Linear Systems and Control - Week 6

Block Reduction of Complex Systems

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Contents that we have covered till now

We studied the following topics till now:

- Converting state-space to transfer function using formula
- Converting transfer functions to state-space models using canonical forms
- Obtaining transfer functions and state-space models for RLC circuits
- Obtaining transfer functions and state-space models for mechanical system
- Analyzing step responses of first order systems (time constant and dc-gain)
- Analyzing step responses of second order systems (underdamped, undamped, over damped, critically damped)

Contents that we will cover before mid term exam

We will study the following topics before mid term exam

- Block reduction of complex systems (today lecture)
- Stability of systems in time-domain and transfer function domain

Block reduction algebra

First we analyze a simple transfer function block.

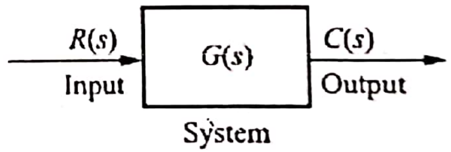


Figure: Transfer function block

The input signal is denoted by $R(s)$ and output signal by $C(s)$. We can write the following:

$$C(s) = G(s)R(s)$$

Sometimes, we skip the term (s) and write the following abusive notation:

$$C = GR$$

Block reduction algebra - Summer

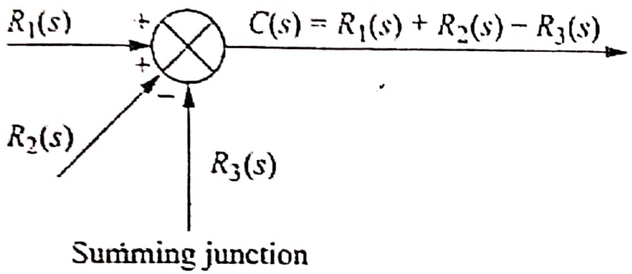


Figure: Summing Junction Symbol

Block reduction algebra - Pick off point

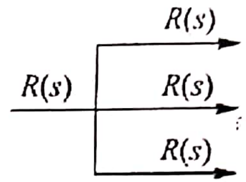


Figure: Pick Off point

Types of known interconnections

There are 3 types of known interconnections which are

- Series interconnection
- Parallel interconnection
- Feedback interconnection

Let us study each interconnection first, and then we study a few operations on transfer functions

Series Interconnection

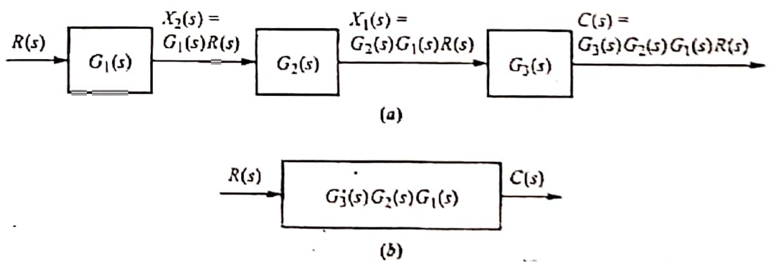


Figure: Series Interconnection of transfer functions

We can write the equivalent transfer function as $G_e = G_3G_2G_1$

Parallel Interconnection

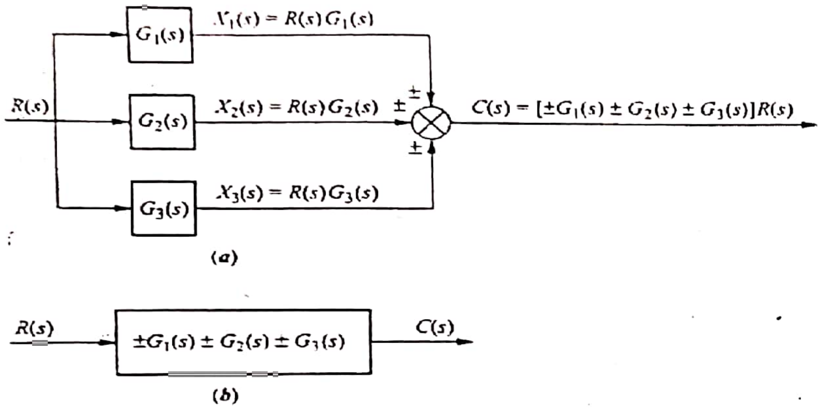


Figure: Parallel Interconnection of transfer functions

We can write $G_e = \pm G_3 \pm G_2 \pm G_1$.

Few Important Points

Series interconnection involves product of transfer functions.

In parallel interconnection, be careful to identify the transfer functions correctly.

Two blocks are in parallel if they have same input signal and the output goes towards same summing junction.

Parallel interconnection involves sum or different of transfer functions.

Feedback Interconnection

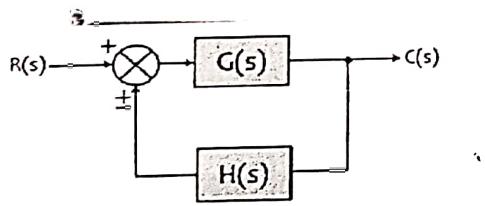
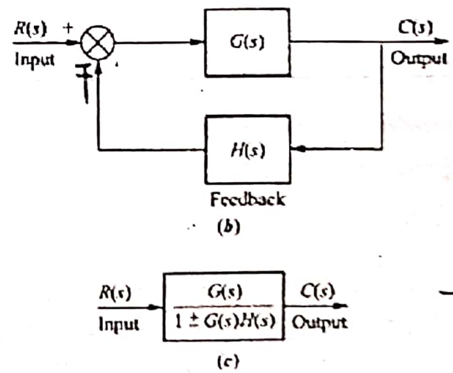


Figure: Feedback Interconnection of transfer functions

We write the following:

$$G_e = \frac{G}{1 \mp GH}$$

Feedback Interconnection



$$\frac{G}{1 \pm GH}$$

Figure: Feedback Interconnection of transfer functions

We can write $G_e = \frac{G}{1 \pm GH}$

Operations with transfer functions

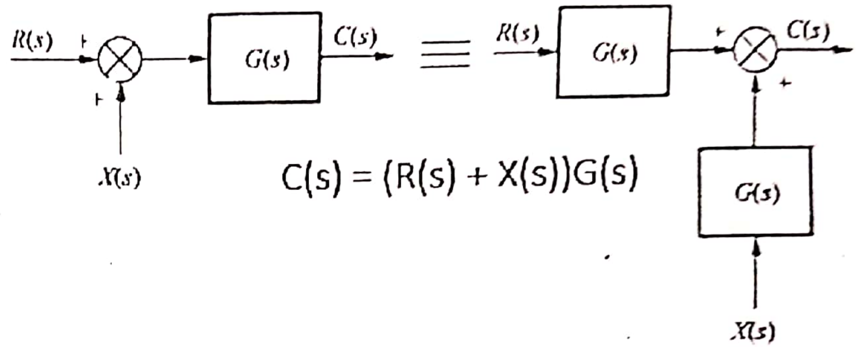
Sometimes in a complex system we can NOT easily identify series or parallel or feedback interconnections.

In those cases, we need to perform operations on transfer function block by either moving it before a point or summing junction or after a point/summing junction

Performing operations on transfer functions involve mathematics which we will study now.

There are 4 types of operations

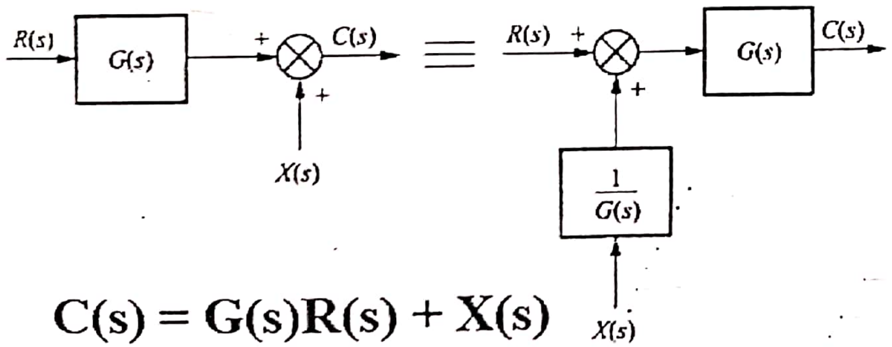
Moving summing junction after transfer function



$$C(s) = (R(s) + X(s))G(s)$$

Figure: Moving a summing junction after transfer function

Moving summing junction before transfer function



$$C(s) = G(s)R(s) + X(s)$$

Figure: Moving a summing junction before transfer function

Moving before pickoff point

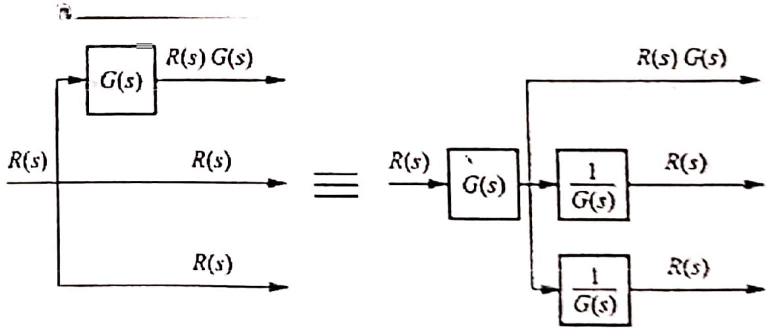


Figure: Moving before a pick-off point

Moving after pickoff point

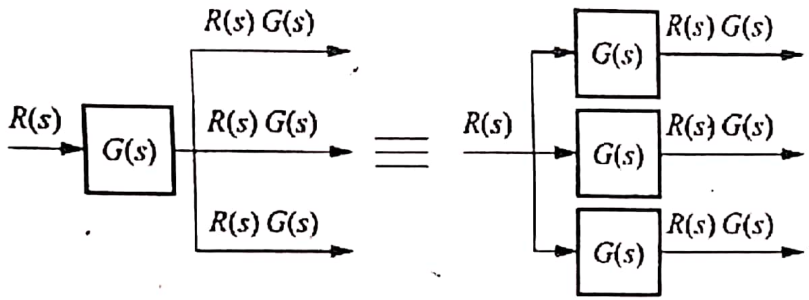


Figure: Moving after a pick-off point

Example 1 - Problem to solve

Can you obtain the transfer function. $\frac{C(s)}{R(s)}$?

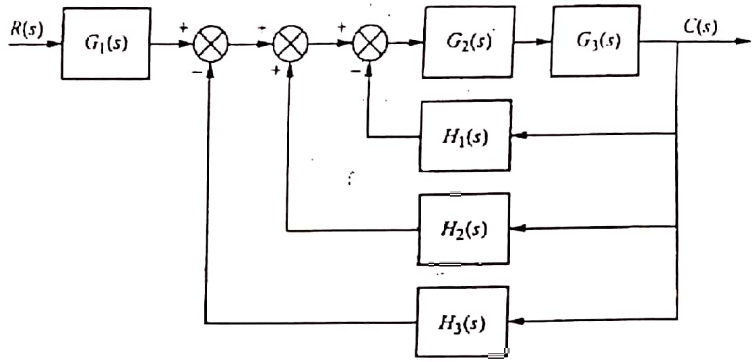
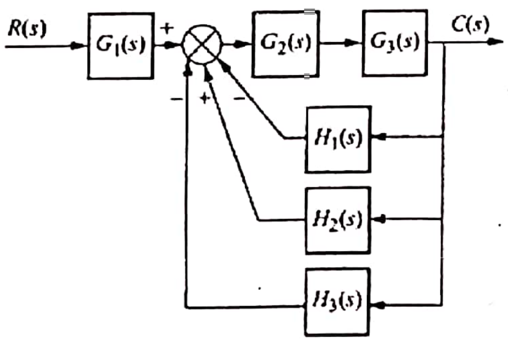


Figure: Example 1

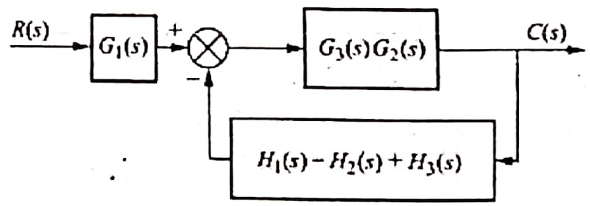
Example 1 - Solution part a



(a)

Figure: Example 1 - Solution part a

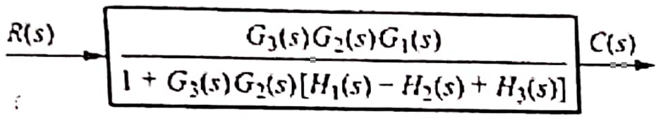
Example 1 - Solution part b



(b)

Figure: Example 1 - Solution part b

Example 1 - Solution part c



(c)

Figure: Example 1 - Solution part c

Example 2 - Problem to solve

Can you obtain the transfer function $\frac{C(s)}{R(s)}$?

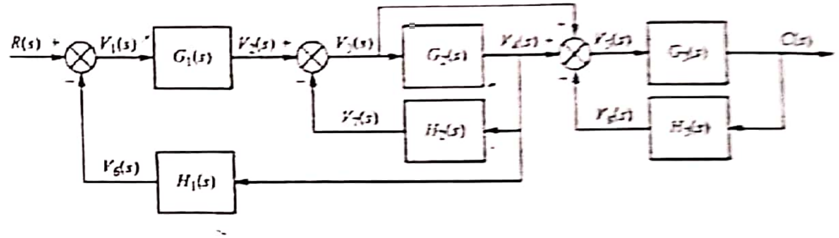


Figure: Example 2

Example 2 - Solution part a

Example 2 - Solution part b

Example 2 - Solution part c

Example 2 - Solution part d

Example 2 - Solution part e