Control Systems - 7^{th} Semester - Week 4

Step Response of Systems

Dr. Salman Ahmed

Topics of last week

In last weeks, we studied about obtaining state-space models from differential equations

Then we studied about converting state-space model to transfer functions using formula

We also studied on converting transfer function to state-space model using canonical forms

Model - Recalling concepts

A model is representation or abstraction of reality/system.

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This means the more knowledge a person has, the better he/she can write a model

What is mathematical model?

Model - Recalling concepts

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What is mathematical model? A set of equations (linear or differential) that describes the relationship between input and output of a system.

Types of Model

There are three types of mathematical models

- Black Box
- Grey Box
- White Box

Introduction to Transient Analysis

Sometimes we cannot write white box models for the systems

Either we do NOT know what is inside the system or either the system is too complex to verify the components

Perhaps sometimes the components are not easily identifiable and their configuration or layout is not readable

So another way to obtain model of a system is to apply a *test* input signal and obtain the output signal

Standard Input Signals

Though there are many possible combinations of input signals, the following are famous or popular input signals

- Impulse Signal
- Step Signal.
- Ramp Signal
- Parabolic Signal

Step Signal

The step signal is used to imitate the sudden change of a signal

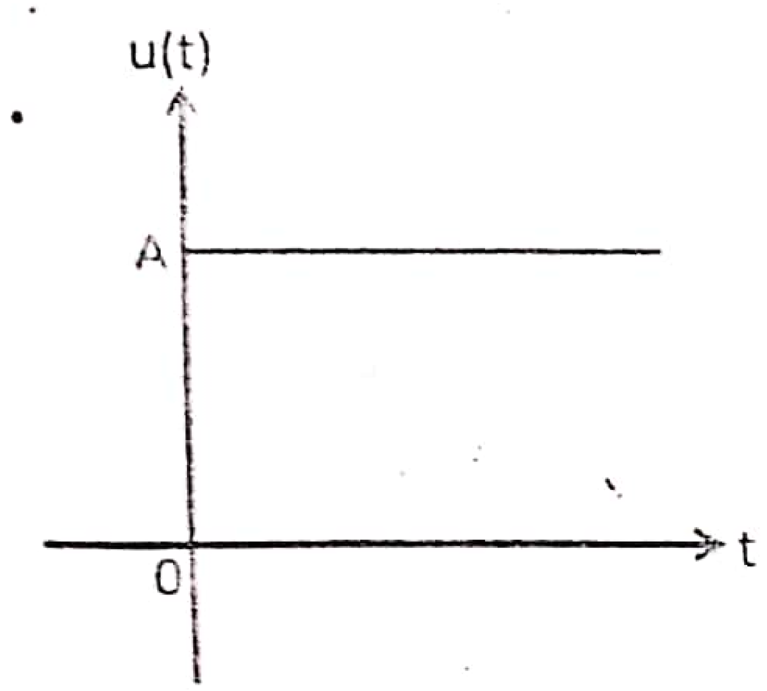

Figure: Step Signal

$$u(t) = \begin{cases} A & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

If $A = 1$, the step signal is called unit step signal

Step Response

Impulse response is the best response because transfer function is defined as impulse response of a system

However, in practical life generating impulse signal is not easy

We use unit step signal as test input signal and then analyze the output of the system

First Order System

A first order system has a single pole (irrespective of number of zeros).

Many systems are of first order. Examples include

- velocity of a car on road
- control of angular velocity of rotating systems
- an RLC circuit with only one capacitor and no inductor
- an RLC circuit with only one inductor and no capacitor
- fluid flow in a pipeline
- level control in a tan
- pressure control in a

First Order System - Examples

Examples of first order system from computing system domain:

- speed control of dc motor in hard disk system
- time taken by queries in database management system, e.g. `SELECT * from STUDENT where ATTENDANCE_PERCENTAGE > 75;`
- time taken to read a temperature sensor interfaced with an embedded system
- energy consumed by an IoT device
- control of in networks e.g. CSMA/CA
- time taken by PS4 or Xbox one device to boot

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Step Response of First Order System

A general first order system without zeros can be written as follows:

$$G(s) = \frac{b}{s+a}$$

Let $C(s)$ be the output of a system having transfer function $G(s)$ (expressed above). If the input to $G(s)$ is a unit step, then the output can be expressed as follows:

$$\text{Output Signal} = \text{Input Signal} \times \text{Transfer function}$$

We can further write the following:

$$C(s) = \text{Unit step signal} \times G(s)$$

$$C(s) = \frac{1}{s} \times \frac{b}{s+a} = \frac{b}{s(s+a)}$$

Step Response of First Order System

The term a is an important term. The inverse of a is called time constant i.e.

$$\tau = \frac{1}{a}$$

where τ is called time-constant of first order systems. For example compute τ of the following system:

$$G(s) = \frac{3}{s+2}$$

Step Response of First Order System

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$$G(s) = \frac{3}{s+2}$$

Here $\tau = \frac{1}{2} = 0.5$ and gain K is computed as $\frac{3}{2} = 1.5$

The value of gain K indicates the final steady-state value of the step response

Step Response of First Order System

In order to compute transfer function from a plot, we need to define a few more terminologies

Rise Time: T_r, time taken to reach 90% or 0.9 of final value from 10% or 0.1. Mathematically:

$$T_r = \frac{2.2}{a}$$

Settling Time: T_s, time taken to stay within 2% of its final value (or reach 98% of final value). Mathematically:

$$T_s = \frac{4}{a}$$

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Step Response of First Order System

Can you compute the transfer function?

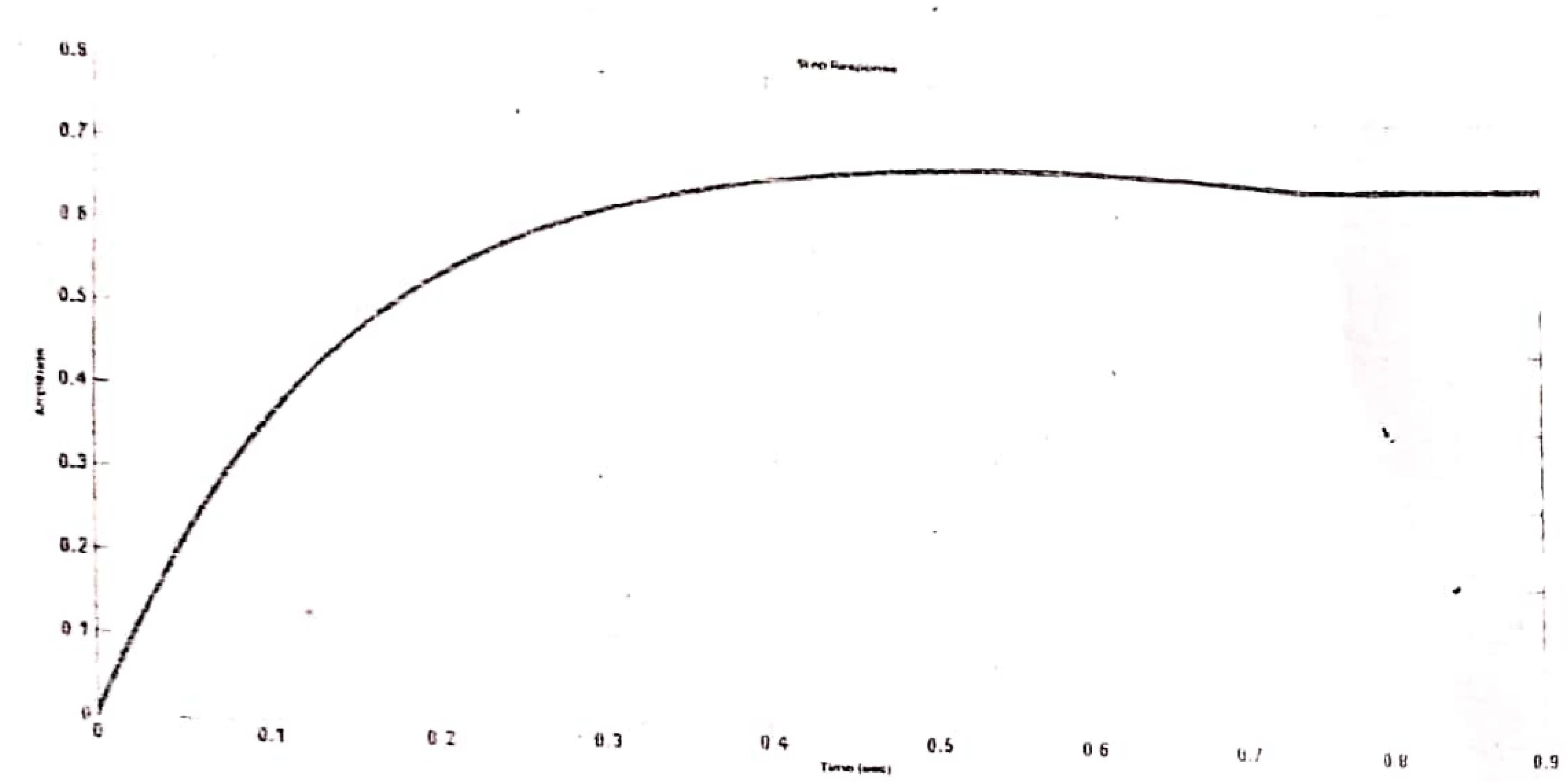

Figure: Step Response of a transfer function

Step Response of First Order System

Time constant: Time to reach 63% of final value. Compute the transfer function from the previous plot.

Step Response of First Order System

Time constant: Time to reach 63% of final value. Compute the transfer function from the previous plot.

Final value = steady-state value = gain $K = 0.72$
63% of final value is $0.63 \times 0.72 = 0.4464$
Time taken to reach 0.45 value is 0.15 seconds
The final transfer function is

$$G(s) = \frac{\frac{0.72}{0.15}}{s + \frac{1}{0.15}}$$

OR

Pole is inverse constant which comes out to be $\frac{1}{0.15} = 6.67$

Another way of the transfer function is

$$G(s) = \frac{4.802}{s + 6.67}$$

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Step Response of First Order System

The previous step-response was obtained for the following actual transfer function:

$$G(s) = \frac{5}{s+7}$$

MATLAB code for obtaining step response

```
num = [5] ;

den = [1 7] ;

step(num,den)
```

Step Response of First Order System

Effects of decreasing time constant

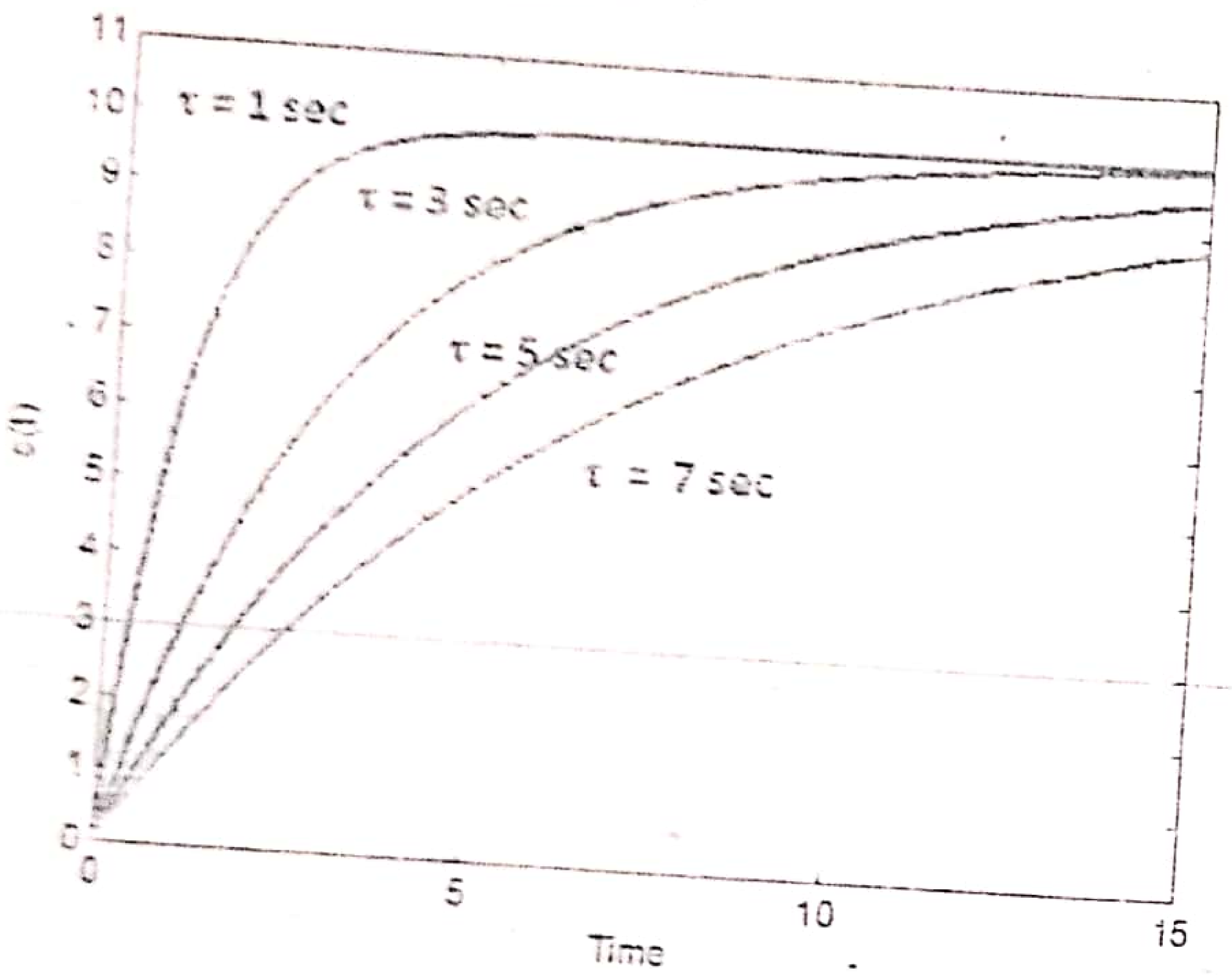

Figure: Effects of decreasing time constants of first order transfer function

Step Response of First Order System

Effects of increasing gains (remember its K not the term b)

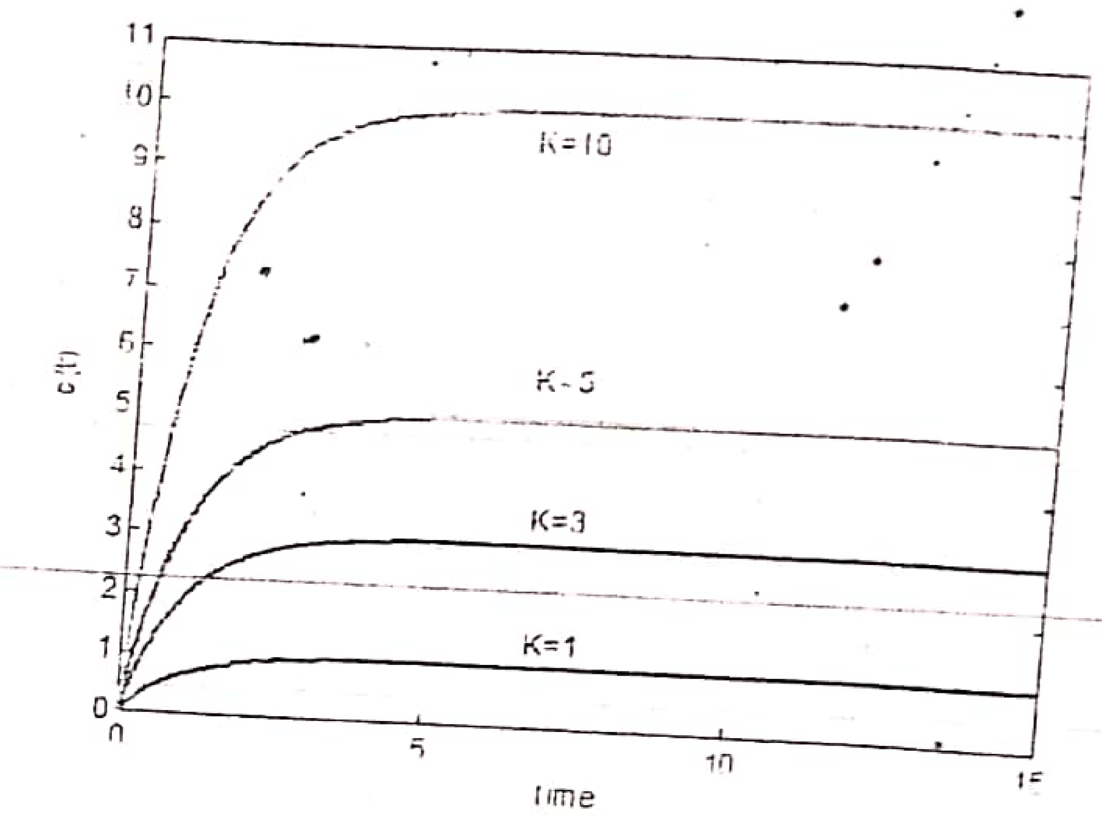

Figure: Effects of increasing gains of first order transfer function

Step Response of First Order System

Question: NADRA manages the registration database of Pakistani citizens. Previously, till 2001 people had 11 digit NIC numbers. Each citizen of Pakistan is issued a 13 digit CNIC number. The first 5 digits in a CNIC are based on a citizen locality, the next 7 numbers are random, and the final last digit is gender based (even for females, and odd for males). The current database entries in NADRA are estimated as 90 million. If a person details are required and his/her CNIC is entered in the NADRA database, its take 1 min to search 98% of the records in a NADRA database. Assuming the query search process to be a first-order system, find the time constant.

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Control Systems - Week 5

Block Reduction of Complex Systems

Dr. Salman Ahmed

Contents that we have covered till now

We studied the following topics till now:

- Converting state-space to transfer function using formula
- Converting transfer functions to state-space models using canonical forms
- Analyzing step responses of first order systems (time constant and dc-gain)

We will study the following topics before mid term exam

- Block reduction of complex systems (today lecture)
- Analyzing step responses of second order systems (underdamped, undamped, over damped, critically damped)

Block reduction algebra

First we analyze a simple transfer function block.

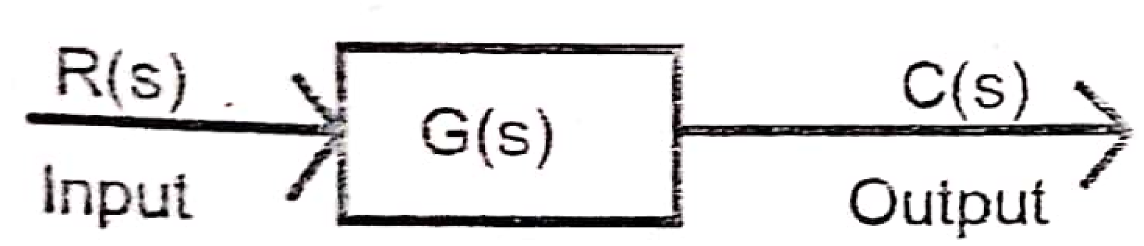


Figure: Transfer function block.

The input signal is denoted by $R(s)$ and output signal by $C(s)$. We can write the following

$$) = G(s)R(s)$$

Sometimes, we skip the term ... the following abusive notation:

Block reduction algebra

There are 3 types of interconnections in control systems:

- Series Interconnection
- Parallel Interconnection
- Feedback Interconnection

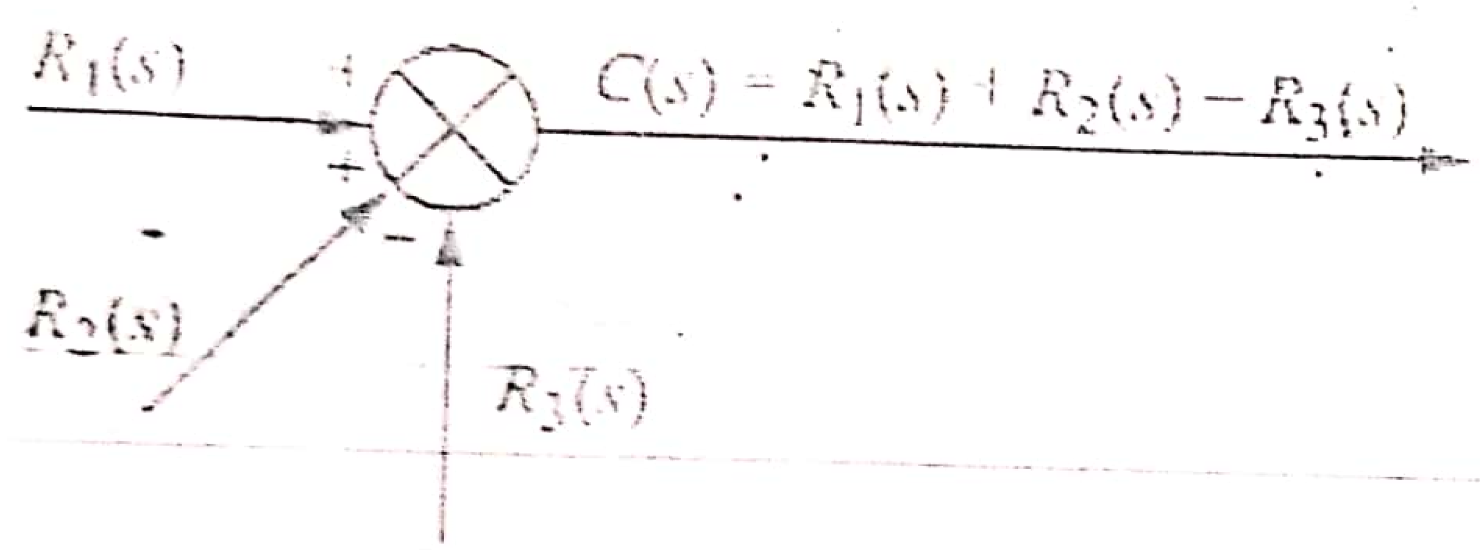
Besides, there are 4 operations which are as follows:

- Moving summing junction after transfer function
- Moving summing junction before transfer function
- Moving before pickoff point
- Moving after pickoff point

Let us introduce a summing junction or summer first, and then pick off point.

⋯ reduction algebra - Summer or Summing Junction

⋯ner or summing junction adds (or subtracts) two or more signals. The default + in a summer or summing junction.



Summing junction

Figure: Summing Junction Symbol

Block reduction algebra - Pick off point

Pick off point: A point where the same signal has to propagate through more than one paths

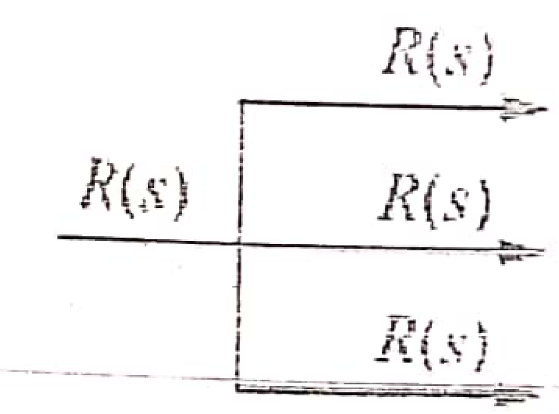


Figure. Pick Off point

⋯nterconnection: Series Interconnection

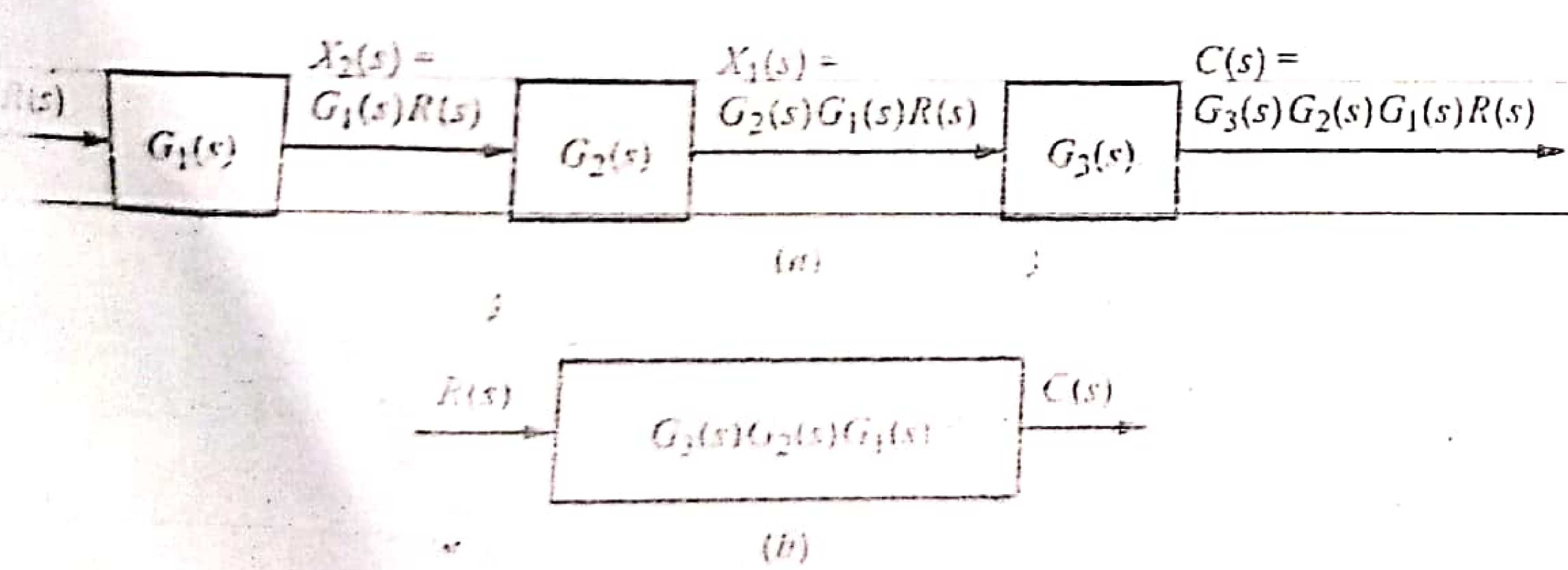


Figure: Series Interconn⋯n of transfer functions

⋯n write $G_e = G_3G_2G_1$

Second interconnection: Parallel Interconnection

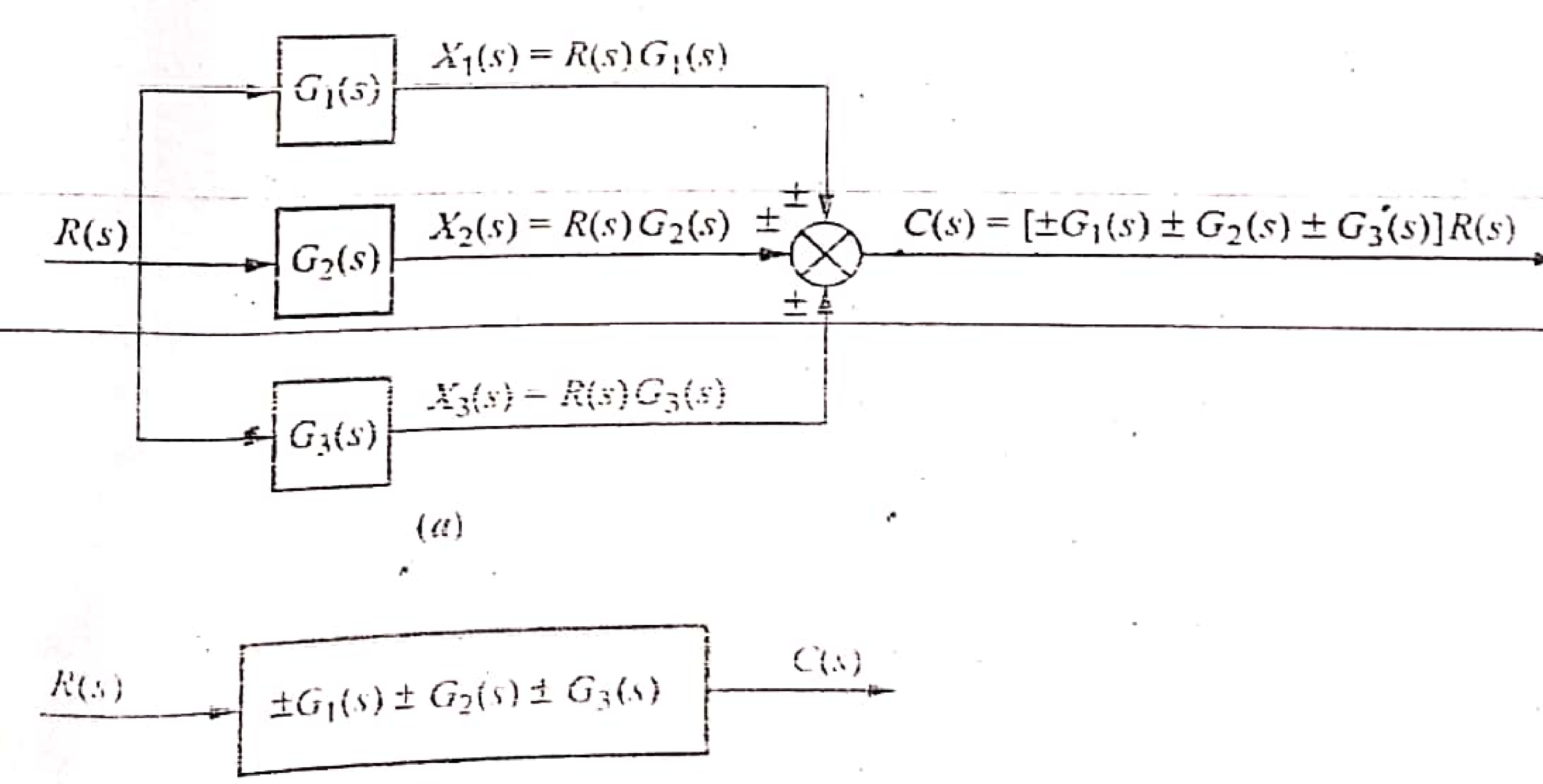


Figure: Parallel Interconnection of transfer functions

We can write $G_e = \pm G_3 \pm G_2 \pm G_1$

Few Important Points

Series interconnection involves product of transfer functions

In parallel interconnection, be careful to identify the transfer functions correctly.

Two blocks are in parallel if they have same input signal and the output goes towards same summing junction.

Parallel interconnection involves sum or different of transfer functions

Third interconnection: Feedback Interconnection

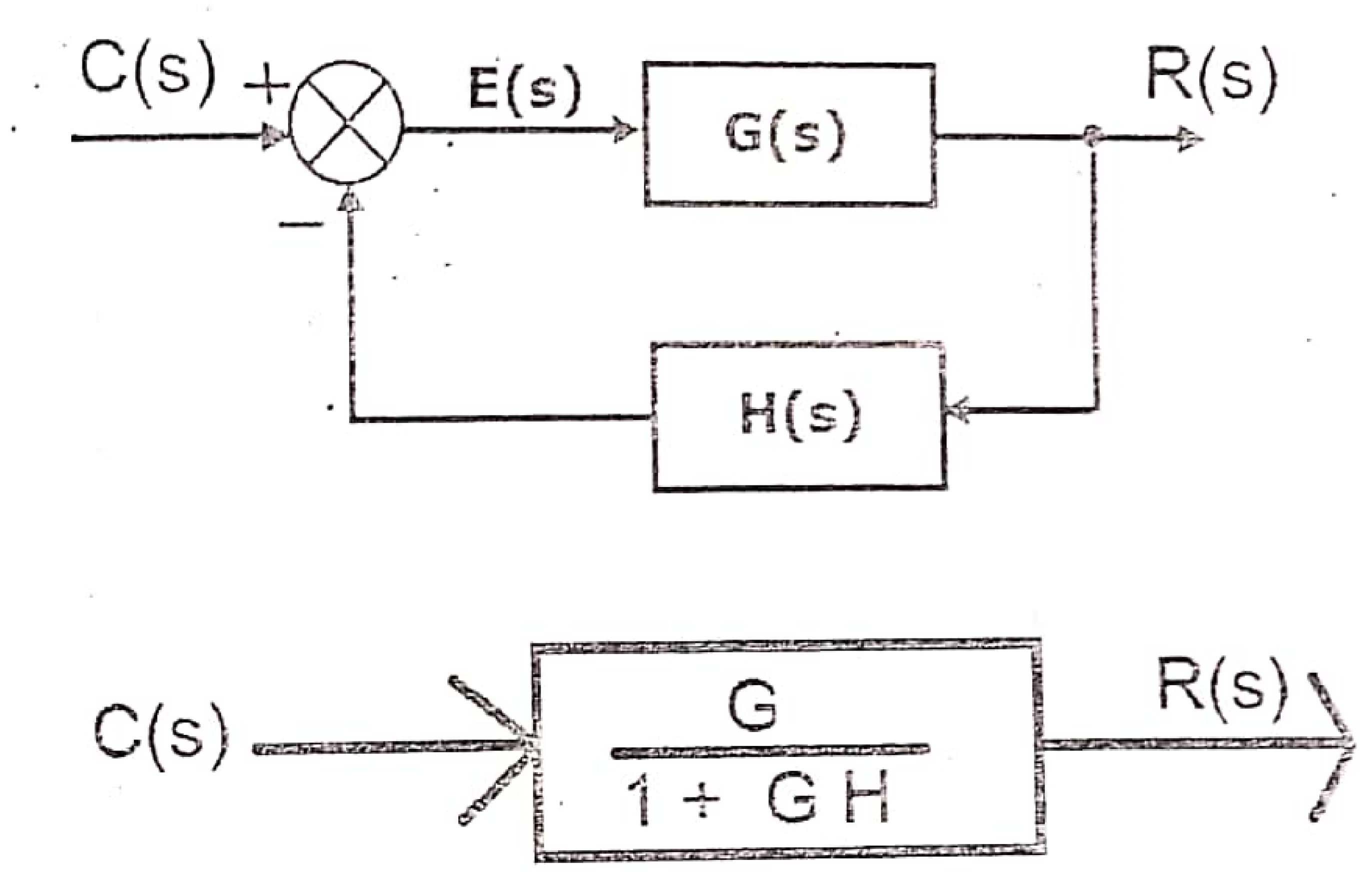
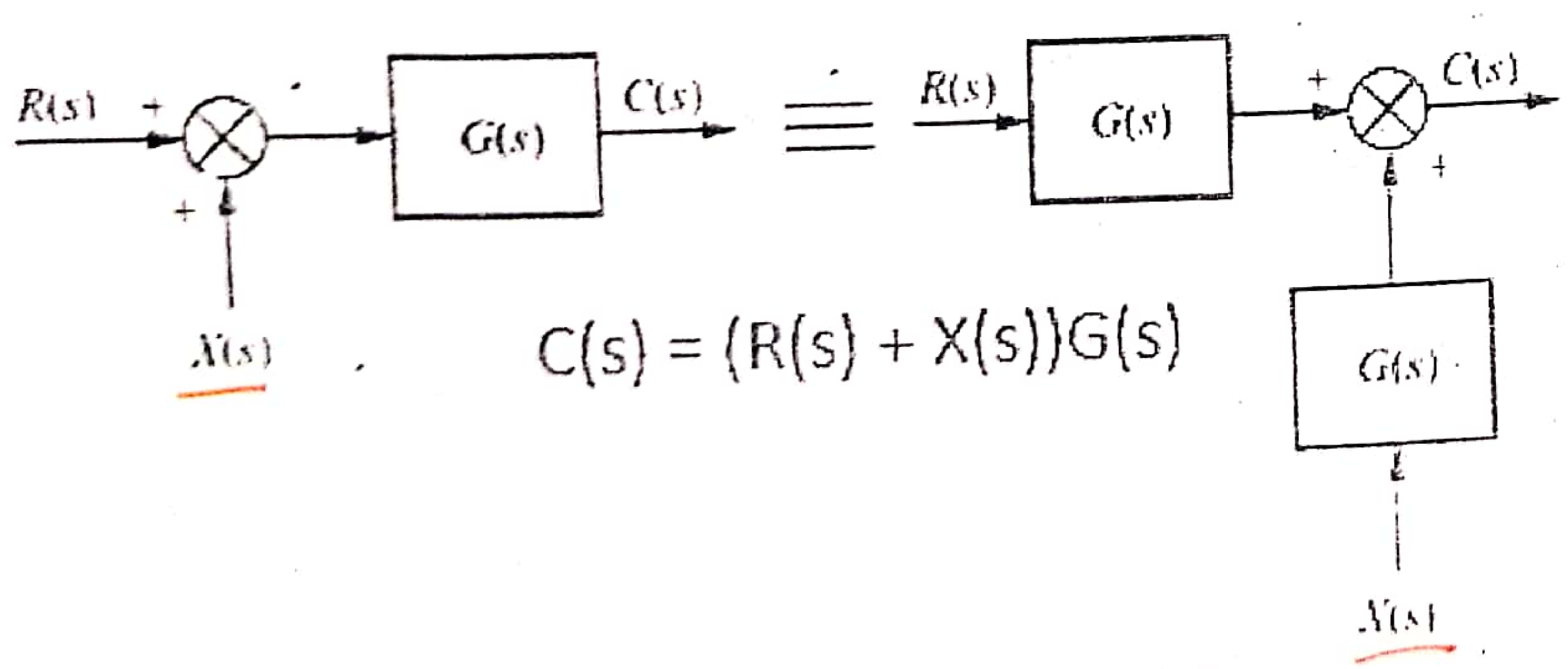


Figure: Feedback Interconnection of transfer functions

We can write $G_e = \frac{G}{1 \pm GH}$

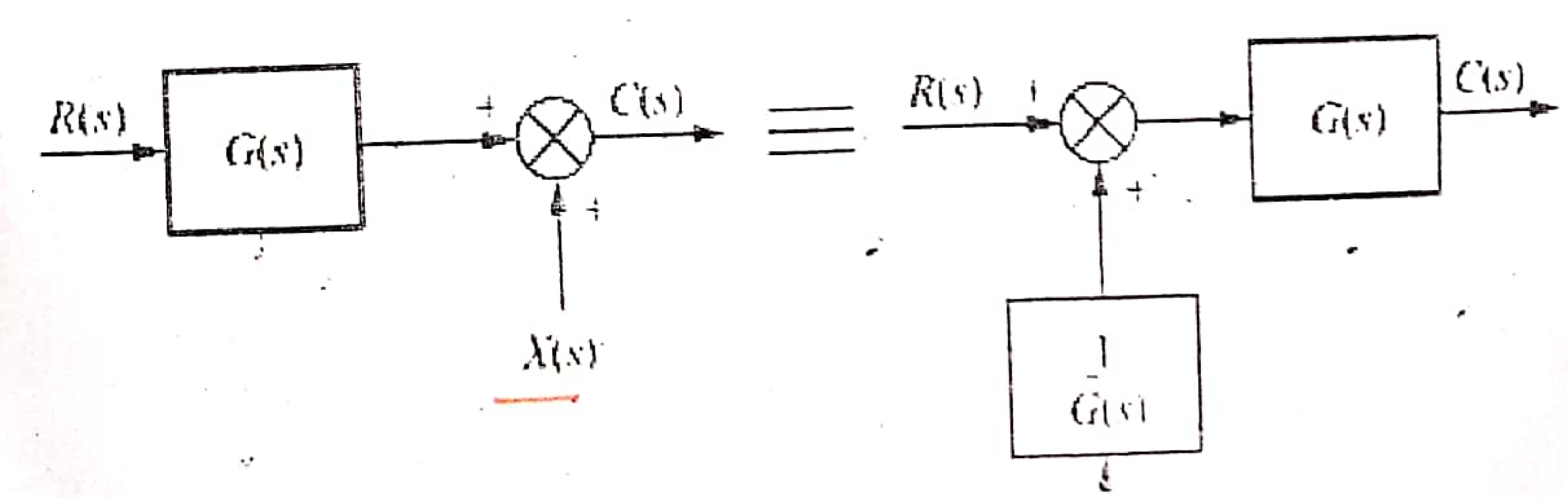
Operation 1: Moving summing junction after transfer function



$$C(s) = (R(s) + X(s))G(s)$$

Figure: Moving a ... on after transfer function

Operation 2: Moving summing junction before transfer function



$$C(s) = G(s)R(s) + X(s)$$

X(s)

Figure: Moving a summing junction before transfer function

...tion 3: Moving before pickoff point

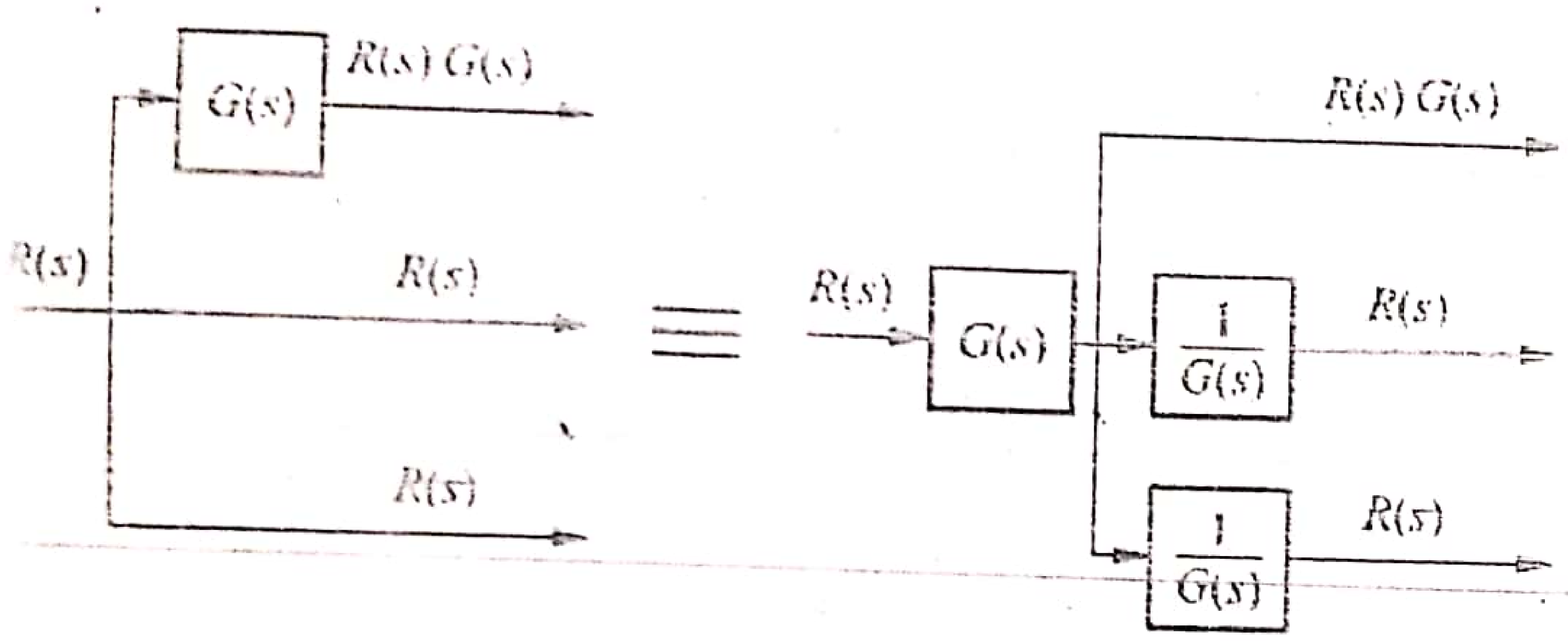


Figure: Moving before a pick-off point

Operation 4: Moving after pickoff point

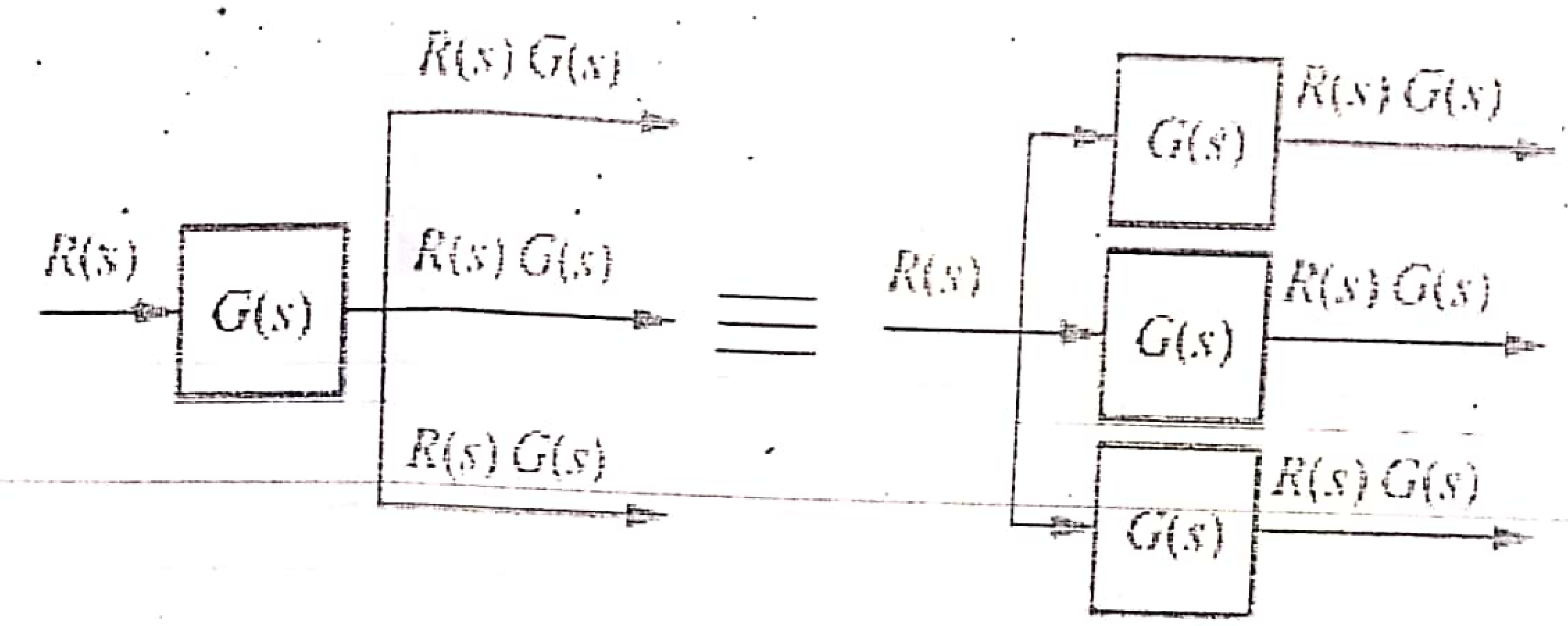


Figure: Moving after a pick-off point

...mary of block reduction rules

...ll use the knowledge about these 3 interconnections, and 4 operations to ... complex systems.

...will be given a complex interconnection schematic, plus input and output, and ... asked to apply this knowledge to reduce or simplify complex systems.

Example 1 - Problem to solve

Can you obtain the transfer function, $\frac{C(s)}{R(s)}$?

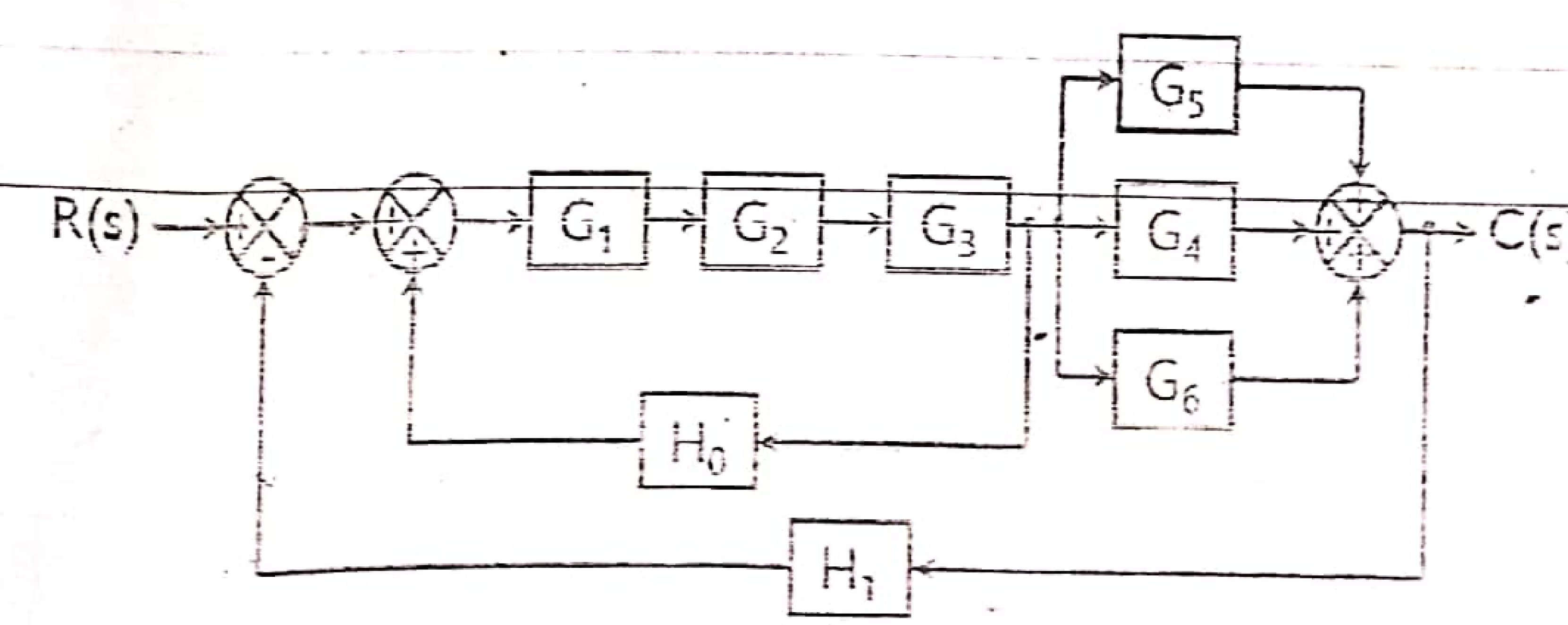


Figure. Example 1

...mple 1 - Solution
...nterconnection

xample 1 - Solution part a - Simplify series interconnection

Example 1 - Solution part b - Simplify parallel interconnection

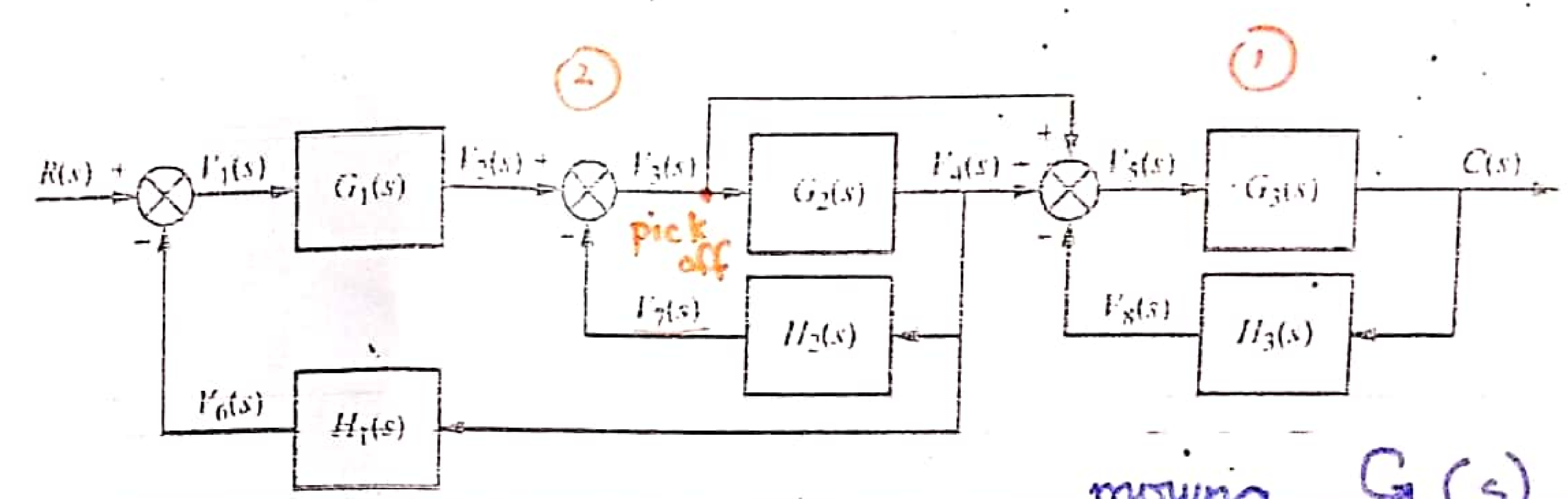
Example 1 - Solution part c - Simplify inner feedback interconnection

Example 1 - Solution part d

...nple 1 - Final Solution

Example 2 - Problem to solve

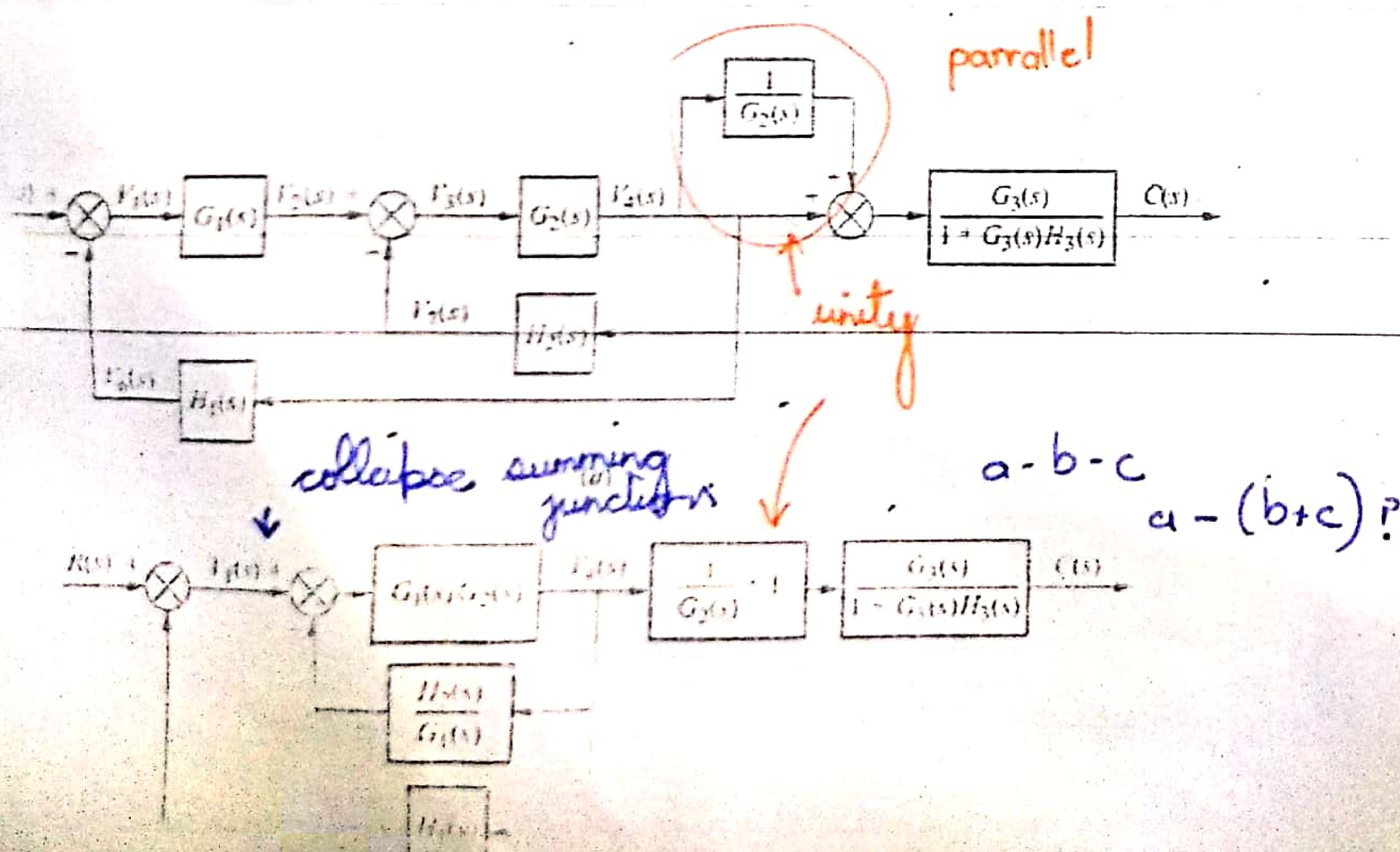
Can you obtain the transfer function, $\frac{C(s)}{R(s)}$?



moving G(s) to the left of the pick off point

Figure: Example 2

...nple 2 - Condensed Solution Part 1



Example 2 - Condensed Solution Part 2

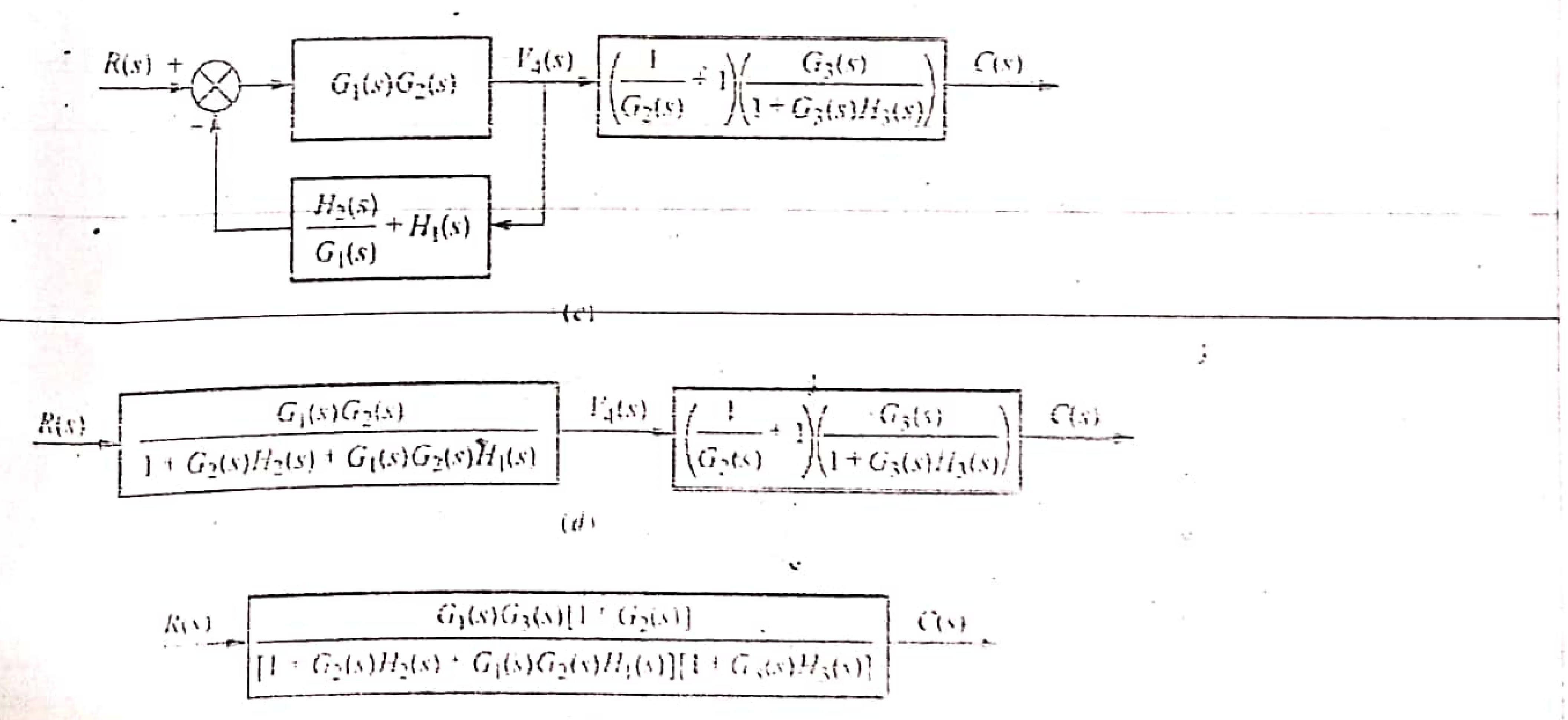


Figure: Example 2 - Final Solution

...fe example of feedb...

...us consider a water-tank level co...
...ne water level remains the same. Ca...

life example of feedback interconnection

Let us consider a water-tank level control systems. The objective is to ensure that the water level remains the same. Can you draw a block diagram of this system?

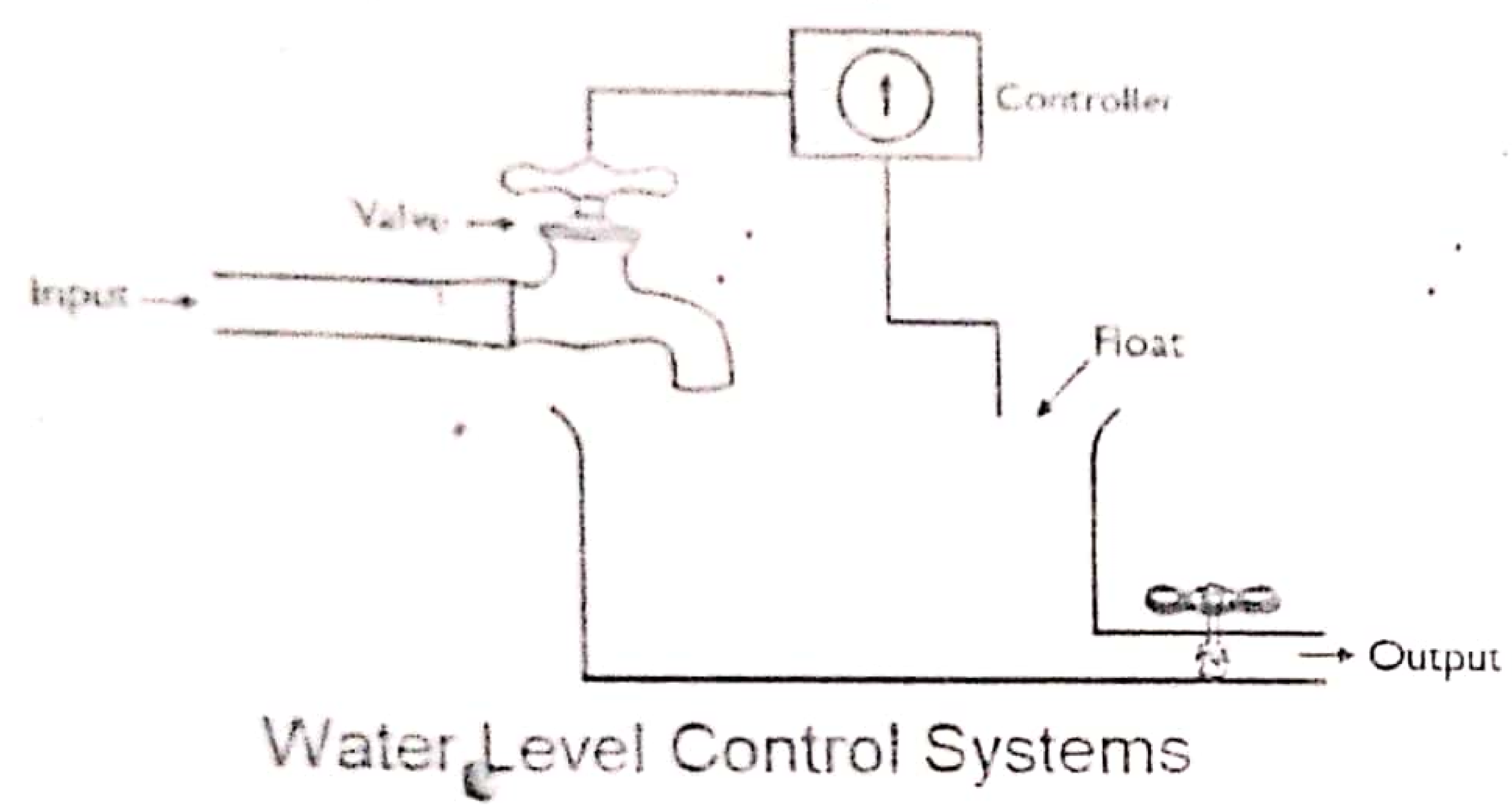


Figure: Example of Water-Level Control Systems

Let us first differentiate between real-world input and output AND control-systems input and output

Real life example of feedback interconnection