Control Systems Presentation Report.

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Introduction

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Consider the following state-space model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0.4 \end{bmatrix} x$$

a. Check the stability of the system using all methods that you know.

b. Compute the controllability and observability for the system. If the system is unstable, design a suitable controller for it.

c. Simulate the system using the controller (that you design) and show all the responses.

d. Design a PID controller and show the response of the system using PID Controller. Compare the results obtained in parts d and c.

e. Compute the steady state error before and after designing controller.

f. Design a tracking controller for step tracking of amplitude $\mathbf{5u(t)}$ and ramp tracking of $\mathbf{5ut(t)}$

a. Check the stability of the system using all ...

There are several methods to check the stability of the system some of them are the following. **Method-1: Poles of Transfer function**

Convert SS to TF and then find the poles to check the system's stability using the following formula.

$$G(s) = D + C[(sI - A)^{-1}B]$$

$$G(s) = \frac{0.52s{-}0.64}{s^2 - 5s - 2}$$

From the above equation poles are:

$$p_1 = -0.372, p_1 = 5.37$$

These poles shows that the system is unstable.

Method-2: Eigenvalues

To find eigenvalues, use the following formula: .

$$\boldsymbol{det(\lambda I - A) = 0}$$

$$\boldsymbol{\lambda^2 - 5\lambda - 2 = 0}$$

Using this formula gives me the same eigenvalues as above.

Method-3: step response

A system is unstable if the output of a system is unbounded for the step function.

Method-4: Routh-Hurwitz

The routh-Hurwitz table:

s^2	1	2	0
s^1	-5	0	0
s^0	$-\frac{1}{-5} \times \begin{vmatrix} 1 & 2 \\ -5 & 0 \end{vmatrix} = 2$		

As there is sign changes in the first column, thus the system is unstable.

Stability Analysis of the System

From the above methods, we get eigenvalues and poles like below. The eigenvalues of the system are:

$$\boldsymbol{\lambda_1 = -0.37, \lambda_2 = 5.37}$$

The poles of the system are:

$$\boldsymbol{p_1 = -0.37, p_2 = 5.37}$$

Controllability

From the matrix, A above the order is 2 so

$$\mathbf{n = 2}$$

Let's find the controllability matrix **P** below:

$$\boldsymbol{P = \begin{bmatrix} B & AB \end{bmatrix}}$$

$$\boldsymbol{P = \begin{bmatrix} 0.4 & 1 \\ 0.3 & 2.4 \end{bmatrix}}$$

Controllability Test

For the controllability test first I have to check the rank of the matrix **P**. To check the rank **P**, let's first convert it to row echelon form as below.
Rank of **P**:

$$\boldsymbol{P = \begin{bmatrix} 1 & 0.4 \\ 2.4 & 0.3 \end{bmatrix} C1 \leftrightarrow C2}$$

$$\boldsymbol{P = \begin{bmatrix} 2.4 & 0.96 \\ 2.4 & 0.3 \end{bmatrix} R1 = 2.4R1}$$

$$\boldsymbol{P = \begin{bmatrix} 2.4 & 0.96 \\ 0 & -0.66 \end{bmatrix} R2 - R1}$$

As non-zero rows are **2** so the rank of matrix **P=2**

Observability Test

In my case $\mathbf{n = 2}$

Let's find the observability matrix **Q** as below:

$$\boldsymbol{Q = \begin{bmatrix} C \\ CA \end{bmatrix}}$$

$$\boldsymbol{Q = \begin{bmatrix} 1 & 0.4 \\ 2.2 & 0.3 \end{bmatrix}}$$

$$\boldsymbol{Q = \begin{bmatrix} 1 & 0.4 \\ 2.2 & 3.6 \end{bmatrix}}$$

For the observability test first I have to check the rank of the matrix **Q**. To check the rank **Q**, let's first convert it to row echelon form as below.

Rank of **Q**:

$$Q = \begin{bmatrix} \mathbf{2.2} & \mathbf{0.88} \\ \mathbf{2.2} & \mathbf{3.6} \end{bmatrix} \boldsymbol{R1 = 2.2R2}$$

$$Q = \begin{bmatrix} \mathbf{2.2} & \mathbf{0.88} \\ \mathbf{0} & \mathbf{2.72} \end{bmatrix} \boldsymbol{R2 - R1}$$

As non-zero rows are **2** so the rank of matrix **Q=2**

As C is not an identity matrix and all the prerequisites are passed so in my case the suitable controller is the Observer state feedback controller

Observer Design

To design an observer I will follow the following steps.

$$\boldsymbol{L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}}$$

$$\boldsymbol{LC = \begin{bmatrix} l_1 & 0.4l_1 \\ l_2 & 0.4l_2 \end{bmatrix}}$$

$$\boldsymbol{(A - LC) = \begin{bmatrix} 1 - l_1 & 2 - 0.4l_1 \\ 3 - l_2 & s + 0.4l_2 - 4 \end{bmatrix}}$$

$$\boldsymbol{det(sI - (A - LC)) = s^2 + (0.4l_2 + l_1 - 5)s + 1.6l_2 - 2.8l1 - 10}$$

Now comparing the above equation with the following to get values of l_1 and $\boldsymbol{l_2}$

$$\boldsymbol{(s + 10)(s + 80) = s^2 + 90s + 800}$$

$$\boldsymbol{L = \begin{bmatrix} -62.06 \\ 392.65 \end{bmatrix}}$$

Conclusions

There are two main types of control systems.

1. Stable
2. Unstable

If the system is stable then there is no need to design a controller and if the system is unstable then keeping some prerequisites in mind we need to design a suitable controller for it. For this kind of system, we can check steady-state errors and can also design a PID or any other tracking controller.

If the system is neither stable nor any prerequisites are satisfied i.e no controller is designed able then we can not check any steady state error or can design any PID or tracking controller.