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Modelling Meta-Agreement through an Agent Based Model

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Declaration of Authorship

I, Amir Sahrani, declare that this thesis, entitled 'Modelling Meta-Agreement through an Agent Based Model' and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at the University of Amsterdam.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
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Date: February 2025

"The majority, standing in for the people, wills everything and therefore wills nothing"

Joshua Cohen

Abstract

Include your abstract here Abstracts must include sufficient information for reviewers to judge the nature and significance of the topic, the adequacy of the investigative strategy, the nature of the results, and the conclusions. The abstract should summarize the substantive results of the work and not merely list topics to be discussed.

Length 200-400 words.

Acknowledgements

Thank the people that have helped, supervisors family etc.

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Abbreviations

CSL	**C**omputational **S**ceince **L**ab
UvA	**U**niversitiet **v**an **A**msterdam

Symbols

N	The set of all voters
X	The set of all alternatives
$\succ$	A preference relation ship
$\mathcal{D}$	A domain of possible profiles
$\lhd$	A geometric order over candidates

CHAPTER 1

Introduction

Though Black's result [1] is a famous positive result, it is far from the only positive result relating to domain restrictions. We first look into various domain restrictions and their properties. To fully understand why single peakedness is specifically desirable we outline the political and philosophical reasons first, after which we elaborate on the mechanism through which deliberation should result in single peakedness according to List [2]. Finally, for completeness' sake, we mention critiques of this theory.

CHAPTER 2

Preliminaries

We first proceed by giving a short introduction of social choice. We first outline the basic model, as well as establishing the notation for most mathematical objects.

2.1 The basic model

To model election, or more generally voting games, we represent voters by the set N consisting of n voters. The possible outcomes of an election, we represent with the set A consisting of $|A|$ possible outcomes, for convenience we will refer to the outcomes of an election as alternatives. Each voter can represent their preferred outcome through a preference relation $\succcurlyeq_i$, for example if voter 2 prefers outcome a to outcome b, we write $a \succcurlyeq_i b$. If, however, this preference is known to be strict, we instead write $a \succ_i b$. We call the set of all strict preferences over the alternatives $\mathcal{L}(a)$, the set of weak preferences is denoted by $\hat{\mathcal{L}}(a)$. Finally, we need a rule F by which we decide the outcome of the election. We discuss the specifics of these rules in section section 2.2.

Given some profile $\mathcal{L}(a)$ can construct a *majority relationship* as follows, for each pair of alternatives x, y, we ask how many people prefer x to y, if the number of people who prefer x to y is greater than the other way around we write $x \succ_{\text{maj}} y$. We proceed with an example.

EXAMPLE 1: *Majority relation*

1	2	3
a	b	a
b	c	c
c	a	b

Given the profile on the left, we first start by comparing a to b, both voters 1 and 3 prefer a to b, thus the majority has prefers a to b. Comparing b to c, we see again that the majority prefers b to c. Finally, comparing a to c we see a is again preferred. Thus, the majority relation is $a \succ_{\text{maj}} b \succ_{\text{maj}} c$

2.2 Social Choice Functions

In order to decide the outcome of an election, we pick a social choice function F, this function should map all possible profiles to an outcome. More formally we have $F : \mathcal{L}(a)^n \mapsto A$. A famous example of a SCF is the plurality rule, which simply elects the alternatives voted into first place most often. Since the outcome of our SCF is only allowed to be a single alternative, the plurality rule also needs to be equipped with a tie breaking mechanism in order to be a valid SCF, we require the tie-breaking to be deterministic. Non-deterministic voting rules are a part of probabilistic social choice theory and are outside the scope of this work. With all these definitions in place we know can define an election.

2.2.1 Axioms

The axiomatic approach specifies desirable axioms which our voting rule should abide by. One such axiom is the axiom of neutrality, stating that the voting rule should be neutral with respect to the outcomes. In this work three main axioms are of importance.

Surjective A SCF is surjective, if for every alternative, there exists a profile R such that $F(R)$ elects it.

Non-Dictatorial A SCF is non-dicatorial, if there does not exists a voter i such that $F(R) = \text{top}(i, R)$ for all profiles R, where $\text{top}(i, R)$ is the function that extracts voter i's most preferred alternative from profile R.

Strategyproof. A SCF is strategy proof if, for any voter $i \in N$, i cannot report a "false" preference, and thereby cause the outcome of the elective to improve for them. For example, given an election with 3 alternatives a, b, c, a, b are very similar, such that some voters have $a \succ b$ and some have $b \succ a$, but all voters that have $a \succ c$ must also have $b \succ c$ and vice versa. in this case, c could win because the voters are split on whether a or b is better. But given a close enough election, ...

Another way to interpret strategyproofness is that the SCF should ideally maximize the outcome for all voters, as such it is clear that if you report something which is not truly your preference, the outcome would not be better than if you were truthful.

These are some of many possible axioms one could wish their voting rule to satisfy, however, in the next section we show some negative results regarding these axioms.

2.3 Negative results

Classic social choice theory has many negative results, one such example is the Condorcet cycle. This is a specific profile that results in a cycle in the majority relation, as shown in the following example.

EXAMPLE 2: *Condorcet cycle*

1	2	3
a	b	c
b	c	a
c	a	b

Voters 1 and 3 prefer a to b, forming a majority, next voters 1 and 2 prefer b to c, forming another majority. However, voters 2 and 3 prefer c to a forming a majority, and thus creating a cycle.

It is not hard to convinces oneself that under weak preferences the Condorcet cycle can occur anytime there are 3 or more alternatives and voters. While under strict preferences this can occur anytime the number of alternatives is odd and greater than 3, with the number of voters being a multiple of the number of alternatives.

The Condorcet cycle is an example of a broader notion of cyclic profiles. A cyclic profile is any profile in which there exists three alternatives x, y, z such that they form a Condorcet cycle in this profile. A trivial example would be to extend the Condorcet cycle with 1 more alternative which is unanimously ranked last.

One of the major negative results in social choice is that of the Gibbard Satherswaite theorem [3, 4].

Theorem 2.1 (Gibbard-Satherswaite)**.** *There exists no resolute Social Choice Function for elections with $|A| \geq 3$ that is surjective, strategyproof, and non-dictatorial.*

Proof. ... □

2.4 Domain Restrictions

Many negative results are a consequence of a few ill-behaved profiles, if one can argue such profiles do not occur in the real election, there is some hope of constructing

SCF's satisfying our axioms. To speak more formally about profiles "not occuring", we introduce Domain restrictions.

DEFINITION 1: *Domain*

Given a set of voters N, alternatives A, and conditions C, the domain $\mathcal{D}$ of an election is the set of all profiles R such that all conditions C are satisfied.

When we consider the domain of an election, one particular profile is the source of many impossibility results in social choice, namely, the Condorcet cycle.

One property of the majority relation, that is both desirable, yet violated by the Condorcet cycle is that of transitivity. We define the transitivity on the majority relation as follows

DEFINITION 2: *Transitivity*

A (majority) relation is transitive, if for any triplet $a, b, c \in A$, if $a \succ_{\text{maj}} b$ and $b \succ_{\text{maj}} c$, then $a \succ_{\text{maj}} c$.

Clearly this profile presents problems, as each possible outcome, would also have a majority of voters preferring another. Naturally one might consider if this profile might even come up in practice, since though conceivable it seems generally unlikely that there exists a perfect split in opinions. Quite naturally one of the first "solutions" one might consider is when the number of voters is not a multiple of the number of alternatives, though this is hardly a solution, if this is the case, it is in fact possible to pick a winner through a simple rule such as the plurality rule [CITATION NEEDED]. This is the first example of a domain restriction, we define it as follows

DEFINITION 3: $\mathcal{D}_{\text{No-tie}}$

Let X be the set of alternatives and N be the set of voters, of size n such that $n \neq k \cdot |X|$. We call the domain of all outcomes $\mathcal{D}_{\text{No-tie}}$.

This allows us to state our first proposition.

Proposition 2.2. *The plurality rule never returns a $|X|$-way tie between alternatives when applied to $\mathcal{D}_{\text{No-tie}}$*

Proof. Assume, for the sake of contradiction, the plurality in fact does return a tie this must mean that all alternatives were ranked first an equal number of times, call this k, necessarily then, we have need exactly $k \cdot |X|$ voters, but this leads to a contradiction, as this would no longer be inside $\mathcal{D}_{\text{No-tie}}$. □

This is a simple result, but it leads to way to more interesting ones. For this we need to specify more clearly in what ways we can restrict our domains. Gaertner [5] establishes 2 ways in which a domain can be restricted. Firstly we can restrict the domain to a number of voters or alternatives, which is what we did in $\mathcal{D}_{\text{No-tie}}$. Secondly, the domain can be restricted to have a certain structure, such as being single-peaked. Furthermore, Elkind et al. [6] establish the *hereditary*

2.4.1 Single-Peaked profiles

CHAPTER 3

Literature review

Though Black's result [1] is a famous positive result, it is far from the only positive result relating to domain restrictions. We first look into various domain restrictions and their properties. To fully understand why single peakedness is specifically desirable we outline the political and philosophical reasons first, after which we elaborate on the mechanism through which deliberation should result in single peakedness according to List [2]. Finally, for completeness' sake, we mention critiques of this theory.

3.1 Domain Restrictions

A voting domain $\mathcal{D}$ is the domain of all possible voting profiles R given some number of voters N and some number of alternatives $|X|$. Intuitively, this is simple the space of all possible outcomes of some election. Put more formally, we get.

Definition 4: *Domain*

Given a set of voters N, alternatives A, and conditions C, the domain $\mathcal{D}$ of an election is the set of all profiles R such that all conditions C are satisfied.

When we consider the domain of an election, one particular profile is the source of many impossibility results in social choice, namely, the Condorcet cycle. To understand why this profile is problematic, let us first define a notion of aggregation, the *majority relation* is the preference relation we get when we compare all alternatives pairwise, and construct a preference profile from this.

One property of the majority relation, that is both desirable, yet violated by the Condorcet cycle is that of transitivity. We define the transitivity on the majority relation as follows

Definition 5: *Transitivity*

A (majority) relation is transitive, if for any triplet $a, b, c \in A$, if $a \succ_{\text{maj}} b$ and $b \succ_{\text{maj}} c$, then $a \succ_{\text{maj}} c$.

v_1	v_2	v_3
a	b	c
b	c	a
c	a	b

Table 3.1: The Condorcet cycle, showing all alternatives in each position

Clearly this profile presents problems, as each possible outcome, would also have a majority of voters preferring another. Naturally one might consider if this profile might even come up in practice, since though conceivable it seems generally unlikely that there exists a perfect split in opinions. Quite naturally one of the first "solutions" one might consider is when the number of voters is not a multiple of the number of alternatives, though this is hardly a solution, if this is the case, it is in fact possible to pick a winner through a simple rule such as the plurality rule [CITATION NEEDED]. This is the first example of a domain restriction, we define it as follows

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Let X be the set of alternatives and N be the set of voters, of size n such that $n \neq k \cdot |X|$. We call the domain of all outcomes $\mathcal{D}_{\text{No-tie}}$.

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Proposition 3.1. *The plurality rule never returns a $|X|$-way tie between alternatives when applied to $\mathcal{D}_{\text{No-tie}}$*

Proof. Assume, for the sake of contradiction, the plurality in fact does return a tie this must mean that all alternatives were ranked first an equal number of times, call this k, necessarily then, we have need exactly $k \cdot |X|$ voters, but this leads to a contradiction, as this would no longer be inside $\mathcal{D}_{\text{No-tie}}$. □

This is a simple result, but it leads to way to more interesting ones. For this we need to specify more clearly in what ways we can restrict our domains. Gaertner [5] establishes 2 ways in which a domain can be restricted. Firstly we can restrict the domain to a number of voters or alternatives, which is what we did in $\mathcal{D}_{\text{No-tie}}$. Secondly, the domain can be restricted to have a certain structure, such as being single-peaked. Furthermore, Elkind et al. [6] establish the *hereditary*

Definition 7: *Hereditary* (Elkind et al. [6])

A domain restriction onto $\mathcal{D}$ is *hereditary* if, for every profile $P \in \mathcal{D}$, and every profile P', that can be obtained by deleting voters and alternatives from P, P' is also in $\mathcal{D}$

3.1.1 Condorcet Domain

If our goal is to prevent Condorcet cycles, or in general have transitive majority relations, the best we could hope to do is to apply our domain restriction such that our domain contains all profiles P such that P has a (weak) Condorcet winner. We call this domain $\mathcal{D}_{\text{Condorcet}}$. Under this domain, let $f_{Condorcet}$ be the Condorcet Rule, which picks a Condorcet winner. Then $f_{Condorcet}$ is strategyproof over $\mathcal{D}_{\text{Condorcet}}$ [6].

Proof. (Elkind et al. [6]).e Assume, for the sake of a contradiction, we have profiles $P = (\succ_1 \dots \succ_i \dots \succ_n)$ and $P' = (\succ_1 \dots \succ_{i'} \dots \succ_n)$ such that:

$$f_{Condorcet}(P) = a, \quad f_{Condorcet}(P') = b, \quad \text{and } a \neq b$$

Then under P a strict majority $N' \subseteq N$ have $a \succ b$, but $i \notin N'$, thus in P', N' is still a majority preferring a to b, but this is in contradiction to b winning in P'. □

Though this result is positive, $\mathcal{D}_{\text{Condorcet}}$ is not hereditary, this is easy to see through an example:

Example 3: $\mathcal{D}_{\text{Condorcet}}$ *is not hereditary*

v_1	v_2	v_3	v_4
a	b	c	a
b	c	a	c
c	a	b	b

We can see that in this example, a is the weak Condorcet winner, as it beats b and is tied with c, however if we remove voter 4, we return to the original Condorcet cycle.

A domain not being hereditary means that the nice properties of the domain can be unstable, as the number of voters and alternatives might not be known or could be manipulated. Instead, we might want to look at hereditary domains.

3.1.2 Hereditary Domains

The first hereditary domain we present, will also be the main focus of this thesis. This is the domain of all single peaked profiles.

DEFINITION 8: *Single Peaked Profiles*

A profile P is single peaked, if given some ordering $\triangleleft$ over the alternatives, it holds that for all voters i, and all $a, b, c \in X$, if $a \triangleleft b \triangleleft c$, then either $a \succ_i b$ or $c \succ_i b$, but never both.

Note that in a voter is allowed to prefer b best, and then choose a or c in any order. It is clear to see that if any voter or alternative is deleted, this property is satisfied.

Proposition 3.2. (Elkind et al. [6]). *$\mathcal{D}_{\text{SP}}$ is hereditary.*

Proof. (Voter Deletion). If we remove a voter, this does not affect the other voters, thus the property is satisfied. ✓

(Alternative Deletion). Consider a voter i and their single peaked vote, if we remove some alternative x, this voter all alternatives which voter i preferred to x stay in the same position, while all other alternatives move up one rank, thus preserving the order. ✓ □

A similar notion to single peaked profiles is that of single caved profiles, which is equivalent, but instead a voters peak representing their most preferred option, they have a valley, which represents the worst option. Single caved profile are hereditary as well, but for a voting rule on them to be strategy proof, only two possible alternatives can be chosen, the left and right most alternatives according to $\triangleleft$.

Instead of ordering the alternatives, we can imagine instead ordering the voters, such that we have a leftmost and rightmost voters, and all other voters can be placed between them based on their difference. In this case, a profile is single crossing if, for any alternative a, its preference relation to another any alternative b flips at most once when traversing the voters in order $\triangleleft$.

DEFINITION 9: *Single Crossing Profiles* (Elkind et al. [6])

A profile P is single crossing w.r.t. some ordering $\triangleleft$, if for any $a, b\ inX$, $\{i \in N : a \succ b\}$ and $\{i \in N : b \succ b\}$ are both intervals over $[n]$. A profile P is single crossing if the votes can be permuted such that it is single crossing w.r.t. a given ordering.

Similar to single peaked profiles, the domain of single crossing profiles, $\mathcal{D}_{\text{SC}}$ is also hereditary

Proposition 3.3. $\mathcal{D}_{\mathrm{SC}}$ *is hereditary*

Proof. (Voter Deletion). Deleting a voter preserves the ordering between voters, as such this cannot introduce a new crossing between alternatives. ✓

(Alternative Deletion). If we remove an alternative the voters' rankings of the other alternatives does not change, thus preserving single crossing. ✓ □

3.2 The History of Deliberation and Meta-Agreement

We have provided an overview of different domain restriction and their properties, mainly showing how they avoid Condorcet cycles. Some argue however, that Condorcet cycle are empirically rare. The next section is dedicated to explaining why this is so through examining the historical ideas around deliberation and deliberative democracy, as well as that of Meta-Agreement.

3.2.1 Deliberation

Though deliberation is intuitively familiar, namely the process of multiple people talking through a problem with the goal of coming to an agreement, compromise or solution, providing a definition that is both clear and consistent with the literature in Political Science, Philosophy and Social choice is difficult. Though the intuition is completely incorrect, it leaves some of the reasons for and goals of deliberation, as state in the literature, unmentioned.

Freeman [7] gives an overview of deliberative democracy, in which he shares the intuitive idea that a deliberative democracy contains open discussion, open legislative deliberation and a pursuit of the common good. He also notes that there is no common agreement on the central features of a deliberative democracy, one account is that of deliberative democracy simply involving discussion among the public before voting. Another similar account is that this voting must not only be preceded by deliberation, but also general communication, all of which intended to change people's preferences. He further proceeds to give a more detailed conception of deliberative democracy, according to which a deliberative democracy is one in which political agents or their representatives

1. Aim to collect, deliberate and vote
2. Represent their sincere and informed judgements
3. Vote and deliberate on measures beneficial to the common good on the citizens
4. Are seen and see each other as political equals

5. Have Constitutional right and social means enables them to participate in public life
6. Are individually free, such that they have their own freely determined conceptions of the good
7. Have diverse and disagreeing conceptions of the good
8. Recognize and accept their duty as democratic citizens, and do not engage in public argument on the basis of their particular moral views incompatible with public reason.
9. Agree reason is public is so much as it is related to and advances common interests of citizens
10. Agree that their common interest lies primarily in freedom, independence and equal status as citizens.

These features allow us to be more precise when we talk about a deliberative democracy, and in turn be more careful about what deliberation must entail. Cohen [8] further argues that deliberation is needed for democratic legitimacy. By this he means that without deliberation, a democracy is simply the will of the majority, but since majority rule is unstable, it is simply a reflection of the particular institutional constrains at the time. He further goes on to describe the *ideal deliberative procedure* as follows

1. Ideal deliberation is *free*, participants regard themselves as only bound by the results of the deliberation, and the preconditions thereof. Participants act in accordance with the decision made through deliberation, and it being agreed on is sufficient reason to do so.
2. Ideal deliberation is *reasoned*, parties are required to state their reasons for advancing proposals.
3. In ideal deliberation, parties are *equal*, but formally and substantively. There are no rules that single individuals out, and existing distributions of power to no lend a party the opportunity to contribute to deliberation.
4. Ideal deliberation aims to arrive at *consensus*, which can be rationally defended.

3.2.2 Meta-Agreement

A goal of deliberation could be to reach consensus, which is sometimes referred to as substantive agreement, Elster [9] argues that this is not only the goal, but through unanimity this process completely replaces voting, thereby circumventing Arrow's impossibility theorem: "Or rather, there would not be any need for an aggregation mechanism, since a rational discussion would tend to produce unanimous preferences." (p. 112). Though it would be desirable to circumvent Arrow's impossibility theorem, in practice people, even after deliberation might not, indeed often do not, come to full substantive agreement. List [2] instead proposed another lens through which we can analyze deliberation and the type of agreement it induces.

Under *Meta-agreement* individuals do not need to agree on the their most preferred outcome, instead they only need to agree on the dimensions of the problem. To contrast this with substantive agreement, under which individuals do not need to conceive of the problem in the same way, all they need is to agree on the same outcome. This means that under substantive agreement, voters can agree outcome $a \succ b$ for different reasons, while under meta-agreement, if voters disagree on $a \succ b$ it must be for the same reason.

According to List [2] there are three hypothesis that need to be satisfied for deliberation to induce meta-agreement:

- D1 Deliberation leads people to discover a single *issue*-dimension
- D2 Deliberation lets people place all possible alternatives in this *issue*-dimension
- D3 After this deliberation, people update their preferences by picking a preferred (peak) outcome, and all other rankings are based on structure of the *issue*-dimension

All these are necessary conditions for meta-agreement, from this is it also clear to see that, given that there is exactly 1 *issue*-dimension, single peaked profile are, by definition, a direct consequence. This is the main reason meta-agreement is desirable, as it lets us circumvent the Gibberd- Satterthwaite theorem ([3] [4])through restricting the domain of preference profiles to the single peaked domain $\mathcal{D}_{\mathrm{SP}}$

List et al. [10] provide empirical evidence for this theory of deliberation, showing deliberation increases proximity to single peakedness, which they define as $S = \frac{m}{n}$ where $m = |M|$ is the largest subset of voters such that their profile is single peaked. Furthermore, they also introduce the notion of salience, which represents to what extent a topic is salient in the voting population. In order to test whether deliberation increases single-peakedness *through* meta-agreement, they test the following four hypotheses: (H1) deliberation increases proximity to single peakedness. (H2')[1] high salience issues show less increase in PtS then low salience issues. (H3) Effective deliberation, in the sense that more is learned during deliberation, results in bigger increases. (H4) All things equal, the increase is largest for issues with natural *issue*-dimensions.

Meta-agreement is not without its critiques, however. Ottonelli and Porello [11] show meta-agreement to be much strong of a requirement than it may seem at a first glance. Firstly for (D1) to hold, the *issue*-dimension must hold some semantic meaning, as

[1]This is a test for a corollary. H2 states that the rate of increase of proximity to single peakedness decreases. Since high salience means some sort of deliberation has happened before, we expect this to have the same affect.

otherwise it unclear how people can exchange conceptualization of the problem otherwise. Furthermore, the issues must consist of 2 semantic issue, otherwise with only 1 dimension voters simply reach substantive agreement. A further restriction on these two dimensions is that they need to be opposite, with opposite justifications. If this is not the case, a voter can agree with both justifications, and thereby introduce a new dimension "balance", which then violates the conditions under which single peaked profiles guarantee the existence of strategyproof voting rules. D2 requires that all voters share the exact same semantic understand of the dimension, and the outcome associated with each alternative. Finally D3 requires D1 and D2 to have happened before in order. Clearly D3 is the weakest of the three.

Thus, meta-agreement is still quite restrictive, needing multiple forms of unanimity, and only applying to problems with certain properties. Nonetheless, in this work we investigate its explanatory power on prevention of Condorcet cycles.

3.3 Related Work

Rad and Roy [12] model deliberation and its effect on single peakedness, though they argue single plateauedness is a more accurate term. To this end, they model each voter to have preferences order, and deliberation being the process of all voters announcing their preferences, after which all other voters update their current preference towards that of the announced ranking, in doing so they might have a bias towards their own preference, as such they try to minimize the distance between their current preference and the announced one. This process repeats until all voters have announced their opinion once, and possibly occurs for multiple rounds. The preference a voter adopts when updating must lie between their current profile and the announced profile, which profiles are considered to be "between" is define by the distance metric used. They considered three metrics, the Kemeny-Snell (KS) [13], Duddy-Piggins (DP) [14], and Cook-Seiford (CS) [15]. Both KS and DP depend on the judgement set resulting from the voters preferences, the KS distance is then defined as the number of binary swaps a judgement set needs to undergo before it becomes the target judgement set, an example for such a swap would be going from $(a \succ b)$ to $\neg(a \succ b)$. The DP distance is defined on the graph of judgement sets, where 2 sets share an edge if there is no judgement set between them. Since KS and DP share their notion of betweenness, we introduce betweenness as follows.

Definition 10: *J-Betweenness*

A judgement set J_i is between profiles J_j and J_k if for all $\succ \in J_i \quad \succ \in J_j \vee \succ \in J_k$

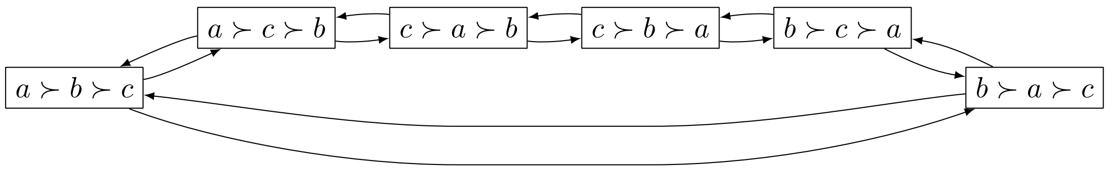

FIGURE 3.1: The graph of judgement sets for all preferences over three alternatives.

Figure 3.1 shows a graph used for the DP distance in the case of 3 alternatives, for simplicity the associated profiles are used to label the judgement sets.

The CS distance is simpler and is simply defined as the number of positions two voters disagree on, and a profile is between two others if for each position it agrees with one of the two profiles.

Each distance has different trade-offs, CS is the simplest, but might exaggerate the distance when there are many alternatives, for example if 2 voters agree on the relative ranking of all but 1 alternative, which one voter happens to rank first, thereby shifting all other profiles right. The KS distance, using judgement sets instead of raw profiles captures this more effectively, while still being relatively easy to compute, but in cases of more disagreement, it is likely to over count the distance, since the binary changes to not capture logical necessities. For example, swapping $(a \succ b)$ to $\neg(a \succ b)$ must result in $(a \succ b)$ becoming true (in the case of strict preferences), thus one might reasonably conclude this should only count as 1 step. DP improves upon this, but in doing so becomes much harder to compute, mainly through the cost of constructing the full graph of judgement sets, which grows in $\mathcal{O}(n!)$ in the number of vertices, where n is the number of alternatives.

CHAPTER 4

METHODS

We determine whether our profiles are single peaked using the algorithm by ..., implemented in `Ocaml` as follows

```
let agent = 1 in
let text = "Testing" in
(* This is a comment *)
print_int agent
```

CHAPTER 5

Experiments and results

CHAPTER 6

Discussion

CHAPTER 7

Conclusion and future work

CHAPTER 8

Ethics and Data Management

A new requirement for the thesis is that there must be a short section in which you reflect on the ethical aspects of your project. This requirement is related to one of the final objectives that a graduated student of the Master of Computational Science must meet: “The graduate of the program has insight into the social significance of Computational Science and the responsibilities of experts in this field within science and in society”. You don’t need to devote an entire chapter to this; a short section or paragraph is sufficient.

I acknowledge that the thesis adheres to the ethical code (https://student.uva.nl/en/topics/ethics-in-research) and research data management policies (https://rdm.uva.nl/en) of UvA and IvI.

The following table lists the data used in this thesis (including source codes). I confirm that the list is complete and the listed data are sufficient to reproduce the results of the thesis. If a prohibitive non-disclosure agreement is in effect at the time of submission ”NDA” is written under ”Availability” and ”License” for the concerned data items.

Short description (max. 10 words)	**Availability (e.g., URL, DOI)**	**License (e.g., MIT, GPL, Creative Commons)**
Example dataset 1	<github url>or Figshare	GPL
Example source code	DOI (from Zenodo)	MIT
Example sensitive data	NDA	NDA

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