- **مهلت تحویل:** جمعه، ۳ اسفندماه ۱۴۰۳، تا ساعت ۲۳:۵۹

P1 - To review the basic topics of vector mathematics, rotation matrices, and their properties, answer the following questions.

(a) Does a rotation matrix R(θ) preserve the length (magnitude) of a vector? Prove this property.
(b) Is the dot product of two vectors independent of the reference frame in which they are defined? If yes, prove it.
(c) Prove that a rotation does not alter the distance between points, that is $\|p_1 - p_2\| = \|Rp_1 - Rp_2\|$

P2 - Check if the following matrices can represent a rotation matrix. If they can represent a rotation matrix, calculate the unknown elements and perform the following sections.

$$({}^{A}R_{B})_1 = \begin{bmatrix} 0.5 & x & 0.8624 \\ y & 0.7866 & z \\ -0.7071 & w & 0.3536 \end{bmatrix}, \quad ({}^{A}R_{B})_2 = \begin{bmatrix} -0.1464 & x & 0.5 \\ 0.8536 & 0.5342 & y \\ z & w & 0.7017 \end{bmatrix}$$

(a) If the position vector ${}^{B}P = [0.7071 \quad -0.7071 \quad 0]^T$, define ${}^{A}P$.
(b) If the position vector ${}^{A}P = [0.5774 \quad -0.5774 \quad -0.5774]^T$, define ${}^{B}P$.
(c) Give the equivalent pitch-roll-yaw Euler angles for this rotation.

P3 - Express the correct order of the final rotation matrix R for the procedure below. do not compute the rotation multiplication, express using $R_{i,j}$ in which i and j denote axis of rotation and rotation angle correspondingly.

(1) A rotation of θ about the current x-axis.
(2) A rotation of ϕ about the current z-axis.
(3) A rotation of α about the fixed y-axis.
(4) A rotation of β about the fixed z-axis.
(5) A rotation of δ about the fixed x-axis.

P4 - Consider the rotation of a rigid body using the following quaternion representation

$$\epsilon = [0.149 \quad 0.149 \quad 0.149 \quad 0.966]^T$$

Find the rotation matrix in this case.

P5 - Assume that to reach the coordinate system embedded on one of the robot's joints, we must transform the base coordinate system in a specific sequence. This sequence is defined with ZXY Euler set. Initially, rotate by the angle ψ around the Z-axis, then rotate by the angle ϕ around the X-axis, and finally rotate by the angle θ around the new Y-axis. If the angular velocity is expressed as $[p \quad q \quad r]^T$ in the final coordinate system relative to the base coordinate system, show that:

$$\begin{Bmatrix} p \\ q \\ r \end{Bmatrix} = \begin{pmatrix} cos\theta & 0 & -sin\theta cos\phi \\ 0 & 1 & sin\phi \\ sin\theta & 0 & cos\theta cos\phi \end{pmatrix} \begin{Bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{Bmatrix}$$

P6 - Consider a robot arm mounted on a wheeled mobile platform moving inside a room, with a camera fixed to the ceiling. Frames 2 and 3 are attached to the wheeled platform and the end-effector of the robot arm, respectively, while frame 4 is attached to the camera. Frame 1 is the global reference frame and the objective of the robot is to pick up an object (Frame 5) using its end-effector.

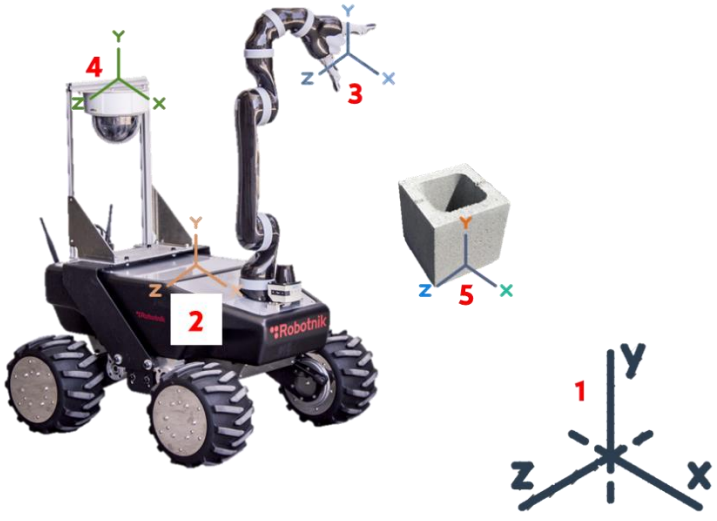

Fig 1- Assignment of reference frames

Given Data:

(1) The transformation matrices T_{42} and T_{45} can be computed from the measurements obtained via the camera.
(2) The transformation matrix T_{23} is derived from the robot arm's joint-angle data.
(3) The transformation matrix T_{14} is assumed to be predetermined and known.

$$T_{42} = {}^{4}T_{2} = \begin{bmatrix} 0 & 0 & -1 & 250 \\ 0 & -1 & 0 & -150 \\ -1 & 0 & 0 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad T_{45} = {}^{4}T_{5} = \begin{bmatrix} 0 & 0 & -1 & 300 \\ 0 & -1 & 0 & 100 \\ -1 & 0 & 0 & 120 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_{23} = {}^{2}T_{3} = \begin{bmatrix} 0 & -1/\sqrt{2} & -1/\sqrt{2} & 30 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} & -40 \\ 1 & 0 & 0 & 25 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad T_{14} = {}^{1}T_{4} = \begin{bmatrix} 0 & 0 & -1 & 400 \\ 0 & -1 & 0 & 50 \\ -1 & 0 & 0 & 300 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Objective:

Using the provided transformation matrices, determine the configuration of the object relative to the robot hand, T_{35}.