Linear Systems and Control - Week 8

Controller Design - Full State Feedback Controller - MATLAB Code

Motivation for Controller Design

A system is unstable if:

- Any/all eigenvalue(s) of matrix A is/are non-negative
- Any/all pole(s) of transfer function is/are non-negative
- Step response is unbounded

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Solution:

- Check the pre-requisites of controller (if pre-requisites full-filled then goto next step)
- Design a suitable controller and
- Integrate/connect the controller with the system.

Types of Controller

There are 3 types of techniques to design controllers which are:

- Full-state feedback controller or state feedback controller
- Observer-based state feedback controller
- Proportional, Integral and Derivative (PID) controller

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In today lecture, we will design and simulate full-state feedback controller.

Example that we did last time :

Consider a system having the following state space model:

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Check the following:

- Do we need a controller?
- If we need a controller, identify which controller to design
- Design that controller and place the eigenvalues at $(-3, -5)$.

MATLAB and Simulink Code division

There are four main segments in programming/coding using MATLAB and Simulink

1. Declare the variables and matrices using MATLAB
2. Check the stability using MATLAB (i.e. the answer to question: do we need a controller)
3. Design the controller using MATLAB
4. Simulate the system with controller connected to the system using Simulink

Segment 1 - Declaring the necessary variables and matrices

Let us declare the matrices in MATLAB

```
% Sample Code
clear;
clc;
close all;

A=[2 3; 0 5];
B=[1; 2];
C=[1 0; 0 1];
D=[0;0];

% Please ensure that matrix D has same columns as matrix B
% and same rows as matrix C
```

Figure: Variables and Matrices initialization in MATLAB

Segment 2 - Checking the stability of the system

```
% We check stability here
% There are 5/6 ways (3 we studied till now)
% Step response, eigen values, poles
% root-locus, nyquist, routh-hurwitz


% % Task 1 - Check stability of the system

% Method 1 - poles of tf
[n,d]=ss2tf(A,B,C,D);
poles_of_transfer_ftn=roots(d);
disp('The poles of the transfer function are ');
poles_of_transfer_ftn
```

Figure: Poles computation for stability information

Segment 2 - Checking the stability of the system

```
% Method 2
eigen_values=eig(A);
disp('The eigenvalues of matrix A are');
eigen_values


% Method 3
% Step response
step(A,B,C,D)
title('Step response of the system')
ylabel('Amplitude in response to unit step');
```

Figure: Eigenvalues and step response plot

Segment 2 - Checking the stability of the system

```
% Method 4 - root locus
figure;
rlocus(A,B,[1 0],0)
title('Root locus of the system for first output')

figure;
rlocus(A,B,[0 1],0)
title('Root locus of the system for second output')
```

Figure: Root locus plot - you have covered this is lab

Pre-requisite check before controller design

```
% Task 2 Check controlability
P=ctrb(A,B);
rank_of_ctrb_matrix=rank(P);
disp('The rank of controllability matrix
rank_of_ctrb_matrix


order_of_system=size(A,1);
disp('The order of the system is')
order_of_system
```

Figure: Controllability matrix computation and rank check

Full-state static feedback controller design

```
% Design controller
desired_eqnvalues=[-3 -5];
K=place(A,B,desired_eqnvalues);
```

Figure: Full-state static feedback controller design

Simulation using MATLAB

```
% TASK 3
% we will do this simulation in SIMULINK
% Now let us see the step response
A_clp=A-B*K;
B_clp=B;
C_clp=C;
D_clp=D;

figure;
step(A_clp,B_clp,C_clp,D_clp);
title('Step response of close loop system');

% Let us compute the final value of the step response
final_value_of_close_loop_system=dcgain(A_clp,B_clp,C_clp,D_c
disp('The DC-gain of the system OR the final value should be'
final_value_of_close_loop_system
```

Construct the system using Simulink

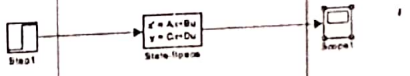


Figure: Construction of the system using Simulink

Construct the system using Simulink

Double-click the state-space block and make these changes

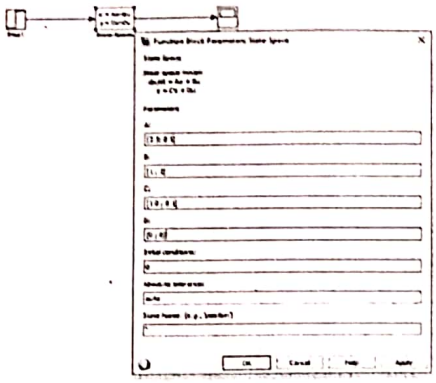
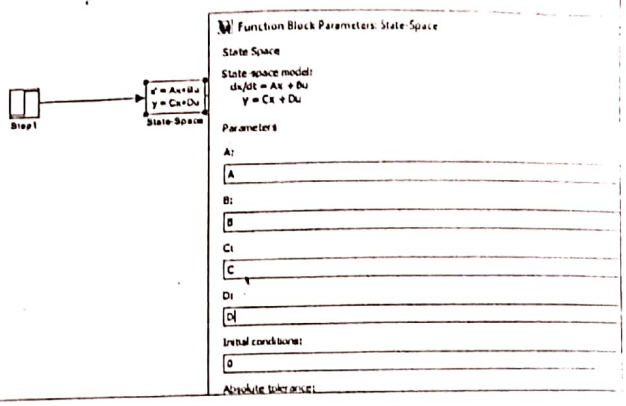


Figure: State-space block initialization to make Simulink understand our

Construct the system using Simulink

An easier way (provided these variables are declared in MATLAB workspace)



Trying to run the simulation using Simulink

Running the simulation from this button - You can increase or decrease the time of simulation

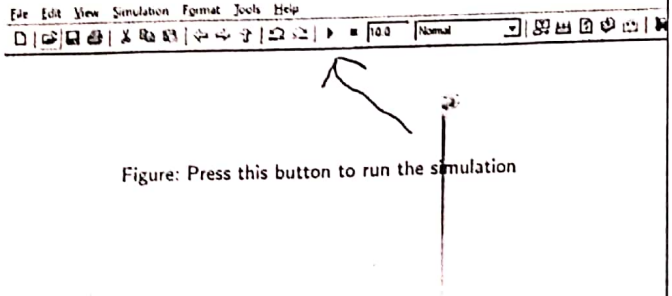


Figure: Press this button to run the simulation

Step response directly in Simulink

Step response of unstable system - its unbounded (going towards infinity)

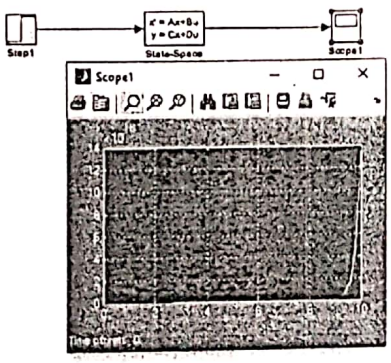
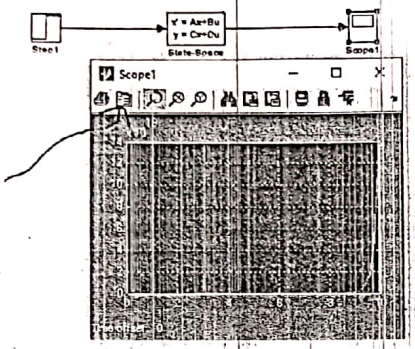


Figure: Step response of unstable system

Export of step response from Simulink to MATLAB

Now we need to export this data to MATLAB (for better plotting)



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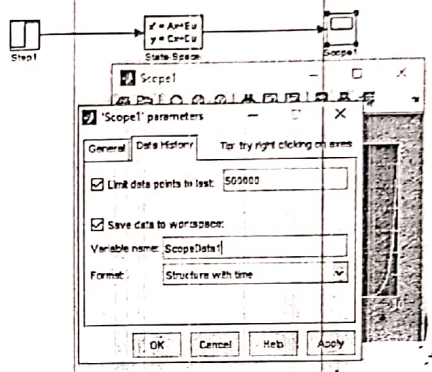

Figure: Step response of unstable system

Plot using MATLAB

Now we need to plot the exported data using MATLAB

```
>> plot(ScopeData1.time, ScopeData1.signals.values(:,1))
```

Figure: MATLAB code for plotting the first step responses

Plot using MATLAB

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```
>> plot(ScopeData1.time, ScopeData1.signals.values(:,1))
>> figure
>> plot(ScopeData1.time, ScopeData1.signals.values(:,2))
>>
```

Figure: MATLAB code for plotting both the step responses

Simulation of augmented system using Simulink

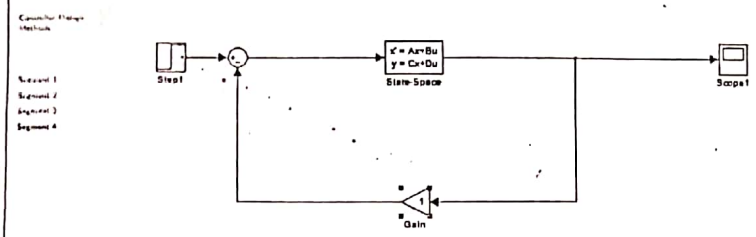

Figure: Sketch of augmented / close loop system

Simulation using Simulink

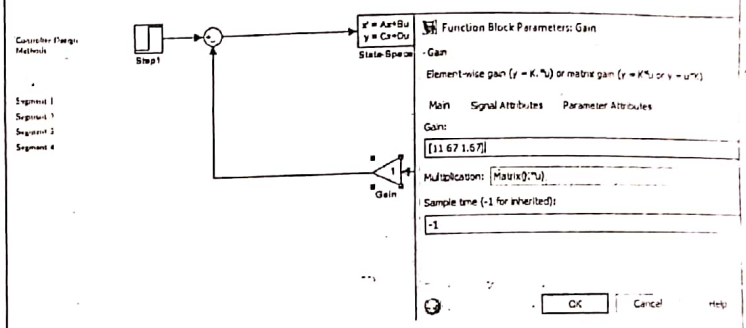

Figure: Sketch of augmented / close loop system

Simulation using Simulink

Now run the simulation and obtain plots of stable system. Check if the steady-state value is the same as the one obtained using MATLAB code