects. The violin plots show the distribution of strategic variation (*SV* in equation 4). 206 Subjects that were instructed to go to the goal location. 162 Subjects that were instructed to take the shortest path to the goal location.

Taken together, these simulations suggest that not only do subjects have varying navigation abilities, but also subjects vary their navigation behavior both within and between trials. In the next section, we use another set of simulations to investigate why this might occur.

3.2 Strategy Variation due to Cost of Traversal

To test whether the cost of traversing over an environment might impact a subject's navigation strategy, we generated maps where the ratio of the traversable paths to learned route varied (see Figure 2). Five maps were used and there were 16 trials per map. As described in the methods section, the cost ratio of traversable path to the learned route ranged from 16:1, 8:1, 4:1, 2:1, to 1:1. These ratios were sufficient to capture the range of behaviors observed in the human data.

To mimic subject behavior, we applied the survey path planner to the map with the learned route. The route and reverse route path planners followed the learned route forwards and backwards, respectively. The survey and topological path planners were applied to a map where all traversable locations were set to 1. The effect was that these planners used map and graph knowledge without relying on the learned route. As expected, in the 1:1 case, the preferred strategy was always a survey strategy since there was no cost advantage for using the learned route. The 8:1 and 4:1 case showed a variation of strategies somewhere between the 16:1 and 2:1 cases. Therefore, we further analyzed the more extreme 16:1 and 2:1 cases. As with the human subject data, the Frechet distance metric was used to measure which path planner most closely resembled the simulated subject (i.e., the survey path planner on the map with the learned route).

Figure 9 shows a representative example of a simulation trial where the path varied due to differences in the cost of traversing a location. On the left side of Figure 9, the survey path planner was applied to the 16:1 map. The trajectory most resembled a route strategy of a subject that was instructed to go to the goal. On the right side of Figure 9, the same survey path planner was applied to the 2:1 map. In this case, the trajectory resembled a survey strategy where the subject takes a shortcut. Similar behavioral differences were observed on the different maps and different trials.

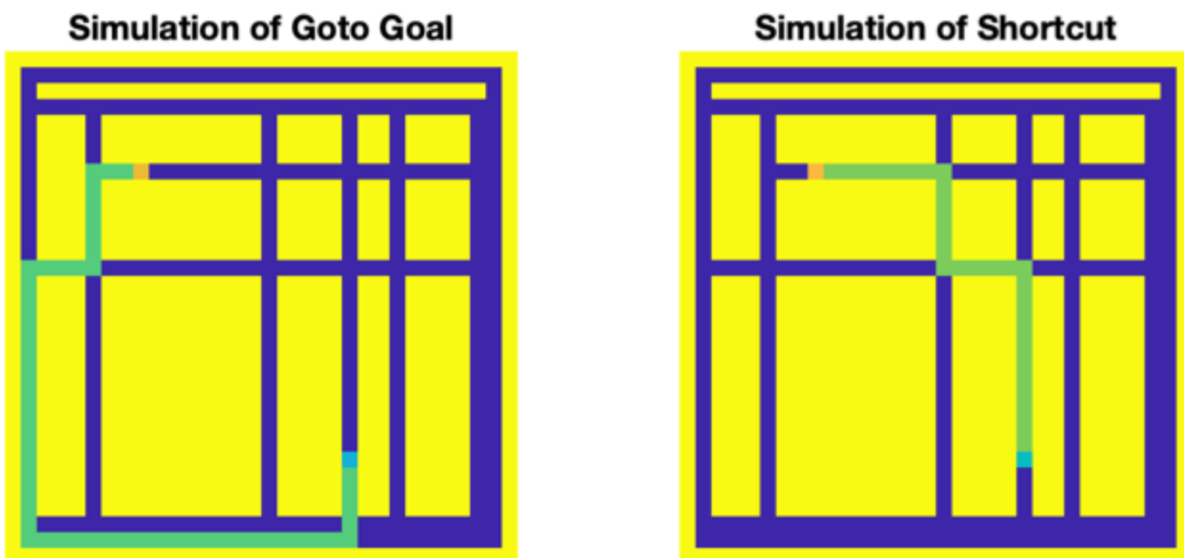

Figure 9. Simulation of navigation strategies with different cost maps. The green pixels denote the path. The cyan pixel is the start location, and the orange pixel is the goal location. The left side shows the path the survey path planner takes when the learned route is 16 times less costly than the other traversable areas. The right side shows the path taken when the learned route is 2 times less costly than the other traversable areas.

The simulations showed similar variation in preferred strategies to that of the human subjects. Figure 10 shows the distributions of preferred strategies (i.e., minimum Frechet distance) taken for the 16:1 map (Figure 10A) and the 2:1 map (Figure 10B). Comparing these distributions to the subject data depicted in Figure 6, when the learned route was less costly (Go to Goal condition), the total proportion of taking Rte or Rev strategies (31.25%), is not significantly different from the total proportion of taking Rte or Rev strategies by human participants (37.98%), $X^2(1) = 1.24$, $p = .27$. Conversely, when the learned route was only slightly less costly than other traversable regions (Shortcut condition), the total proportion of taking Rte or Rev strategies (17.50%) dropped and is not significantly different from the total proportion of taking Rte or Rev strategies by human participants (21.31%), $X^2(1) = .47$, $p =$

.49. Note that in these simulations the agent does not have knowledge of route direction. Therefore, route (Rte) and reverse route (Rev) can be considered together. It is also of interest that the mixed strategies of route then survey (RteSur and RevSur) in the simulations (Goto Goal: 8.75 %; Shortcut: 5.00%) showed similar proportions to those in the human subject data (Goto Goal: 7.99 %; Shortcut: 9.17%), Goto Goal: $X^2(1) = .002$, $p = .97$; Shortcut: $X^2(1) = 1.18$, $p = .28$.

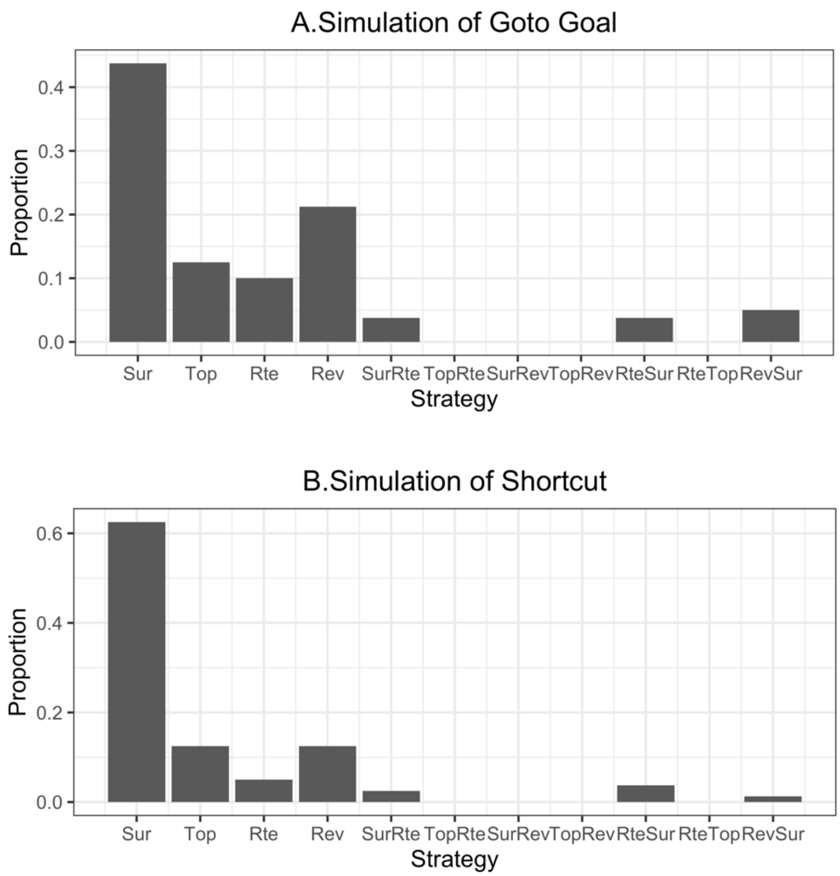

Figure 10. Simulations of preferred navigation strategies. **A.** Goto goal instructions were simulated using the 16:1 map (the number of total trials is 80). **B.** Shortcut instructions were simulated using the 2:1 map (the number of total trials is 80).

4. Discussion

The present paper suggests that the variation observed in human navigation is both between subjects and on a trial-to-trial basis for an individual, and that this variability might be explained by taking the cost of traversing an environment into consideration. This has implications for both cognitive science and for autonomous navigation systems.

Although it has been suggested that people vary in their ability to navigate and some tend to demonstrate route or survey knowledge (Boone et al., 2018; Boone et al., 2019; Chrastil & Warren, 2015; Weisberg & Newcombe, 2018), we found that people varied their strategy on a trial-by-trial basis (see Figures 7 and 8). Furthermore, we found that subjects can change strategies within a trial (see 6 and 7). It was interesting that within a trial approximately 8% to 9% of the time a subject would start on the learned route and then switch to an allocentric strategy by using survey knowledge to take shortcuts. This suggests that at some point, they felt confident enough to leave the route and take a shortcut. Moreover, the opposite mixed strategy rarely occurred. That is, subjects did not start a trajectory using an allocentric map, and then switch to the route once connecting with the route. It may suggest that participants use a less costly strategy, due to cognitive load or environmental conditions, to go to the sub-region where they are comfortable with using their survey knowledge. Previous research on hierarchical structure of environmental knowledge has shown the mental map of the whole environment contain many sub-regions and the mental map, as a part, is contained in larger regions (McNamara, 2013). The present results suggest that the strategy people use to go across the sub-regions may differ from the strategy used for navigating within a sub-region. Wiener and Mallot have also shown one useful wayfinding heuristic called fine-to-coarse heuristic, where people plan a route to the region containing the goal and only once inside that region, they will determine the subsequent specific route (Wiener & Mallot, 2003). It has been recently shown that once freely navigating bats choose a shortcut using vector navigation, they do not vary their path as they approach a goal (Harten et al., 2020; Toledo et al., 2020).

The cost of traversing portions of the environment might be able to explain the observed variability. Previous research on executive control and decision making has demonstrated that people tend to conserve cognitive effort, which is referred to as the law of less work or demand avoidance (Kool et al., 2010). He and Hegarty have demonstrated that different people have various preferences in putting mental efforts in solving navigational problems. All these add variability in navigational strategies within and between different individuals (He & Hegarty, 2020). In a set of simulations, we were able replicate the variability within and between subjects by varying the ratio between the cost of taking a learned route and the cost of taking shortcuts. When the cost of taking the learned route was relatively low, the simulated agents utilized a route strategy fairly often (see Figure 10A). However, when the cost of taking the route was relatively high with respect to other potential paths, the simulated agent utilized a survey strategy (see Figure 10B. In the case of Boone and colleagues (Boone et al., 2019), an instruction to take the shortest path was able to tip this balance in favor of survey knowledge suggesting that subjects had this capability but did not express it unless told to do so.

It remains an open question why subjects might choose to use one type of knowledge over another when navigating. One answer may be related to cognitive load. It might take more mental effort to choose a never experienced shortcut than to follow a route known by habit. An instruction, such as take the shortest path, might force a person to make that effort. It could also be related to confidence in the current navigation strategy (Oess et al., 2017). If subjects were confident about their allocentric position within the environment, they might rely on a survey strategy, or if they were confident that taking a known route would lead them to the desired goal, they might use route knowledge. Another answer may be environmental. The availability and salience of different cues as well as the complexity of the layout influence the quality of acquired environmental knowledge and how people apply the knowledge (Carlson et al., 2010; Chai & Jacobs, 2010; He et al., 2020, November; He et al., 2020). In a series of robot navigation tasks, variations in the cost of traversal (e.g., how costly the path planner considered traversing over uneven terrain) could shift the robot from taking a direct but bumpy and hilly path from one location to another, to taking a longer but smoother path on a paved sidewalk (Hwu et al., 2018).

Variations in neuromodulatory signaling may explain population variability during navigation. For example, depending on dopaminergic or serotonergic activation from the orbitofrontal cortex or the medial prefrontal cortex, rodents may shift from putting more effort to get a larger, but more difficult to obtain reward, to taking an easier but less rewarding choice (Rudebeck et al., 2006). Natural variations in the dopaminergic COMT gene, can result in risk taking variability (Roussos et al., 2008). This might have implications for navigation as well. Whereas a shortcut might be a risky endeavor, following a known route might be the safer option. Similarly, there are natural variations in the serotoninergic system that may affect behavior (Caspi et al., 2010). For example, recent optogenetic experiments with mice showed that altering the level of serotonin could vary how long an animal would wait for a reward (Miyazaki et al., 2018). In human studies, individual differences in risk-taking traits as well as spatial anxiety experienced during navigation have been demonstrated to be influential in human navigation behaviors (He & Hegarty, 2020; Lawton & Kallai, 2002; Pingel, 2012). People who are more likely to avoid taking risks and experience more spatial anxiety tend to use route strategy more often than survey strategy. This population variability is explained from the perspective of evolution such that different roles in hunting and gathering, mating competition as well as fertility-and-parental-care all may lead to a wide range of variation in tackling risks and costs of navigation (Cashdan & Gaulin, 2016; Gagnon et al., 2016; Gagnon et al., 2018).

We recently applied the idea that serotonin levels affect patience to a robot navigation task (Xing et al., 2020). The robot was tasked to find a set of GPS waypoints in an outdoor park. If the simulated serotonin levels were high, the robot patiently searched until it found the waypoints. However, when the simulated serotonin levels were low, the robot gave up searching for hard to find waypoints, due to difficult terrain or weak GPS signals, and took a shortcut to another waypoint. Such variability could be advantageous in self-driving vehicles. For example, being assertive at a 4-way stop might break a deadlock, or impatience due to changing traffic patterns may result in re-routing the vehicle's path.

Recently, a neurobiologically inspired model was proposed to explain when an agent may shift navigation strategies (Edvardsen et al., 2020). It was suggested that the place cell system supported topological navigation and the grid cell system supported vector navigation. They suggested that vector navigation, similar to a survey strategy, was used until the agent got stuck. Once stuck, hippocampal replay of place cells was invoked to choose an alternative route. Interestingly, this is different than what was observed in our analysis and simulation of human subjects. Subjects analyzed in the present study tended to start with the route strategy and then occasionally switched to a survey strategy. The opposite order was rarely observed. However, the two environmental conditions are different. In the case of Edvardsen and colleagues, the agent began using survey knowledge, but was subsequently blocked by an unexpected obstacle. In future work, it will be of interest to see if the algorithms in present paper can also demonstrate this remapping given similar environmental constraints.

We suggest that populational variability may confer advantages for information gathering by biological organisms and artificial systems. Heterogenous teams of agents have been used to solve a wide range of problems (Engelbrecht, 2010; Hara et al., 2012; Valle et al., 2008). In a heterogeneous robotic swarm, certain tasks can be solved efficiently through cooperation and functional specialization (Dorigo et al., 2013). Therefore, in future studies it will be important to take this variability into consideration as a benefit rather than a liability.

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References

Boone, A. P. (2019). The Influence of the Human Stress Response on Navigation Strategy and Efficiency University of California, Santa Barbara].

Boone, A. P., Gong, X., & Hegarty, M. (2018, Aug). Sex differences in navigation strategy and efficiency. Mem Cognit, 46(6), 909-922. https://doi.org/10.3758/s13421-018-0811-y

Boone, A. P., Maghen, B., & Hegarty, M. (2019, Oct). Instructions matter: Individual differences in navigation strategy and ability. Mem Cognit, 47(7), 1401-1414. https://doi.org/10.3758/s13421-019-00941-5

Carlson, L. A., Hölscher, C., Shipley, T. F., & Dalton, R. C. (2010, 2010/10/01). Getting Lost in Buildings. Current Directions in Psychological Science, 19(5), 284-289. https://doi.org/10.1177/0963721410383243

Cashdan, E., & Gaulin, S. J. (2016, Mar). Why Go There? Evolution of Mobility and Spatial Cognition in Women and Men : An Introduction to the Special Issue. Hum Nat, 27(1), 1-15. https://doi.org/10.1007/s12110-015-9253-4

Caspi, A., Hariri, A. R., Holmes, A., Uher, R., & Moffitt, T. E. (2010, May). Genetic sensitivity to the environment: the case of the serotonin transporter gene and its implications for studying complex diseases and traits. Am J Psychiatry, 167(5), 509-527. https://doi.org/10.1176/appi.ajp.2010.09101452

Chai, X. J., & Jacobs, L. F. (2010, Apr 2). Effects of cue types on sex differences in human spatial memory. Behav Brain Res, 208(2), 336-342. https://doi.org/10.1016/j.bbr.2009.11.039

Chrastil, E. R., & Warren, W. H. (2015). Active and passive spatial learning in human navigation: Acquisition of graph knowledge. Journal of Experimental Psychology: Learning, Memory, and Cognition, 41(4), 1162-1178. https://doi.org/10.1037/xlm0000082

Danziger, Z. (2020). Discrete Frechet Distance. . https://www.mathworks.com/matlabcentral/fileexchange/31922-discrete-frechet-distance

Dorigo, M., Floreano, D., Gambardella, L. M., Mondada, F., Nolfi, S., Baaboura, T., Birattari, M., Bonani, M., Brambilla, M., Brutschy, A., Burnier, D., Campo, A., Christensen, A. L., Decugniere, A., Caro, G. D., Ducatelle, F., Ferrante, E., Forster, A., Gonzales, J. M., Guzzi, J., Longchamp, V., Magnenat, S., Mathews, N., Oca, M. M. d., Grady, R. O., Pinciroli, C., Pini, G., Retornaz, P., Roberts, J., Sperati, V., Stirling, T., Stranieri, A., Stutzle, T., Trianni, V., Tuci, E., Turgut, A. E., & Vaussard, F. (2013). Swarmanoid: A Novel Concept for the Study of Heterogeneous Robotic Swarms. Ieee Robotics & Automation Magazine, 20(4), 60-71. https://doi.org/10.1109/MRA.2013.2252996

Edvardsen, V., Bicanski, A., & Burgess, N. (2020, 2020/03/01). Navigating with grid and place cells in cluttered environments. Hippocampus, 30(3), 220-232. https://doi.org/10.1002/hipo.23147

Eiter, T., & Mannila, H. (1994). Computing discrete Fréchet distance. A. Rep. CD-TR 94/64.

Engelbrecht, A. P. (2010, 2010//). Heterogeneous Particle Swarm Optimization. Swarm Intelligence, Berlin, Heidelberg.

Gagnon, K. T., Cashdan, E. A., Stefanucci, J. K., & Creem-Regehr, S. H. (2016, Mar). Sex Differences in Exploration Behavior and the Relationship to Harm Avoidance. Hum Nat, 27(1), 82-97. https://doi.org/10.1007/s12110-015-9248-1

Gagnon, K. T., Thomas, B. J., Munion, A., Creem-Regehr, S. H., Cashdan, E. A., & Stefanucci, J. K. (2018, Nov). Not all those who wander are lost: Spatial exploration patterns and their relationship to gender and spatial memory. Cognition, 180, 108-117. https://doi.org/10.1016/j.cognition.2018.06.020

Gallistel, C. R. (1990). The organization of learning. The MIT Press.

Gallistel, C. R. (2017, Feb 6). Navigation: Whence Our Sense of Direction? Current Biology, 27(3), R108-R110. https://doi.org/10.1016/j.cub.2016.11.044

Hara, A., Shiraga, K., & Takahama, T. (2012, 20-24 Nov. 2012). Heterogeneous Particle Swarm Optimization including predator-prey relationship. The 6th International Conference on Soft Computing and Intelligent Systems, and The 13th International Symposium on Advanced Intelligence Systems,

Harten, L., Katz, A., Goldshtein, A., Handel, M., & Yovel, Y. (2020). The ontogeny of a mammalian cognitive map in the real world. Science, 369(6500), 194. https://doi.org/10.1126/science.aay3354

He, C., Boone, A. P., & Hegarty, M. (2020, November). The Influence of Environmental Structure on Measures of Spatial Navigation. Psychonomic Society's 61st Annual Meeting, He, C., Boone, A. P., & Hegarty, M. (2020, October). Ability to Navigate by Novel Efficient Paths with Imprecise Spatial Knowledge. 3rd Interdisciplinary Navigation Symposium,

He, C., & Hegarty, M. (2020, 2020/10/01/). How anxiety and growth mindset are linked to navigation ability: Impacts of exploration and GPS use. Journal of Environmental Psychology, 71, 101475. https://doi.org/https://doi.org/10.1016/j.jenvp.2020.101475

He, Q., Han, A. T., Churaman, T. A., & Brown, T. I. (2020, Sep 14). The role of working memory capacity in spatial learning depends on spatial information integration difficulty in the environment. J Exp Psychol Gen. https://doi.org/10.1037/xge0000972

Hegarty, M., He, C., Boone, A. P., & Chrastil, E. R. (2020, November). Individual Differences in Wayfinding Strategies in Real and Virtual Environments. . Psychonomic Society's 61st Annual Meeting.,

Hegarty, M., Montello, D. R., Richardson, A. E., Ishikawa, T., & Lovelace, K. (2006, 2006/03/01/). Spatial abilities at different scales: Individual differences in aptitude-test performance and spatial-layout learning. Intelligence, 34(2), 151-176. https://doi.org/https://doi.org/10.1016/j.intell.2005.09.005

Hegarty, M., & Waller, D. A. (2005). Individual Differences in Spatial Abilities. In The Cambridge Handbook of Visuospatial Thinking. (pp. 121-169). Cambridge University Press. https://doi.org/10.1017/CBO9780511610448.005

Hwu, T., Wang, A. Y., Oros, N., & Krichmar, J. L. (2018, Jun). Adaptive Robot Path Planning Using a Spiking Neuron Algorithm With Axonal Delays. Ieee Transactions on Cognitive and Developmental Systems, 10(2), 126-137. https://doi.org/10.1109/Tcds.2017.2655539

Ishikawa, T., & Montello, D. R. (2006, Mar). Spatial knowledge acquisition from direct experience in the environment: individual differences in the development of metric knowledge and the integration of separately learned places. Cogn Psychol, 52(2), 93-129. https://doi.org/10.1016/j.cogpsych.2005.08.003

Kohlbrecher, S., von Stryk, O., Meyer, J., & Klingauf, U. (2011, 1-5 Nov. 2011). A flexible and scalable SLAM system with full 3D motion estimation. 2011 IEEE International Symposium on Safety, Security, and Rescue Robotics,

Kool, W., McGuire, J. T., Rosen, Z. B., & Botvinick, M. M. (2010, Nov). Decision making and the avoidance of cognitive demand. J Exp Psychol Gen, 139(4), 665-682. https://doi.org/10.1037/a0020198

Laughlin, P. R., Hatch, E. C., Silver, J. S., & Boh, L. (2006, Apr). Groups perform better than the best individuals on letters-to-numbers problems: effects of group size. J Pers Soc Psychol, 90(4), 644-651. https://doi.org/10.1037/0022-3514.90.4.644

LaValle, S. M. (2011a, Mar). Motion Planning Part I: The Essentials. Ieee Robotics & Automation Magazine, 18(1), 79-89. https://doi.org/10.1109/Mra.2011.940276

LaValle, S. M. (2011b, Jun). Motion Planning Part II: Wild Frontiers. Ieee Robotics & Automation Magazine, 18(2), 108-118. https://doi.org/10.1109/Mra.2011.941635

Lawton, C. A., & Kallai, J. (2002, 2002/11/01). Gender Differences in Wayfinding Strategies and Anxiety About Wayfinding: A Cross-Cultural Comparison. Sex Roles, 47(9), 389-401. https://doi.org/10.1023/A:1021668724970

Liker, A., & Bokony, V. (2009, May 12). Larger groups are more successful in innovative problem solving in house sparrows. Proc Natl Acad Sci U S A, 106(19), 7893-7898. https://doi.org/10.1073/pnas.0900042106

McNamara, T. P. (2013). Spatial memory: Properties and organization. In Handbook of spatial cognition. (pp. 173-190). American Psychological Association. https://doi.org/10.1037/13936-010

Miyazaki, K., Miyazaki, K. W., Yamanaka, A., Tokuda, T., Tanaka, K. F., & Doya, K. (2018, Jun 1). Reward probability and timing uncertainty alter the effect of dorsal raphe serotonin neurons on patience. Nat Commun, 9(1), 2048. https://doi.org/10.1038/s41467-018-04496-y

Mur-Artal, R., Montiel, J. M. M., & Tardos, J. D. (2015, Oct). ORB-SLAM: A Versatile and Accurate Monocular SLAM System. Ieee Transactions on Robotics, 31(5), 1147-1163. https://doi.org/10.1109/Tro.2015.2463671

Oess, T., Krichmar, J. L., & Rohrbein, F. (2017, Feb 7). A Computational Model for Spatial Navigation Based on Reference Frames in the Hippocampus, Retrosplenial Cortex, and Posterior Parietal Cortex. Frontiers in Neurorobotics, 11. https://doi.org/10.3389/fnbot.2017.00004

Pingel, T. J. (2012, 2012/07/01/). Characterizing the role of strategic disposition and orientation to risk in wayfinding. Transportation Research Part F: Traffic Psychology and Behaviour, 15(4), 427-437. https://doi.org/https://doi.org/10.1016/j.trf.2012.03.003

Roussos, P., Giakoumaki, S. G., Pavlakis, S., & Bitsios, P. (2008, Jan 31). Planning, decision-making and the COMT rs4818 polymorphism in healthy males. Neuropsychologia, 46(2), 757-763. https://doi.org/10.1016/j.neuropsychologia.2007.10.009

Rudebeck, P. H., Walton, M. E., Smyth, A. N., Bannerman, D. M., & Rushworth, M. F. (2006, Sep). Separate neural pathways process different decision costs. Nat Neurosci, 9(9), 1161-1168. https://doi.org/10.1038/nn1756

Toledo, S., Shohami, D., Schiffner, I., Lourie, E., Orchan, Y., Bartan, Y., & Nathan, R. (2020). Cognitive map–based navigation in wild bats revealed by a new high-throughput tracking system. Science, 369(6500), 188. https://doi.org/10.1126/science.aax6904

Valle, Y. d., Venayagamoorthy, G. K., Mohagheghi, S., Hernandez, J., & Harley, R. G. (2008). Particle Swarm Optimization: Basic Concepts, Variants and Applications in Power Systems. IEEE Transactions on Evolutionary Computation, 12(2), 171-195. https://doi.org/10.1109/TEVC.2007.896686

Weisberg, S. M., & Newcombe, N. S. (2016, May). How do (some) people make a cognitive map? Routes, places, and working memory. J Exp Psychol Learn Mem Cogn, 42(5), 768-785. https://doi.org/10.1037/xlm0000200

Weisberg, S. M., & Newcombe, N. S. (2018, Aug). Cognitive Maps: Some People Make Them, Some People Struggle. Curr Dir Psychol Sci, 27(4), 220-226. https://doi.org/10.1177/0963721417744521

Wiener, J. M., & Mallot, H. A. (2003). 'Fine-to-Coarse' Route Planning and Navigation in Regionalized Environments. Spatial Cognition and Computation, 3(4), 331-358. https://doi.org/10.1207/s15427633scc0304_5

Xing, J., Zou, X., & Krichmar, J. L. (2020). Neuromodulated Patience for Robot and Self-Driving Vehicle Navigation. IEEE International Joint Conference on Neural Networks (IJCNN), Glasgow.