Project Report

Control Systems

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Problem

Bicycles are convenient, environment friendly and efficient transportation devices. An interesting control problem is to stabilize a bicycle while riding it. A bicycle can be modeled by taking various parameters into account such as its geometry, tires, elasticities and the rider. If a bicycle is stabilized, then it can be ridden without hands. Considering the handlebar torque as the input, and the steering angle and tilt angle as the state-variables, assume we can obtain the following linearized state-space model of the system:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \tag{1}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{2}$$

a. Check the stability of the system using all methods that you know.
b. Compute the controllability and observability for the system. If the systemis unstable, design a suitable controller for it.
c. Simulate the system using the controller (that you design) and show all theresponses.
d. Design a PID Controller and show the response of the system using PIDController. Compare the results obtained in part d and c.
e. Compute the steady state errors before and after designing controller.

1 Solution

In this report, we describe the above problem stepwise and discuss each part in details such as its stablility using distinct methods, compute controllability and observability and if the system is unstable we design a controller to make the system stabilized.

1.1 State-space Representation of the System

The state-space representation of the system is given below:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \tag{3}$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{4}$$

1.2 Stability analysis of the system

After solving the state-space model of the system the transfer function is:

$$\lambda^2 - 2\lambda + 1 \tag{5}$$

In this section, we analyze stability of the system. The stability can be checked using distinct ways such as eigen values, poles, step response, root locus and RH-stability criterion. In our case, the system is of 2nd order, so there would be two eigen values. Let λ_1 and λ_2 represent the eigen values of the system. We have already computed the eigen values for our problem. The eigen values of the system are:

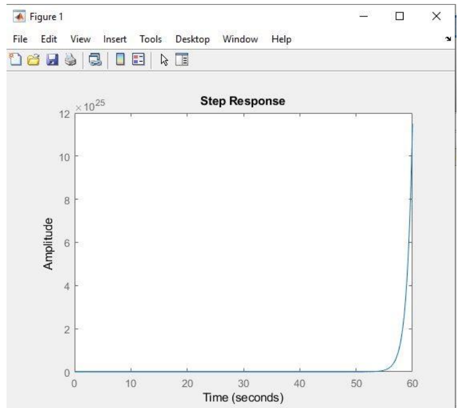
$$\lambda_1 = 1, \lambda_2 = 1 \tag{6}$$

As we can see both of the eigen values are positive, which infers that the system is unstable. Further, we also prove the same fact by observing the poles of the system. Let p_1 and p_2 represent the poles of the system. The computed values of the poles are:

$$p_1 = 1, p_2 = 1 \tag{7}$$

We observe here again that the poles are positive, which shows that the system is unstable. Further, we verify the same fact by looking at the stepresponse of the system. The step-response of open-loop system is shown in the following

Figure.



From Figure, we can do the following analysis:

$$Risetime(T_r) = N/A$$
$$Settlingtime(T_s) = N/A$$
$$PeakV\,alue(Amplitude) = 1.14e + 26$$
$$FinalV\,alue = inf$$
$$\%Overshoot = NaN$$

Next, we construct a Routh-Hurwitz table to check the stability of the system.

$s2$	1	1	0
$s1$	-2	0	0
$s0$	$-\begin{vmatrix} 1 & 1 & 1 & 1 \\ -2 & 0 & 2 & 0 \end{vmatrix} = 1$	0	0

Since there is a sign change in the first column of the RH table, so the system is unstable.
Thus from all of the above four methods we can clearly see that the system is unstable.

1.3 Designing a Controller

To design a controller we first need to decide which controller should be designed as in our case there is no need for state feedback controller since matrix C is not equal to identity matrix and it fails the pre-requisite number 1, so we have to design an Observer feedback controller.

For Observer feedback controller the system has to pass the following two more pre-requisites. These are pre-requisite number 2 and pre-requsite number 3.

Contollability Test.
Observability Test.

1.4 Controllability analysis of the system

Since the order of the System is : n = 2, so the P mtrix would be

$$P = \begin{bmatrix} B & AB \end{bmatrix} \tag{8}$$

or,

$$P = \begin{bmatrix} 1 & 1 \\ 0 & -0.5 \end{bmatrix} \tag{9}$$

$$\text{rank(p) = 2} \tag{10}$$

Hence, it passes the Controllability tests.

1.5 Observability analysis of the system

Since the order of the System is : n = 2, so the Q mtrix would be

$$Q = \begin{bmatrix} C \\ CA \end{bmatrix} \tag{11}$$

or,

$$Q = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix} \tag{12}$$

$$rank(Q) = 1 \tag{13}$$

Since it fails the Observability test, so no controller can be designed.

1.6 Controller Design for the system

As it fails the Observability test so the system is Uncontrollable.

2 Results

As we have observed that no controller can be designed for the system, so the bicycle cannot be stabilized.