Control Systems Project

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Section B

Department: CSE

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Introduction

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Consider the following state-space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0.4 \end{bmatrix} x$$

a. Check the stability of the system using all methods that you know.

b. Compute the controllability and observability for the system. If the system is unstable, design a suitable controller for it.

c. Simulate the system using the controller (that you design) and show all the responses.

d. Design a PID controller and show the response of the system using PID Controller. Compare the results obtained in parts d and c.

e. Compute the steady state error before and after designing controller.

f. Design a tracking controller for step tracking of amplitude $\mathbf{5u(t)}$ and ramp tracking of $\mathbf{5ut(t)}$

a. Check the stability of the system using all ...

Method-1: Poles of Transfer function

Convert SS to TF and then find the poles to check the system's stability using the following formula.

$$G(s) = D + C[(sI - A)^{-1}B]$$

$$G(s) = \frac{0.52s-0.64}{s^2 - 5s - 2}$$

From the above equation poles are:

$$p_1 = -0.372, p_1 = 5.37$$

These poles shows that the system is unstable.

Method-2: Eigenvalues

To find eigenvalues, use the following formula: .

$$\boldsymbol{det(\lambda I - A) = 0}$$

$$\boldsymbol{\lambda^2 - 5\lambda - 2 = 0}$$

Using this formula gives me the same eigenvalues as above.

Method-3: step response

A system is unstable if the output of a system is unbounded for the step function.

Method-4: Routh-Hurwitz

The routh-Hurwitz table:

s^2	1	2	0
s^1	-5	0	0
s^0	$-\frac{1}{-5} \times \begin{vmatrix} 1 & 2 \\ -5 & 0 \end{vmatrix} = 2$		

As there is sign changes in the first column, thus the system is unstable.

Stability Analysis of the System

The eigen values of the system are:

$$\boldsymbol{\lambda_1 = -0.37, \lambda_2 = 5.37}$$

The poles of the system are:

$$\boldsymbol{p_1 = -0.37, p_2 = 5.37}$$

Stability Analysis of the System

The step response of the system is:

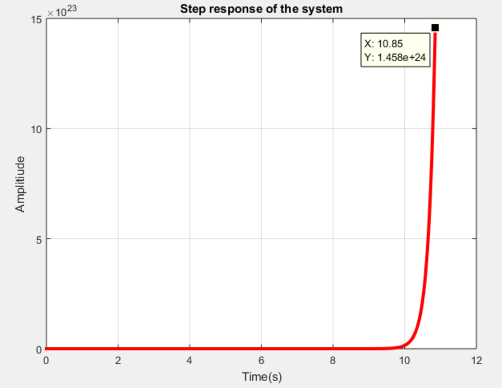

Figure: Plot of the step response.

Stability Analysis of the System

The root lucus of the system is:

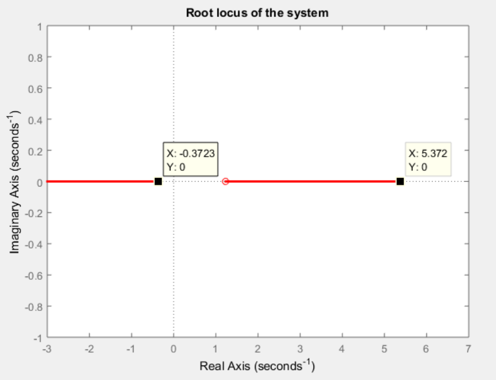

Figure: Root lucus

Controllability

Order of matrix **A = 2** which is **n**.

Let's find the controllability **P** matrix as:

$$\boldsymbol{P = \begin{bmatrix} B & AB \end{bmatrix}}$$

$$\boldsymbol{P = \begin{bmatrix} 0.4 & 1 \\ 0.3 & 2.4 \end{bmatrix}}$$

Let's find the observability **Q** matrix as:

$$\boldsymbol{Q = \begin{bmatrix} C \\ CA \end{bmatrix}}$$

$$\boldsymbol{Q = \begin{bmatrix} 1 & 0.4 \\ 2.2 & 0.3 \end{bmatrix}}$$

Controllability Test

In my case $\mathbf{A} = \mathbf{2}$

$$\boldsymbol{P} = \begin{bmatrix} \mathbf{0.4} & \mathbf{1} \\ \mathbf{0.3} & \mathbf{2.4} \end{bmatrix}$$

Rank of **P**:

$$\boldsymbol{P} = \begin{bmatrix} \mathbf{1} & \mathbf{0.4} \\ \mathbf{2.4} & \mathbf{0.3} \end{bmatrix} \boldsymbol{C1} \leftrightarrow \boldsymbol{C2}$$

$$\boldsymbol{P} = \begin{bmatrix} \mathbf{2.4} & \mathbf{0.96} \\ \mathbf{2.4} & \mathbf{0.3} \end{bmatrix} \boldsymbol{R1} = \mathbf{2.4}\boldsymbol{R1}$$

$$\boldsymbol{P} = \begin{bmatrix} \mathbf{2.4} & \mathbf{0.96} \\ \mathbf{0} & \mathbf{-0.66} \end{bmatrix} \boldsymbol{R2} - \boldsymbol{R1}$$

As non-zero rows are **2** so the rank of matrix **P=2**

Observability Test

In my case **A = 2**

$$Q = \begin{bmatrix} \mathbf{1} & \mathbf{0.4} \\ \mathbf{2.2} & \mathbf{3.6} \end{bmatrix}$$

Rank of **Q**:

$$Q = \begin{bmatrix} \mathbf{2.2} & \mathbf{0.88} \\ \mathbf{2.2} & \mathbf{3.6} \end{bmatrix} R1 = 2.2R2$$

$$Q = \begin{bmatrix} \mathbf{2.2} & \mathbf{0.88} \\ \mathbf{0} & \mathbf{2.72} \end{bmatrix} R2 - R1$$

As non-zero rows are **2** so the rank of matrix **Q=2**

As C is not an identity matrix and all the prerequisites are passed so in my case the suitable controller is Observer state feedback controller

Observer Design

To design an observer I will follow the following steps.

$$L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$$

$$LC = \begin{bmatrix} l_1 & 0.4l_1 \\ l_2 & 0.4l_2 \end{bmatrix}$$

$$(A - LC) = \begin{bmatrix} 1 - l_1 & 2 - 0.4l_1 \\ 3 - l_2 & s + 0.4l_2 - 4 \end{bmatrix}$$

$$det(sI - (A - LC)) = s^2 + (0.4l_2 + l_1 - 5)s + 1.6l_2 - 2.8l1 - 10$$

Now comparing the above equation with the following to get values of l_1 and l_2

$$(s + 10)(s + 80) = s^2 + 90s + 800$$

$$L = \begin{bmatrix} -62.06 \\ 392.65 \end{bmatrix}$$

Simulink Design and Graphs

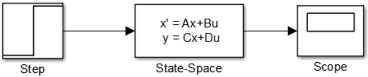

Figure: System without controller

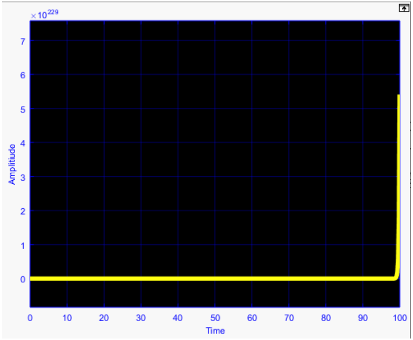

Figure: System step response without controller

Simulink Design and Graphs

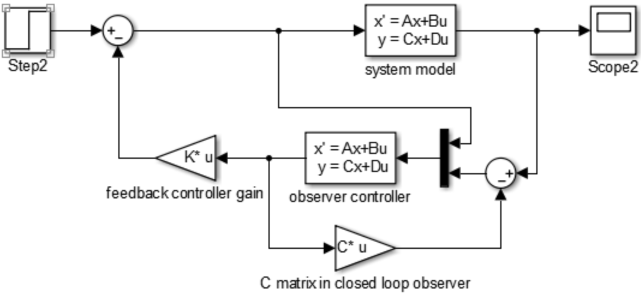

Figure: System with observer controller

Simulink Design and Graphs

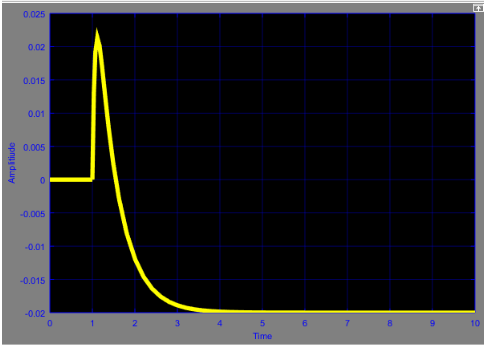

Figure: System step response with observer controller

Simulink Design and Graphs

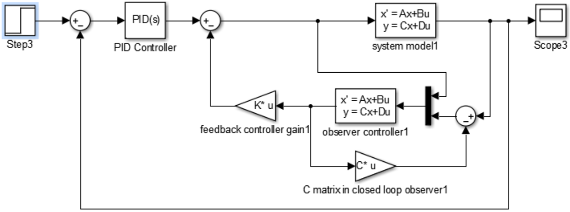

Figure: observer state feedback controller with PID controller

Simulink Design and Graphs

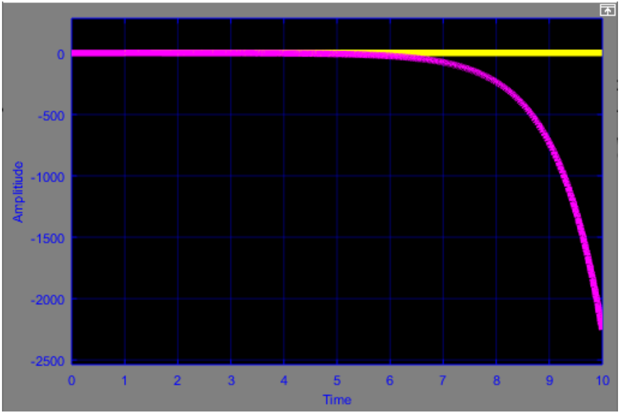

Figure: step response of observer state feedback controller with PID controller

Tracking Controller Design

Tracking controller for step tracking of amplitude **5u(t)**

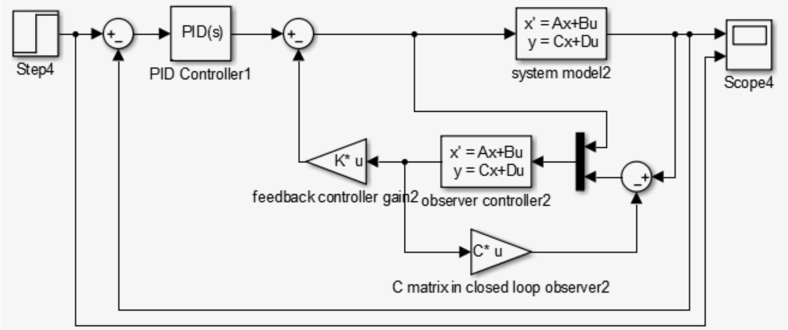

Figure: Tracking controller

Tracking Controller Design

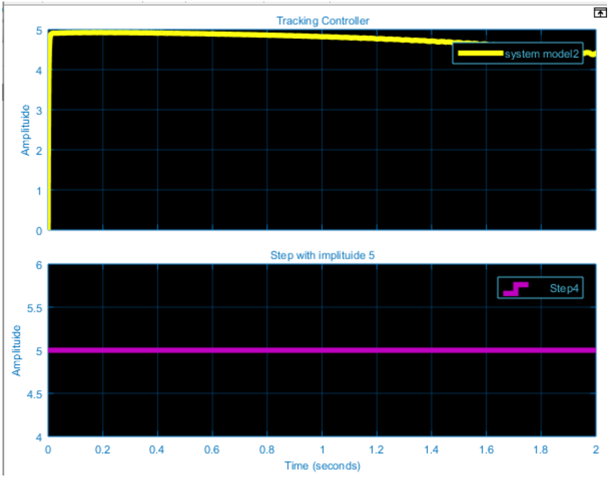

Figure: Tracking controller step response

Tracking Controller Design

Tracking controller for ramp tracking of amplitude **5tu(t)**

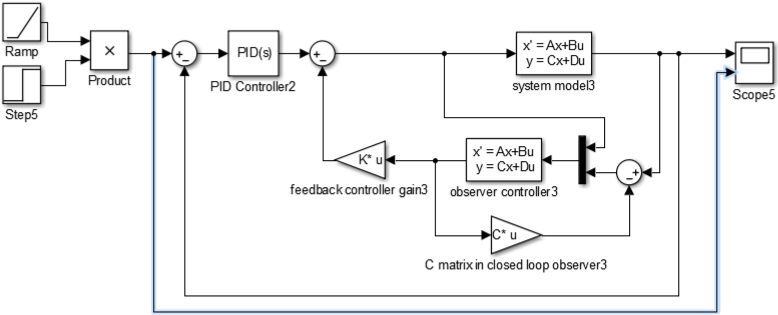

Figure: Tracking controller

Tracking Controller Design

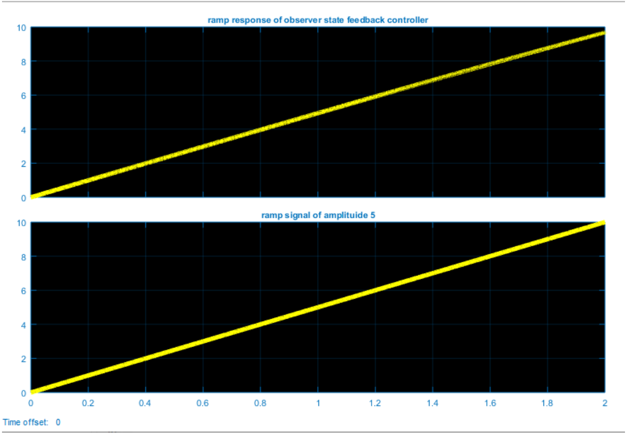

Figure: Tracking controller step response

Conclusions

There are two main types of controll systems.

1. Stable
2. Unstable

If system is stable then there is no need to design a controller and if the system is unstable then keeping some prerequisites in mind we need to design a suitable controller for it . For these kind of systems we can check steady state error and can also design a PID or any other tracking controller.

If the system is neither stable nor any prerequisites are satisfied i.e no controller is design able then we can not check any steady state error or can design any PID or tracking controller.