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K. N. Toosi University of Technology

# Robotics Exam

*Chalmers Phd examination*

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# Contents

<b>1 Abstract</b>	<b>3</b>
<b>2 Mechanics</b>	<b>4</b>
Topology and Mobility Analysis .....	4
2.0.1 Exercise 1 . . . . .	4
2.0.2 Exercise 2 . . . . .	8
Exercise 2 . . . . .	8
2.0.3 Exercise 3 . . . . .	10
Exercise 3 . . . . .	10
Geometry and Kinematics .....	13
2.0.4 Exercise 4 . . . . .	13
2.0.5 Exercise 5 . . . . .	19
2.0.6 Exercise 6 . . . . .	20
Dynamics .....	20
2.0.7 Exercise 7 . . . . .	20
2.0.8 Exercise 8 . . . . .	21
2.0.9 Exercise 9 . . . . .	21
<b>3 Control</b>	<b>21</b>
Exercise 10 .....	22
Exercise 11 .....	22
Exercise 12 .....	22
<b>4 Questionnaire</b>	<b>22</b>
Question 1 .....	23
Question 2 .....	24
Question 3 .....	24
Question 4 .....	24
Question 5 .....	25
Question 6 .....	25
Question 7 .....	25
Question 8 .....	25
Question 9 .....	26
Question 10 .....	26
<b>References</b>	<b>27</b>

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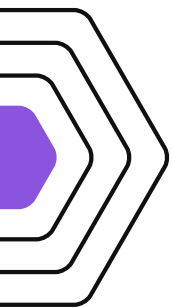
# Abstract

This document contains some basic exercises that evaluate your suitability for working in my research group at Chalmers. Please provide your solutions in the form of PDF (we highly appreciate LaTeX skills) and also working code (wherever asked) in a language of your choice (preferably C++, Matlab, Ruby, Julia or Python1 ).

## Contents

<b>Topology and Mobility Analysis .....</b>	<b>4</b>
2.0.1 Exercise 1 . . . . .	4
2.0.2 Exercise 2 . . . . .	8
Exercise 2 . . . . .	8
2.0.3 Exercise 3 . . . . .	10
Exercise 3 . . . . .	10
<b>Geometry and Kinematics .....</b>	<b>13</b>
2.0.4 Exercise 4 . . . . .	13
2.0.5 Exercise 5 . . . . .	19
2.0.6 Exercise 6 . . . . .	20
<b>Dynamics.....</b>	<b>20</b>
2.0.7 Exercise 7 . . . . .	20
2.0.8 Exercise 8 . . . . .	21
2.0.9 Exercise 9 . . . . .	21

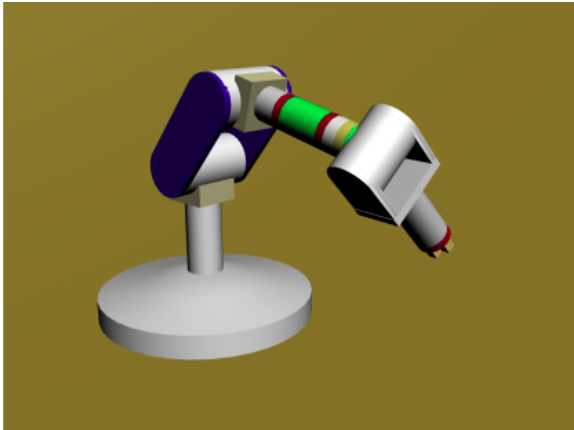
## Mechanics



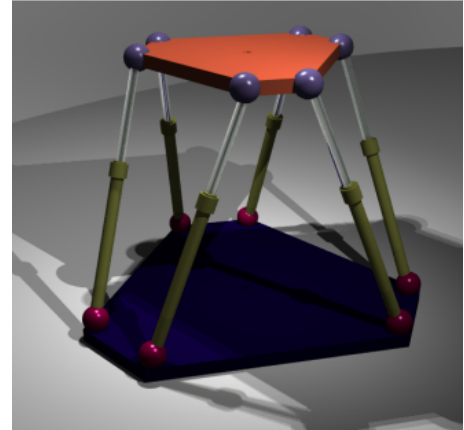
# Topology and Mobility Analysis

### 2.0.1 Exercise 1

In this exercise, we will analyze the topology and mobility of two different robot configurations: a 6-DOF serial robot and a parallel robot (Stewart platform). Figure 2.1 shows a comparison of these two types of robots.



(a) 6-DOF Serial Robot



(b) Parallel Robot

Figure 2.1: Comparison of 6-DOF Serial Robot and Parallel Robot

Let's start by analyzing the 6-DOF serial robot in more detail. Figure 2.2 illustrates the joint axes and links of this robot.

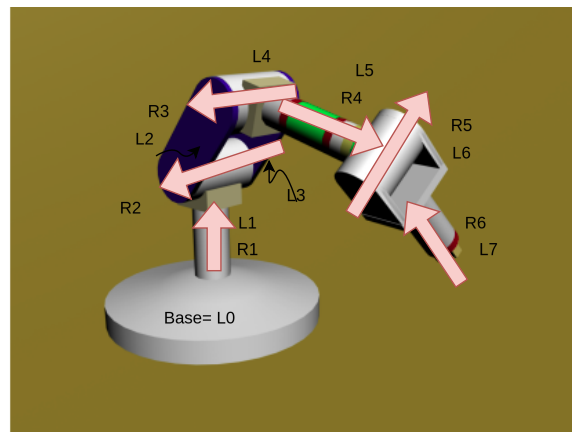


Figure 2.2: Joint axes and links of the 6-DOF serial robot

### **Solution:** 8-Link Serial Robot

The serial robot described in Figure 2.1a and detailed in Figure 2.2 is a typical example of an industrial robotic arm. It consists of the following components:

- **Base ( $L_0$ ):**
  - This is the fixed part of the robot, usually mounted on the ground or a stable platform.
  - It serves as the reference frame for the entire robot structure.
- **Seven Moving Links ( $L_1$  to  $L_7$ ):**
  - These are the rigid bodies that make up the robot's arm.
  - Each link connects to the next via a joint, forming a chain-like structure.
  - $L_7$  is typically the end-effector or the link to which a tool or gripper is attached.
- **Revolute Joints ( $J_0$  to  $J_6$ ):**

- These joints connect the links and allow rotational motion between them.
- Each joint has one degree of freedom, allowing rotation around a single axis.
- The joints are typically labeled  $J_0$ ,  $J_1$ ,  $J_2$ ,  $J_3$ ,  $J_4$ ,  $J_5$ , and  $J_6$ .

- **Connectivity:**

- Base ( $L_0$ ) is connected to  $L_1$  via  $J_0$
- $L_1$  is connected to  $L_2$  via  $J_1$  and to  $L_3$  via  $J_2$
- $L_2$  is connected to  $L_4$  via  $J_3$
- $L_3$  is connected to  $L_4$  via  $J_4$
- $L_4$  is connected to  $L_5$  via  $J_5$
- $L_5$  is connected to  $L_6$  via  $J_6$
- $L_6$  is connected to  $L_7$  via  $J_7$

- **Kinematic Chain:**

- The robot forms an open kinematic chain, starting from the base and ending at the end-effector.
- Each joint adds one degree of freedom to the robot.

- **Degrees of Freedom:**

- With seven revolute joints, this robot has 7 degrees of freedom (7-DOF).
- This allows the end-effector to achieve any position and orientation within its workspace.

### Connectivity Graph

Based on Featherstone's [3] definition, the connectivity graph for this robot represents each link as a node and each joint as an edge connecting the nodes, clearly showing the serial nature of the robot's structure with parallel connections.

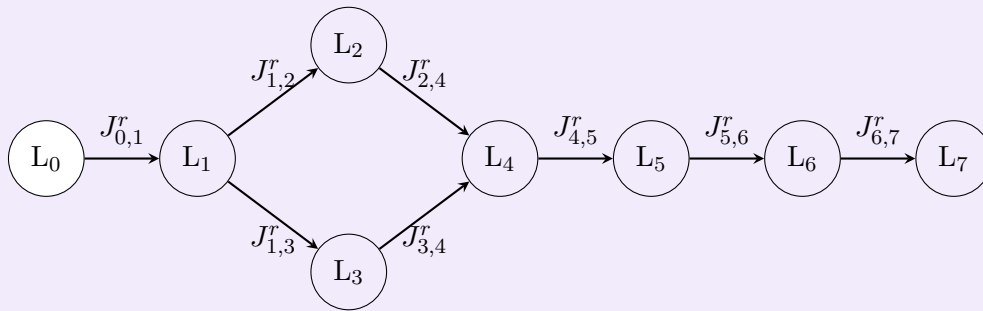


Figure 2.3: Connectivity graph for an 8-link serial robot with parallel connections

In this topological graph:

- Nodes (circles) represent links  $L_0$  to  $L_7$ .
- Edges (arrows) represent joints  $J_0$  to  $J_7$ .
- The white node ( $L_0$ ) represents the fixed base.
- The graph clearly shows the serial chain structure of the robot with parallel connections.

**Solution:** Connectivity graph for the 6-SPS parallel robot (Stewart platform)

Now, let's analyze the connectivity of the parallel robot (Stewart platform) shown in Figure 2.1b. Figure 2.4 illustrates the connectivity graph for this 6-SPS parallel robot.

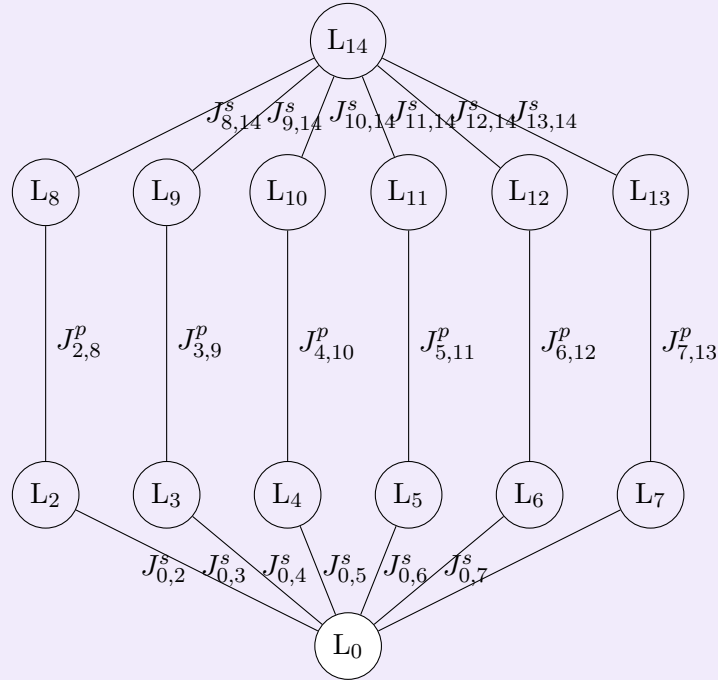


Figure 2.4: Connectivity graph for the 6-SPS parallel robot (Stewart platform)

**Connectivity Graph:** The graph represents the structure of a 6-SPS (Spherical-Prismatic-Spherical) parallel robot, also known as a Stewart platform.

**Components:**

- **Base ( $L_0$ ):** Represented by the white node at the bottom.
- **End-Effector ( $L_{14}$ ):** Represented by the top node, it's the moving platform.
- **Legs:** Six kinematic chains, each composed of two links:
  - Lower links:  $L_2$  to  $L_7$
  - Upper links:  $L_8$  to  $L_{13}$

**Joints:**

- **Spherical Joints (S):**
  - Base to lower links:  $J_{0,i}^s$  where  $i = 2, 3, \dots, 7$
  - Upper links to End-Effector:  $J_{j,14}^s$  where  $j = 8, 9, \dots, 13$
- **Prismatic Joints (P):**
  - Between lower and upper links:  $J_{i,j}^p$  where  $i = 2, 3, \dots, 7$  and  $j = i + 6$

**Structure:** Each of the six legs follows an SPS (Spherical-Prismatic-Spherical) configuration, connecting the base to the end-effector. This arrangement provides the robot with 6 degrees of freedom (3 translational and 3 rotational).

### 2.0.2 Exercise 2

The Chebyshev–Grübler–Kutzbach criterion estimates the degree of freedom (DOF) of a kinematic chain, that is, a coupling of rigid bodies by means of mechanical constraints. The general mobility of a robot can be estimated by the following criteria [7]:

$$F = \lambda(n - j - 1) + \sum_{i=1}^j f_i - f_p, \quad (2.1)$$

where

- $F$  – degrees-of-freedom of the mechanism
- $\lambda$  – degree-of-freedom of the space (= 3 for planar and spherical mechanisms, = 6 for spatial mechanisms)
- $n$  – number of links in the mechanism including the base
- $j$  – number of binary joints of the mechanism
- $f_i$  – degrees of relative motion permitted by joint  $i$
- $f_p$  – total number of passive degrees-of-freedom

#### Solution:

Let's compute the mobility of the two robots shown in Figure 2.1 using Equation 2.1.

#### 1. 6-DOF Serial Robot

For the serial robot (Figure 2.1a):

- $\lambda = 6$  (spatial mechanism)
- $n = 8$  (7 moving links + 1 base)
- $j = 7$  (7 revolute joints)
- $\sum_{i=1}^j f_i = 7$  (1 DOF per revolute joint)
- $f_p = 0$  (no passive degrees-of-freedom)

Applying Equation 2.1:

$$\begin{aligned} F &= \lambda(n - j - 1) + \sum_{i=1}^j f_i - f_p \\ &= 6(8 - 7 - 1) + 7 - 0 \\ &= 6(0) + 7 \\ &= 7 \end{aligned}$$

The mobility of the 6-DOF serial robot is 7, which matches our earlier analysis.

#### 2. 6-SPS Parallel Robot (Stewart Platform)

For the parallel robot (Figure 2.1b):



- $\lambda = 6$  (spatial mechanism)
- $n = 14$  (12 leg links + 1 base + 1 platform)
- $j = 18$  (12 spherical joints + 6 prismatic joints)
- $\sum_{i=1}^j f_i = 42$  (3 DOF per spherical joint  $\times 12$  + 1 DOF per prismatic joint  $\times 6$ )
- $f_p = 6$  (1 passive DOF per leg, rotating around its axis)

Applying Equation 2.1:

$$\begin{aligned}
 F &= \lambda(n - j - 1) + \sum_{i=1}^j f_i - f_p \\
 &= 6(14 - 18 - 1) + 42 - 6 \\
 &= 6(-5) + 42 - 6 \\
 &= -30 + 42 - 6 \\
 &= 6
 \end{aligned}$$

The calculated mobility of the 6-SPS parallel robot is 6, which is correct.

### Simplified Formula for Serial Robots

For serial robots, we can simplify Equation 2.1 because:

- The number of joints is always one less than the number of links ( $j = n - 1$ )
- There are typically no passive degrees-of-freedom ( $f_p = 0$ )

Substituting these into Equation 2.1:

$$\begin{aligned}
 F &= \lambda(n - j - 1) + \sum_{i=1}^j f_i - f_p \\
 &= \lambda(n - (n - 1) - 1) + \sum_{i=1}^j f_i - 0 \\
 &= \lambda(0) + \sum_{i=1}^j f_i \\
 &= \sum_{i=1}^j f_i
 \end{aligned}$$

Therefore, the simplified formula for serial robots is:

$$F = \sum_{i=1}^j f_i \quad (2.2)$$

This means that for serial robots, the mobility is simply the sum of the degrees of freedom of all joints.

### Application to Parallel Mechanisms

The extended Chebyshev–Grübler–Kutzbach criterion (Equation 2.1) works accurately for parallel mechanisms when we account for passive degrees-of-freedom. In the case of the Stewart platform:

1. Each spherical joint contributes 3 DOF to  $\sum_{i=1}^j f_i$ . 2. Each prismatic joint contributes 1 DOF to  $\sum_{i=1}^j f_i$ . 3. Each leg has 1 passive DOF (rotation around its axis), contributing to  $f_p$ .

By including these passive degrees-of-freedom in our calculation, we obtain the correct mobility of 6 for the Stewart platform without needing additional simplifications or corrections.

This demonstrates the importance of considering passive degrees-of-freedom when analyzing the mobility of complex parallel mechanisms.

### 2.0.3 Exercise 3

It seems almost magical that a simple formula like the Chebyshev–Grübler–Kutzbach criterion can be used to estimate the general mobility of a system. Does it always work? Can you find some counter-examples where this formula fails?

#### Solution:

While the Chebyshev–Grübler–Kutzbach criterion is a powerful tool for estimating the mobility of many mechanical systems, it does not always work correctly. This paper [4] addresses most of mechanisms that the mobility criteria failed to predict their DOF. Here we state several situations where the formula can fail to accurately predict the degrees of freedom of a mechanism. Let's explore some of these cases:

#### 1. Overconstrained Mechanisms

Some mechanisms are overconstrained yet still mobile due to special geometric conditions. The Chebyshev–Grübler–Kutzbach criterion often predicts zero or negative mobility for these systems, even though they can move.

**Example: Bennett's Linkage**

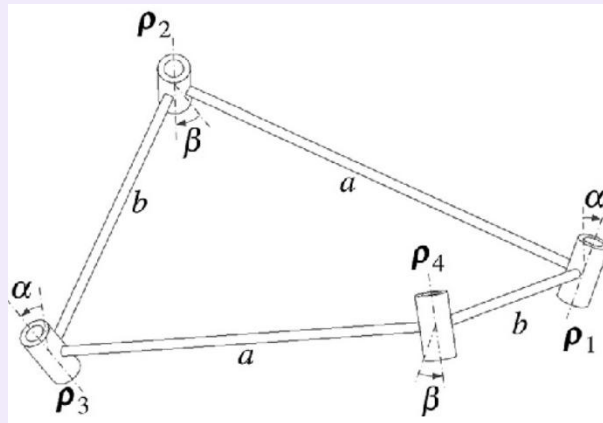


Figure 2.5: Bennett's Linkage [5]

Bennett's linkage is a spatial 4-bar linkage with one degree of freedom. However, applying the Chebyshev–Grübler–Kutzbach criterion:

- $\lambda = 6$  (spatial mechanism)
- $n = 4$  (4 links)
- $j = 4$  (4 revolute joints)
- $\sum_{i=1}^j f_i = 4$  (1 DOF per revolute joint)
- $f_p = 0$  (no passive DOF)

$$\begin{aligned}
 F &= \lambda(n - j - 1) + \sum_{i=1}^j f_i - f_p \\
 &= 6(4 - 4 - 1) + 4 - 0 \\
 &= 6(-1) + 4 \\
 &= -2
 \end{aligned}$$

The formula predicts -2 DOF, but the mechanism actually has 1 DOF due to its special geometry.

## 2. Mechanisms with Redundant Constraints

Some mechanisms have redundant constraints that don't affect mobility but are counted in the formula.

**Example: Parallel Manipulator with Redundant Actuation** Consider a planar parallel manipulator with three legs, each containing an actuated prismatic joint, where only two actuators are needed for full mobility.

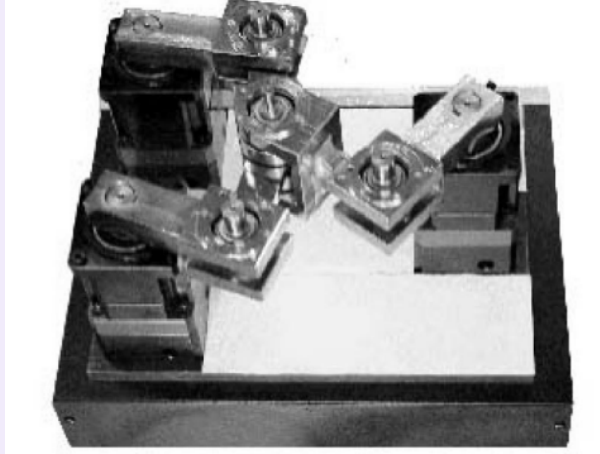


Fig. 6. Two-DOF redundantly actuated parallel manipulator.

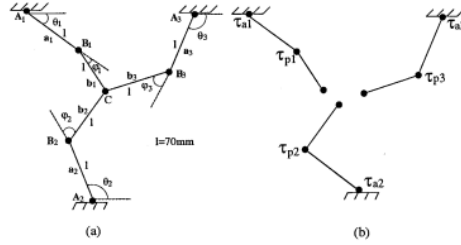


Fig. 7. Two-DOF redundant parallel mechanism and its equivalent open-chain system.

Figure 2.6: Two-DOF redundantly actuated parallel manipulator [2]

Applying the formula:

- $\lambda = 3$  (planar mechanism)
- $n = 8$  (1 base, 1 platform, 6 leg links)
- $j = 9$  (3 prismatic + 6 revolute joints)
- $\sum_{i=1}^j f_i = 9$  (1 DOF per joint)
- $f_p = 0$  (no passive DOF)

$$\begin{aligned}
 F &= \lambda(n - j - 1) + \sum_{i=1}^j f_i - f_p \\
 &= 3(8 - 9 - 1) + 9 - 0 \\
 &= 3(-2) + 9 \\
 &= 3
 \end{aligned}$$

The formula predicts 3 DOF, which is correct, but it doesn't account for the redundant actuation.

### 3. Mechanisms with Special Configurations

Some mechanisms can change their mobility in certain configurations, which the formula doesn't capture.

**Example: Bricard's Flexible Octahedron [1]** : This mechanism can transition between 0 and 1 DOF depending on its configuration, but the Chebychev–Grübler–Kutzbach criterion always predicts the same mobility.

### 4. Mechanisms with Higher Pair Joints

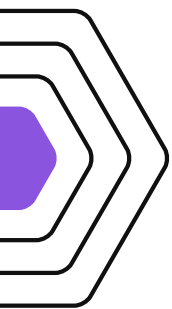
The formula assumes lower pair joints (e.g., revolute, prismatic). It may not accurately predict mobility for mechanisms with higher pair joints (e.g., cam-follower systems).

### Conclusion

While the Chebychev–Grübler–Kutzbach criterion is a valuable tool for initial mobility analysis, it has limitations. It's important to consider:

- Special geometric conditions
- Redundant constraints
- Configuration-dependent mobility
- The nature of the joints involved

In complex or unusual mechanisms, additional analysis methods (e.g., screw theory, instantaneous kinematics) may be necessary to accurately determine mobility.



## Geometry and Kinematics

### 2.0.4 Exercise 4

A serially connected 3 DOF robotic leg with 2 orthogonally intersecting revolute joints and a 1 DOF prismatic joint is shown in Figure 2.7. Given  $q_1$  and  $q_2$  denote the joint angles in the first two revolute joints and  $q_3$  denote the linear displacement in the prismatic joint:

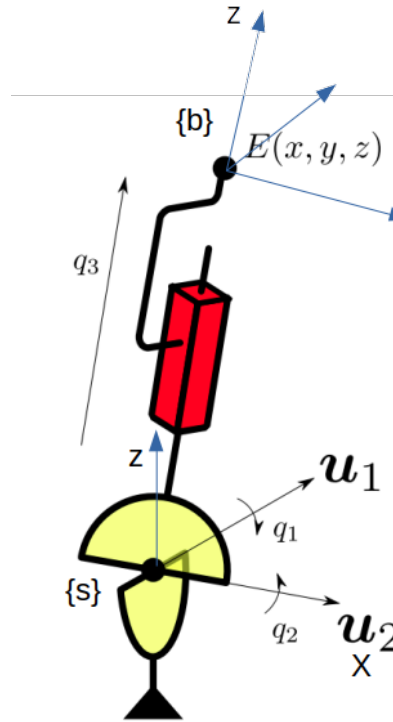


Figure 2.7: A 3 DOF robotic leg with 2 revolute and 1 prismatic joints

### Solution:

#### 1. Geometric object where the end-effector point $E$ lives

The end-effector point  $E$  lives in a three-dimensional Euclidean space,  $\mathbb{R}^3$ . More specifically, it traces out a subset of  $\mathbb{R}^3$  that forms the workspace of the robot.

#### 2. Forward Kinematics using Product of Exponentials (PoE)

Based on Kevin-Lynch's Modern Robotics book [6] the forward kinematics for our 3 DOF robotic leg can be derived using the Product of Exponentials (PoE) formula. This method offers an elegant and intuitive approach, particularly suitable for open kinematic chains like our robot leg. Let's go through the process step-by-step:

##### Step 1: Define Frames

- **Space Frame  $\{s\}$ :** Fixed at the base of the robot.
- **Body Frame  $\{b\}$ :** Attached to the end-effector (point  $E$ ).

##### Step 2: Zero Configuration

Set all joint variables  $(q_1, q_2, q_3)$  to zero. In this configuration, the transformation matrix  $M \in SE(3)$  from  $\{s\}$  to  $\{b\}$  is:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.3)$$

### Step 3: Determine Screw Axes

For our robot, we need to determine the screw axes for each joint  $(\omega_i^T, v_i^T)^T$ . The screw axis is represented by two vectors:  $\omega_i$  (axis of rotation or translation) and  $v_i$  (linear velocity). Let's examine how we calculate  $v_i$  for each type of joint:

**Revolute Joints (Joints 1 and 2):** For a revolute joint,  $v_i$  is calculated using the formula:

$$v_i = -\omega_i \times q_i \quad (2.4)$$

where  $q_i$  is any point on the joint axis. This formula comes from the fact that the linear velocity of any point on a rigid body rotating about an axis is given by the cross product of the angular velocity vector and the position vector from any point on the axis to the point in question.

- **Joint 1:** The axis of rotation is along the y-axis, passing through the origin.

$$\begin{aligned} \omega_1 &= [0, 1, 0]^T \\ q_1 &= [0, 0, 0]^T \text{ (we can choose the origin as our point)} \\ v_1 &= -\omega_1 \times q_1 = -[0, 1, 0]^T \times [0, 0, 0]^T = [0, 0, 0]^T \end{aligned}$$

- **Joint 2:** The axis of rotation is along the x-axis, passing through the origin  $[0, 0, 0]$ .

$$\begin{aligned} \omega_2 &= [1, 0, 0]^T \\ q_2 &= [0, 0, 0]^T \\ v_2 &= -\omega_2 \times q_2 = -[1, 0, 0]^T \times [0, 0, 0]^T = [0, 0, 0]^T \end{aligned}$$

**Prismatic Joint (Joint 3):** For a prismatic joint,  $\omega_i$  is zero (no rotation), and  $v_i$  is simply a unit vector in the direction of positive translation.

- **Joint 3:** The prismatic joint moves along the z-axis of the space frame after the second rotation.

$$\begin{aligned} \omega_3 &= [0, 0, 0]^T \\ v_3 &= [0, 0, 1]^T \text{ (unit vector in the direction of translation)} \end{aligned}$$

This calculation of  $v_i$  ensures that the screw axis correctly represents the motion of each joint, whether it's a rotation around an axis (revolute) or a translation along an axis (prismatic).

#### Step 4: Screw Axis Matrices (Extended Explanation)

In the Product of Exponentials (PoE) formula, we represent each joint's motion using a 4x4 matrix  $[S_i] \in se(3)$ , where  $se(3)$  is the Lie algebra of the Special Euclidean group  $SE(3)$ . This matrix encapsulates both the rotational and translational components of the joint's motion.

The general form of the screw axis matrix  $[S_i]$  is:

$$[S_i] = \begin{bmatrix} [\omega_i] & v_i \\ 0 & 0 \end{bmatrix} \quad (2.5)$$

where  $[\omega_i]$  is the 3x3 skew-symmetric matrix representation of  $\omega_i$ , and  $v_i$  is the 3x1 linear velocity vector we calculated in Step 3.

For a vector  $\omega = [\omega_1, \omega_2, \omega_3]^T$ , its skew-symmetric matrix representation is:

$$[\omega] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \quad (2.6)$$

Now, let's calculate  $[S_i]$  for each joint:

**Joint 1 (Revolute):**  $\omega_1 = [0, 1, 0]^T$ ,  $v_1 = [0, 0, 0]^T$

$$[S_1] = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.7)$$

**Joint 2 (Revolute):**  $\omega_2 = [1, 0, 0]^T$ ,  $v_2 = [0, 0, 0]^T$

$$[S_2] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.8)$$

**Joint 3 (Prismatic):**  $\omega_3 = [0, 0, 0]^T$ ,  $v_3 = [0, 0, 1]^T$

$$[S_3] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (2.9)$$

**Interpretation:** - For revolute joints (1 and 2), the upper-left 3x3 submatrix represents the axis of rotation, while the upper-right 3x1 vector represents the moment of the axis. - For the prismatic joint (3), the upper-left 3x3 submatrix is zero

#### Step 5: PoE Formula

The forward kinematics is given by:

$$T(q_1, q_2, q_3) = e^{[S_1]q_1} e^{[S_2]q_2} e^{[S_3]q_3} M \quad (2.10)$$



### Step 6: Compute Matrix Exponentials

$$\begin{aligned}
 e^{[S_1]q_1} &= \begin{bmatrix} \cos q_1 & 0 & \sin q_1 & 0 \\ 0 & 1 & 0 & 0 \\ -\sin q_1 & 0 & \cos q_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 e^{[S_2]q_2} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos q_2 & -\sin q_2 & 0 \\ 0 & \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
 e^{[S_3]d} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

### Step 7: Final Forward Kinematics

Multiplying these matrices together, we get the final transformation:

$$T(q_1, q_2, q_3) = \begin{bmatrix} \cos q_1 & \sin q_1 \sin q_2 & \sin q_1 \cos q_2 & \cos q_2 \sin q_1 (L_0 + q_3) \\ 0 & \cos q_2 & -\sin q_2 & -\sin q_2 (L_0 + q_3) \\ -\sin q_1 & \cos q_1 \sin q_2 & \cos q_1 \cos q_2 & \cos q_2 \cos q_1 (L_0 + q_3) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2.11)$$

The position of the end-effector  $(x, y, z)$  can be extracted from the last column of this matrix:

$$\begin{aligned}
 x &= \cos q_2 \sin q_1 (L_0 + q_3) \\
 y &= -\sin q_2 (L_0 + q_3) \\
 z &= \cos q_2 \cos q_1 (L_0 + q_3)
 \end{aligned}$$

This result matches our earlier geometric derivation, validating both approaches.

### Advantages of PoE over Denavit-Hartenberg (D-H)

- No need to define individual link frames
- Uniform treatment of revolute and prismatic joints
- More intuitive geometric interpretation via screw axes
- Particularly advantageous for open kinematic chains like our robot leg

The PoE method provides a systematic and elegant approach to deriving forward kinematics, offering both mathematical rigor and geometric intuition.

### 3. Inverse Kinematics

The inverse kinematics expression  $(q_1, q_2, q_3) = f^{-1}(x, y, z)$  can be derived as follows:

$$\begin{aligned} q_1 &= \arctan 2(y, x) \\ q_3 &= \sqrt{(x^2 + y^2 - L_1^2) + (z - L_0)^2} \\ q_2 &= \arctan 2(z - L_0, \sqrt{x^2 + y^2 - L_1^2}) \end{aligned}$$

Note that this solution assumes  $x^2 + y^2 \geq L_1^2$  to ensure a real solution exists.

### 4. Program to verify forward and inverse kinematics

Here's a Python program that verifies the forward and inverse kinematics and provides a simple visualization:

```
import numpy as np
import matplotlib.pyplot as plt
from mpl_toolkits.mplot3d import Axes3D

L0 = 1.0 # Height of first joint
L1 = 0.5 # Length of first link

def forward_kinematics(q1, q2, q3):
    x = np.cos(q1) * (L1 + q3 * np.cos(q2))
    y = np.sin(q1) * (L1 + q3 * np.cos(q2))
    z = L0 + q3 * np.sin(q2)
    return x, y, z

def inverse_kinematics(x, y, z):
    q1 = np.arctan2(y, x)
    q3 = np.sqrt((x**2 + y**2 - L1**2) + (z - L0)**2)
    q2 = np.arctan2(z - L0, np.sqrt(x**2 + y**2 - L1**2))
    return q1, q2, q3

# Verify for random poses
for _ in range(5):
    q1, q2, q3 = np.random.uniform(0, 2*np.pi), np.random.uniform(-np.pi/2,
    x, y, z = forward_kinematics(q1, q2, q3)
    q1_inv, q2_inv, q3_inv = inverse_kinematics(x, y, z)
    print(f"Original: □q1={q1:.2 f}, □q2={q2:.2 f}, □q3={q3:.2 f}")
    print(f"Inverse: □□q1={q1_inv:.2 f}, □q2={q2_inv:.2 f}, □q3={q3_inv:.2 f}")
    print(f"Position: □x={x:.2 f}, □y={y:.2 f}, □z={z:.2 f}\n")

# Visualization
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')
```

```

q1, q2, q3 = np.random.uniform(0, 2*np.pi), np.random.uniform(-np.pi/2,
x, y, z = forward_kinematics(q1, q2, q3)

ax.plot([0, 0, x], [0, 0, y], [0, L0, z], 'r-')
ax.scatter([x], [y], [z], c='b', s=100)

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('3-DOF Robotic Leg')
plt.show()

```

## 5. Workspace plot

To plot the workspace, we can sample the joint space and plot the resulting end-effector positions:

```

# Workspace plot
fig = plt.figure()
ax = fig.add_subplot(111, projection='3d')

q1_range = np.linspace(0, 2*np.pi, 50)
q2_range = np.linspace(-np.pi/2, np.pi/2, 50)
q3_range = np.linspace(0, 1, 10)

for q1 in q1_range:
    for q2 in q2_range:
        for q3 in q3_range:
            x, y, z = forward_kinematics(q1, q2, q3)
            ax.scatter(x, y, z, c='b', s=1)

ax.set_xlabel('X')
ax.set_ylabel('Y')
ax.set_zlabel('Z')
ax.set_title('Workspace of 3-DOF Robotic Leg')
plt.show()

```

This code will generate a plot showing the workspace of the robotic leg.

### 2.0.5 Exercise 5

Morbi luctus, wisi viverra faucibus pretium, nibh est placerat odio, nec commodo wisi enim eget quam. Quisque libero justo, consectetur a, feugiat vitae, porttitor eu, libero. Suspendisse sed mauris vitae elit sollicitudin malesuada. Maecenas ultricies eros sit amet ante. Ut venenatis velit. Maecenas sed mi eget dui varius euismod. Phasellus aliquet volutpat odio. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Pellentesque sit amet pede ac sem eleifend consectetur. Nullam elementum, urna vel imperdiet sodales, elit ipsum pharetra ligula, ac pretium ante justo a nulla. Curabitur tristique arcu eu metus. Vestibulum lectus. Proin mauris. Proin eu nunc eu urna hendrerit faucibus. Aliquam auctor, pede consequat laoreet varius, eros tellus scelerisque

quam, pellentesque hendrerit ipsum dolor sed augue. Nulla nec lacus.

#### Solution:

Suspendisse vitae elit. Aliquam arcu neque, ornare in, ullamcorper quis, commodo eu, libero. Fusce sagittis erat at erat tristique mollis. Maecenas sapien libero, molestie et, lobortis in, sodales eget, dui. Morbi ultrices rutrum lorem. Nam elementum ullamcorper leo. Morbi dui. Aliquam sagittis. Nunc placerat. Pellentesque tristique sodales est. Maecenas imperdiet lacinia velit. Cras non urna. Morbi eros pede, suscipit ac, varius vel, egestas non, eros. Praesent malesuada, diam id pretium elementum, eros sem dictum tortor, vel consectetur odio sem sed wisi.

### 2.0.6 Exercise 6

Sed feugiat. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Ut pellentesque augue sed urna. Vestibulum diam eros, fringilla et, consectetur eu, nonummy id, sapien. Nullam at lectus. In sagittis ultrices mauris. Curabitur malesuada erat sit amet massa. Fusce blandit. Aliquam erat volutpat. Aliquam euismod. Aenean vel lectus. Nunc imperdiet justo nec dolor.

#### Solution:

Etiam euismod. Fusce facilisis lacinia dui. Suspendisse potenti. In mi erat, cursus id, nonummy sed, ullamcorper eget, sapien. Praesent pretium, magna in eleifend egestas, pede pede pretium lorem, quis consectetur tortor sapien facilisis magna. Mauris quis magna varius nulla scelerisque imperdiet. Aliquam non quam. Aliquam porttitor quam a lacus. Praesent vel arcu ut tortor cursus volutpat. In vitae pede quis diam bibendum placerat. Fusce elementum convallis neque. Sed dolor orci, scelerisque ac, dapibus nec, ultricies ut, mi. Duis nec dui quis leo sagittis commodo.

## Dynamics

### 2.0.7 Exercise 7

Aliquam lectus. Vivamus leo. Quisque ornare tellus ullamcorper nulla. Mauris porttitor pharetra tortor. Sed fringilla justo sed mauris. Mauris tellus. Sed non leo. Nullam elementum, magna in cursus sodales, augue est scelerisque sapien, venenatis congue nulla arcu et pede. Ut suscipit enim vel sapien. Donec congue. Maecenas urna mi, suscipit in, placerat ut, vestibulum ut, massa. Fusce ultrices nulla et nisl.

#### Solution:

Etiam ac leo a risus tristique nonummy. Donec dignissim tincidunt nulla. Vestibulum rhoncus molestie odio. Sed lobortis, justo et pretium lobortis, mauris turpis condimentum augue, nec ultricies nibh arcu pretium enim. Nunc purus neque, placerat id, imperdiet sed, pellentesque nec, nisl. Vestibulum imperdiet neque non sem accumsan laoreet. In hac habitasse platea dictumst. Etiam condimentum facilisis libero. Suspendisse in elit quis nisl aliquam dapibus. Pellentesque auctor sapien. Sed egestas sapien nec lectus. Pellentesque vel dui vel neque bibendum viverra. Aliquam porttitor nisl nec pede. Proin mattis libero vel turpis. Donec rutrum mauris et libero. Proin euismod porta felis. Nam lobortis, metus quis elementum commodo, nunc lectus elementum

mauris, eget vulputate ligula tellus eu neque. Vivamus eu dolor.

### 2.0.8 Exercise 8

Nulla in ipsum. Praesent eros nulla, congue vitae, euismod ut, commodo a, wisi. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Aenean nonummy magna non leo. Sed felis erat, ullamcorper in, dictum non, ultricies ut, lectus. Proin vel arcu a odio lobortis euismod. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Proin ut est. Aliquam odio. Pellentesque massa turpis, cursus eu, euismod nec, tempor congue, nulla. Duis viverra gravida mauris. Cras tincidunt. Curabitur eros ligula, varius ut, pulvinar in, cursus faucibus, augue.

#### Solution:

Nulla mattis luctus nulla. Duis commodo velit at leo. Aliquam vulputate magna et leo. Nam vestibulum ullamcorper leo. Vestibulum condimentum rutrum mauris. Donec id mauris. Morbi molestie justo et pede. Vivamus eget turpis sed nisl cursus tempor. Curabitur mollis sapien condimentum nunc. In wisi nisl, malesuada at, dignissim sit amet, lobortis in, odio. Aenean consequat arcu a ante. Pellentesque porta elit sit amet orci. Etiam at turpis nec elit ultricies imperdiet. Nulla facilisi. In hac habitasse platea dictumst. Suspendisse viverra aliquam risus. Nullam pede justo, molestie nonummy, scelerisque eu, facilisis vel, arcu.

### 2.0.9 Exercise 9

Curabitur tellus magna, porttitor a, commodo a, commodo in, tortor. Donec interdum. Praesent scelerisque. Maecenas posuere sodales odio. Vivamus metus lacus, varius quis, imperdiet quis, rhoncus a, turpis. Etiam ligula arcu, elementum a, venenatis quis, sollicitudin sed, metus. Donec nunc pede, tincidunt in, venenatis vitae, faucibus vel, nibh. Pellentesque wisi. Nullam malesuada. Morbi ut tellus ut pede tincidunt porta. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Etiam congue neque id dolor.

#### Solution:

Donec et nisl at wisi luctus bibendum. Nam interdum tellus ac libero. Sed sem justo, laoreet vitae, fringilla at, adipiscing ut, nibh. Maecenas non sem quis tortor eleifend fermentum. Etiam id tortor ac mauris porta vulputate. Integer porta neque vitae massa. Maecenas tempus libero a libero posuere dictum. Vestibulum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Aenean quis mauris sed elit commodo placerat. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos hymenaeos. Vivamus rhoncus tincidunt libero. Etiam elementum pretium justo. Vivamus est. Morbi a tellus eget pede tristique commodo. Nulla nisl. Vestibulum sed nisl eu sapien cursus rutrum.

## Contents

Exercise 10 .....	22
Exercise 11 .....	22
Exercise 12 .....	22

## Control

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### Exercise 10

### Exercise 11

### Exercise 12

## Contents

Question 1.....	23
Question 2.....	24
Question 3.....	24
Question 4.....	24
Question 5.....	25
Question 6.....	25
Question 7.....	25
Question 8.....	25
Question 9.....	26
Question 10 .....	26

## Questionnaire

### Question 1

During your studies, please select if you have taken any formal/related courses in:

- Mechanics or Applied Mechanics
- Mechanism Theory
- Machine Design
- Rigid Body Mechanics or Multi-Body Dynamics
- Modeling and Control of Robot Manipulators
- Humanoid Robotics
- Biomechanics
- Linear Control Theory
- Non-Linear Control Theory
- Introduction to Robotics
- Artificial Intelligence or Machine Learning
- Linear Algebra
- Advanced Calculus

- Probability Theory
- Object Oriented Programming

Additionally, mention your grade along with maximum possible grade in the applicable subjects.



## Question 2

How do you find the overall difficulty level of the exercises? Categorize the exercises.

- Easy
- Medium
- Hard



## Question 3

What kind of additional help did you seek while solving the exercises?

- Textbooks
- Wikipedia
- Research papers
- Other online sources
- Friends/Colleagues



## Question 4

Please categorize your attempt to solve the exercises into:

- Solved without any external reference
- Solved with the help of known textbooks
- Solved with the help of online content (Wikipedia, online blogs etc.)
- Solved after reading research papers

List the exercise numbers in front of each option above.





## Question 5

Please select the programming languages where you have a working knowledge.

- C/C++
- Python
- Matlab
- Julia
- Ruby
- Mathematica



## Question 6

Please specify if you have worked with any symbolic manipulation packages.

- Matlab Symbolic Toolbox
- Maple
- Singular
- Sympy



## Question 7

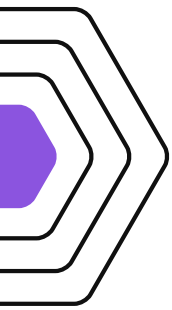
Have you worked with any industrial robot platforms?

- Universal robots
- KUKA robots
- Stäubli robots
- PUMA 560
- Others



## Question 8

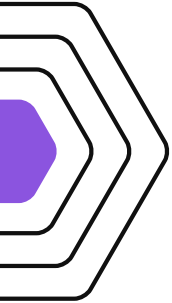
Have you ever built your own robot as a hobby or group project at the University? If yes, describe your robot and your role in the project.



## Question 9

Have you worked with a version control system? If yes, which one?

- Git
- SVN



## Question 10

Do you have LaTeX skills?

---

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