Mansoor Hussain.

Control Systems - 7^{th} Semester - Week 4

Step Response of Systems

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Topics of last week

In last weeks, we studied about obtaining state-space models from differential equations

Then we studied about converting state-space model to transfer functions using formula

We also studied on converting transfer function to state-space model using canonical forms

Model - Recalling concepts

A model is representation or abstraction of reality/system.

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What is mathematical model? A set of equations (linear or differential) that describes the relationship between input and output of a system.

Types of Model

There are three types of mathematical models

- Black Box
- Grey Box
- White Box

Introduction to Transient Analysis

Sometimes we cannot write white box models for the systems

Either we do NOT know what is inside the system or either the system is too complex to verify the components

Perhaps sometimes the components are not easily identifiable and their configuration or layout is not readable

So another way to obtain model of a system is to apply a *test* input signal and obtain the output signal

Standard Input Signals

Though there are many possible combinations of input signals, the following are famous or popular input signals

- Impulse Signal
- Step Signal
- Ramp Signal
- Parabolic Signal

Step Response of First Order System

A general first order system without zeros can be written as follows:

$$G(s) = \frac{b}{s+a}$$

Let $C(s)$ be the output of a system having transfer function $G(s)$ (expressed above). If the input to $G(s)$ is a unit step, then the output can be expressed as follows:

$$\text{Output Signal} = \text{Input Signal} \times \text{Transfer function}$$

We can further write the following:

$$C(s) = \text{Unit step signal} \times G(s)$$

$$C(s) = \frac{1}{s} \times \frac{b}{s+a} = \frac{b}{s(s+a)}$$

of First O r System

The te t term. The inverse a is called time constant i.e.

$$\tau = \frac{1}{a}$$

where τ is called time constant of first order systems. For example compute τ of the following system:

$$G(s) = \frac{3}{s+2}$$

Step Response of First Order System

The term a is an important term. The inverse of a is called time constant i.e.

$$\tau = \frac{1}{a}$$

where τ is called time-constant of first order systems. For example compute τ of the following system:

$$G(s) = \frac{3}{s+2}$$

Here $\tau = \frac{1}{2} = 0.5$ and gain K is computed as $\frac{3}{2} = 1.5$

The value of gain K indicates the final steady-state value of the step response

Step Response of First Order System

In order to compute transfer function from a plot, we need to define a few more terminologies

Rise Time: T_r, time taken to reach 90% or 0.9 of final value from 10% or 0.1. Mathematically:

$$T_r = \frac{2.2}{a}$$

Settling Time: T_s, time taken to stay within 2% of its final value (or reach 98% of final value). Mathematically:

$$T_s = \frac{4}{a}$$

Step Response of First Order System

Can you compute the transfer function?

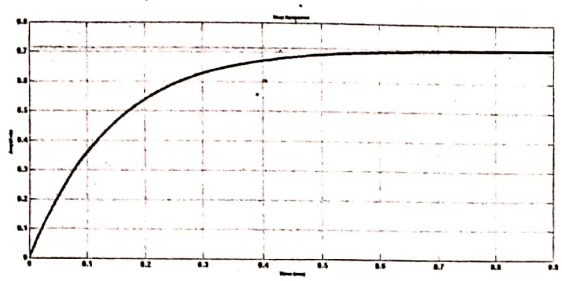

Figure: Step Response of a transfer function

Step Response of First Order System

Time constant: Time to reach 63% of final value. Compute the transfer function from the previous plot.

Final value = steady-state value = gain $K = 0.72$
63% of final value is $0.63 \times 0.72 = 0.4464$
Time taken to reach 0.45 value is 0.15 seconds
The final transfer function is

$$G(s) = \frac{\frac{0.72}{0.15}}{s + \frac{1}{0.15}}$$

OR

Pole is inverse of time constant which comes out to be $\frac{1}{0.15} = 6.67$

Another way of writing the transfer function is

$$G(s) = \frac{4.802}{s + 6.67}$$

5

Step Response of First Order System

Time constant: Time to reach 63% of final value. Compute the transfer function from the previous plot.

Step Response of First Order System

The previous step-response was obtained for the following actual transfer function:

$$G(s) = \frac{5}{s+7}$$

MATLAB code for obtaining step response

```
num = [5] ;

den = [1 7] ;

step(num,den)
```

Step Response of First Order System

Effects of decreasing time constant

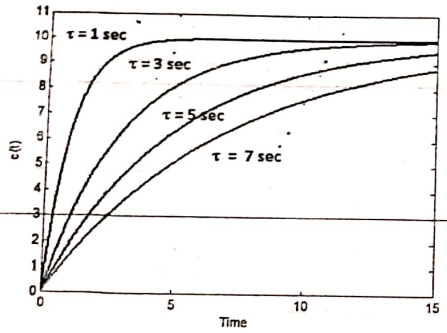

Figure: Effects of decreasing time constants of first order transfer function

Step Response of First Order System

Effects of increasing gains (remember its K not the term b)

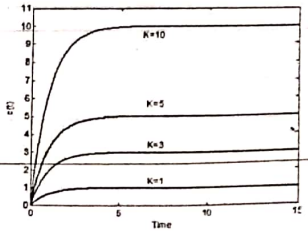

Figure: Effects of increasing gains of first order transfer function

Step Response of First Order System

Question: NADRA manages the registration database of Pakistani citizens. Previously, till 2001 people had 11 digit NIC numbers. Each citizen of Pakistan is issued a 13 digit CNIC number. The first 5 digits in a CNIC are based on a citizen locality, the next 7 numbers are random, and the final last digit is gender based (even for females, and odd for males). The current database entries in NADRA are estimated as 90 million. If a person details are required and his/her CNIC is entered in the NADRA database, its take 1 min to search 98% of the records in a NADRA database. Assuming the query search process to be a first-order time constant.

Step Response of First Order System

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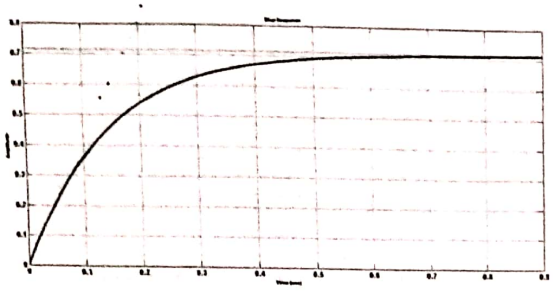

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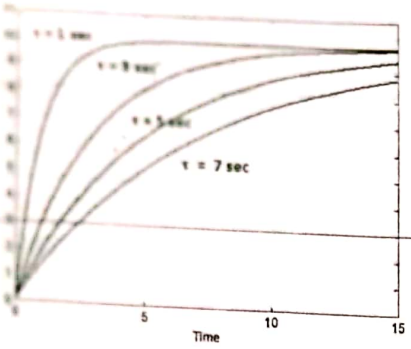

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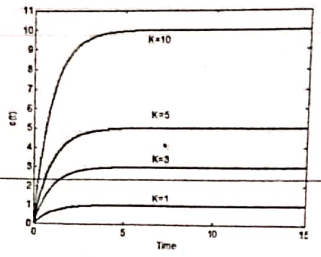

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