Homework 5

February 12, 2016

1 Model

A weight is suspended from a non-ideal spring. In the model to be used in this assignment, if m is the mass of the weight in kilograms, and x is the spring extension in meters, the potential energy of the spring and the weight are, respectively, given by

$$\begin{aligned} E_s(x) &= a\left[\frac{b}{x} + \frac{d^2}{f(x-d)^2} - \exp\left\{\frac{-c(x-b)^2}{2a}\right\}\right] \\ E_w(x) &= -mgx \end{aligned}$$

so that the total energy is given by

$$E_t(x) = E_s(x) + E_w(x).$$

In these expression, the parameters are as follows:

$$\begin{aligned} a &= 1J \quad \text{energy scale} \\ b &= 0.1m \quad \text{length scale} \\ c &= 100N/m \quad \text{spring constant} \\ d &= 0.5m \quad \text{maximum spring extension} \\ f &= 2500 \quad \text{stiffness at maximum extension} \\ g &= 9.8m/s^2 \quad \text{gravitational accelleration.} \end{aligned}$$

The weight will be in equilibrium when x is at a minimum of the energy function $E_t(x)$. It turns out there are two local minima: one regular one, and one at a larger extension, which we interpret as the spring having been stretched beyond its limits.

2 Assignment

For this assignment:

- Write a single function `f_all_min` that, given the mass m of the weight, determines the x positions and energy values of both mimima of E_t. In this function, you are to use the GSL library's implementation of the Brent method for finding minima.
- Use the function `f_all_min` to find the mass-extension relation. Do this by determining the x value of E_t's global minimum (i.e., the lowest of its minima) for a range of 25 mass values between 0 and 0.5 kg. Your program should write this data to a file in column format.

- The maximum load m, i.e. the value of the mass m at which the two local minima have the same energy. One way to implement this is to use one of the GSL root finding algorithms on a function that gives the difference of the energy values of the two minima, which can be obtained from `f_all_min`.

  The value of the maximum load can be printed out to the console.

As in previous assignments, you should still use git version control and a makefile for your code.
For simplicity, use one single git repository for the assignment.
Please submit:

- All source, header and make files.
- The output of your code.
- The output of 'git log' from the development of your codes.

Submit by February 25, 2016. The late policy can be found in the syllabus.